Fast Optical Layer Mesh Protection Using Pre-Cross-Connected Trails

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***Abstract*—Conventional optical networks are based on SONET rings, but since rings are known to use bandwidth inefficiently, there has been much research into *shared mesh protection*, which promises significant bandwidth savings. Unfortunately, most shared mesh protection schemes cannot guarantee that failed traffic will be restored within the 50-ms timeframe that SONET standards specify. A notable exception is the p-cycle scheme of Grover and Stamatelakis. We argue, however, that p-cycles have certain limitations, e.g., there is no easy way to adapt p-cycles to a path-based protection scheme, and p-cycles seem more suited to static traffic than to dynamic traffic. In this paper we show that the key to fast restoration times is not a ring-like topology *per se*, but rather the ability to *pre-cross-connect* protection paths. This leads to the concept of a *pre-cross-connected trail* or PXT, which is a structure that is more flexible than rings and that adapts readily to both path-based and link-based schemes and to both static and dynamic traffic. The PXT protection scheme achieves fast restoration speeds, and our simulations, which have been carefully chosen using ideas from experimental design theory, show that the bandwidth efficiency of the PXT protection scheme is comparable to that of conventional shared mesh protection schemes.**

***Index Terms*—Bandwidth sharing, cage graph, Dijkstra algorithm, experimental design, mesh protection, mesh restoration, online algorithm, p-cycle, self-healing networks, SONET, survivable optical networks.**

I. Introduction

MODERN optical networks carry large amounts of traffic, so it is imperative that they be able to survive an accidental failure such as a fiber cut or a node failure. Traditionally, such survivability has been provided by means of *ring protection*, e.g., SONET unidirectional (UPSR) or bidirectional (BLSR) rings. Rings can recover automatically from any single link or node failure by rerouting traffic around the other side of the ring, and their switch completion times are very low (e.g., 50 ms for a SONET BLSR with a 1200-km circumference, provided that there is no extra traffic), thereby minimizing the amount of data lost. However, rings are known to have low bandwidth efficiency. There has, therefore, been much research into *shared mesh protection*, which promises significant bandwidth savings (see [1]–[12] for a sample—by no means comprehensive—of the literature). These savings come partly from the flexibility of an arbitrary mesh topology, which allows many traffic demands to take more direct routes to their destinations than if they were constrained to a ring topology. More importantly, however, bandwidth savings are achieved through *sharing*. The basic assumption behind sharing is that failures in optical networks are rare enough that the probability of two independent failures occurring simultaneously is negligible. Therefore, if two light paths that carry working traffic do not have any common point of failure, then their protection paths can share the same unit of bandwidth, since they will never request that unit of backup bandwidth simultaneously. Because of sharing, the amount of spare capacity in a mesh-protected network can be as little as 60% (or even less) of the working capacity, whereas in a BLSR or UPSR this fraction is always at least 100%.

Most shared mesh protection schemes can be classified as either *link-based* or *path-based*. We define these terms and compare them in the next section; for now it suffices to observe that link-based and path-based schemes each have their respective pros and cons. Depending on a particular network's characteristics and needs, one or the other approach may be preferable.

Shared mesh protection looks good on paper, but as practical implementions have begun to be built and tested, certain difficulties have surfaced. One of these has been the experimental fact that shared mesh protection, whether link-based or path-based, typically does not restore traffic as quickly as SONET rings do, and therefore, runs the risk of being unable to guarantee the quality of service that is typically required for high-priority traffic. The only obvious way to achieve very fast switch completion times in a mesh network is to give up the idea of sharing and to use *dedicated* (also known as $1+1$) protection instead, but this means giving up the bandwidth savings that motivated mesh protection in the first place.

One of the few mesh protection schemes to address this quandary satisfactorily is the *p-cycle* concept of Grover and Stamatelakis [4]. We say more about p-cycles in a later section; for now let us just remark that the idea is to route the working traffic using an arbitrary mesh routing algorithm, but to constrain the protection routes to lie on certain predetermined "p-cycles" or rings. Grover and Stamatelakis report that p-cycles achieve the "speed of rings with the efficiency of mesh."

Impressive as the results of [4] are, the p-cycle concept does have certain limitations. It is inherently a link-based scheme and is, therefore, saddled with all the usual pros and cons of link-based schemes. More subtly, Grover and Stamatelakis have

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found that in many situations, achieving high bandwidth efficiency requires the deployment of *large* p-cycles. In a real network that carries live traffic, demands are not static but are *dynamic:* they come and go incrementally over time. (Caution: The term *dynamic* here has its usual meaning in the context of optical networking—referring to light paths that are set up and torn down—and does not refer to dynamics at higher layers such as the IP layer.) A network provider faced with a new traffic demand that cannot be handled with existing p-cycles must, therefore, choose between allocating a small p-cycle that meets the current demand cheaply but that may be inefficient over the long run, or allocating a large p-cycle that requires a large investment up front and that may be wasted if expected future demands never materialize. This dilemma can be solved by starting with small p-cycles and periodically reoptimizing their size as traffic increases, but continual reoptimization entails significant management overhead.

In this paper, we generalize the p-cycle concept. We argue that rings and p-cycles restore traffic quickly not because of their circular topology but because their protection routes are *pre-cross-connectable*. We are thus naturally led to consider arranging protection capacity not only into cycles but more generally into *pre-cross-connected trails* (or PXTs; the precise definition will be given later). Like p-cycles, PXTs achieve the "speed of rings with the efficiency of mesh," but they are more flexible than p-cycles. In particular, PXTs can be used in either a link-based or a path-based scheme, and they are well suited to dynamic traffic because they grow and shrink incrementally in a natural way.

II. Link-Based Versus Path-Based Protection

As we mentioned in Section I, some protection schemes are *link-based* while others are *path-based*. In this section, we define these two terms and give a brief comparison of their respective merits.

In link-based protection, the nodes—let us call them A and B—at either end of a failed link are responsible for detecting the failure and switching the traffic from the failed link onto a protection path P that bypasses the failure. The failed link may be utilized by a large number of different light paths, each with a different source and destination. After the failure, these light paths travel from their source node to node A as before, then take the protection path P to get to node B, then finally travel from node B to their final destinations. SONET BLSRs use link-based protection.

In path-based protection, it is the source and destination nodes of each individual working light path that are responsible for switching the traffic onto a protection path. As in link-based protection, a single failed link may cause many different light paths to fail. However, in path-based protection, each one of these light paths is free to travel on a completely different protection path from source to destination. In particular, there is no need for any of them to visit the nodes A and B at the ends of the failed link. SONET UPSRs use path-based protection. Path-based protection is also known as *end-to-end protection*.

There is one more point that should be made about path-based protection. In principle, the source and destination of a given working light path could choose different protection paths depending on which link or node along the working path fails. Although such failure-dependent routing can improve the bandwidth efficiency slightly, it requires additional signaling to isolate the fault. This slows down the recovery process and so is rarely used in practice. In this paper, we assume that the same protection path is always used for a given working path. Naturally, this means that the protection path must be disjoint from the working path.

How does one choose between link-based and path-based protection? There are several factors to consider.

1) **Shared path-based protection tends to use less total bandwidth than shared link-based protection.** One reason is that in link-based protection, there is a backhaul problem. A protection light path may travel to node A and then double back on itself in order to get to node B.
2) **Shared link-based protection tends to be faster than shared path-based protection.** The reason is that in link-based protection, the failure detection and repair happen locally, whereas in path-based protection the signals must travel all the way to the source and the destination. Furthermore, as already mentioned, a single fiber cut usually triggers a large number of alarms in a path-based scheme, and processing all these alarms simultaneously can bog down the network.
3) **It is difficult if not impossible for a link-based scheme to protect against node failures.** Link-based schemes rely on the nodes on either end of a link to perform a protection switch; if one of these nodes fails, then it cannot perform the switch. A path-based scheme can simply choose node-disjoint paths from end to end for all its light paths and then node failures are automatically survivable—unless it is the source or destination node that fails, but in that case it is impossible to recover from the failure anyway. (Note, however, that we said "difficult" rather than "impossible"; for example, a SONET BLSR, which we described as a link-based scheme, *is* able to survive node failures. But this is only because a BLSR has, in addition to its basic mechanism for surviving a single link failure, a complicated system involving ring maps and squelch tables for detecting and coping with multiple failures.)

As we said before, in some situations link-based protection is the right choice and in other situations path-based protection is preferable. Ideally, a comprehensive protection methodology should be flexible enough to provide either link-based or path-based protection, depending on the needs of an individual network.

III. Pre-Cross-Connection: The Key to Fast Protection

A. Branch Points

Why does a conventional shared mesh protection scheme experience a longer restoration time than a SONET BLSR? The basic reason is that in shared mesh protection, the sequence of recovery actions triggered by a failure is longer and more complex than in a ring. For example, in order to achieve high bandwidth efficiency, some shared mesh protection schemes choose different backup paths for a given working path depending on the location of the failure. The backup path may even exploit *stub release*, i.e., it may use bandwidth that was previously used by working paths that have now failed. To accomplish these feats without misconnection or blocking, the network elements must perform fault isolation and must disseminate this information to all the other nodes involved in the recovery. Acknowledgments and hold-off times may also be required, slowing the restoration process.

Now, some shared mesh protection schemes circumvent the above problems by pre-assigning a single backup path for each working path, independent of the location of the failure, and not exploiting stub release; see, for example, [9]. However, even in these schemes, the network elements still need to perform recovery actions that are unnecessary in a ring. Specifically, in a mesh network, the sharing of protection bandwidth by its very nature implies that the correct configuration of the nodes on the backup path can be *different for different failures*. This typically means that when a failure occurs, a notification must be sent along the backup path so that the nodes can configure themselves appropriately. With in-band signaling, each node may need to cross-connect itself in real time before it can pass the message on to the next node. With current optical cross-connect technology, each cross-connection can take 5–10 ms or more, so this effect by itself can easily use up the 50-ms budget. Even if out-of-band signaling is used, so that cross-connecting can be done in parallel in different nodes, a single node that receives several cross-connection requests for different backup paths may only be able to perform these cross-connections sequentially. In contrast, in a BLSR, only the nodes on either side of a failure need to make a real-time switch. The rest of the protection path is *pre-cross-connected*, so that the intermediate nodes on the protection path simply pass through the traffic without having to make a switching decision.

Fig. 1 may clarify this point. Solid lines represent physical links while broken lines represent light paths. The working path *W1* (from *A* to *B*) and the working path *W2* (from *C* to *D*) have no common point of failure, so let us assume that their protection paths *P1* (from *A* to *E* to *B*) and *P2* (from *C* to *A* to *E* to *D*) share protection bandwidth on the link *AE*. Now, if *W1* fails, then node *E* must connect *AE* to *EB*, whereas if *W2* fails, then node *E* must connect *AE* to *ED*. Therefore, in order for node *E* to decide which connection to make, it must be informed of the location of the failure, and this information is available only *after the failure has occurred* and cannot be pre-calculated. Although one could arbitrarily decide to pre-cross-connect *AE* to either *EB* or *ED*, neither choice would entirely eliminate the need for a real-time switch at *E*; if *AE* is pre-cross-connected to *EB*, then *E* must make a real-time switch when *W2* fails, and if *AE* is pre-cross-connected to *ED*, then *E* must make a real-time switch when *W1* fails.

It is convenient to introduce some terminology to describe the above situation. We define a *branch point* to be a node *X* with the property that, no matter how the protection capacity is pre-cross-connected at *X*, there exists a failure scenario for which some needed protection path that has *X* as an intermediate node is not properly pre-cross-connected at *X*. Notice that branch points can arise regardless of whether one uses link-based or path-based shared mesh protection.

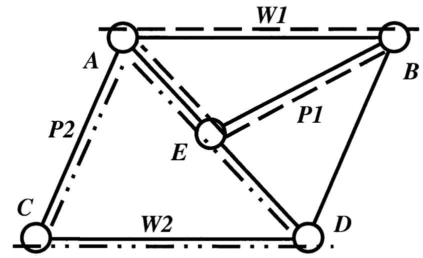

Fig. 1. Protection scheme with a branch point at *E*.

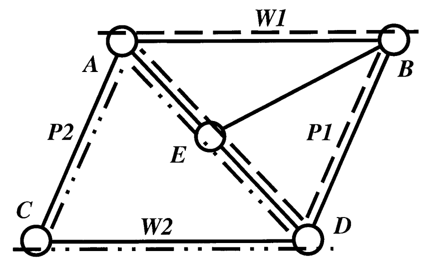

Fig. 2. Pre-cross-connectable protection scheme.

These observations explain why Grover and Stamatelakis's mesh protection scheme [4] succeeds in achieving ring-like speeds. As we mentioned before, in their scheme, all the protection capacity is organized into certain predetermined rings or p-cycles, and therefore, no troublesome branch points can arise. The p-cycles are pre-cross-connected just as in a BLSR. When a failure occurs, the nodes at either end of the failure must react and perform a real-time switch, but all the intermediate nodes on the protection path simply pass through the traffic. The only recovery operations that the nodes need to perform are those that also need to be performed in a BLSR, so comparable speed is achieved.

We now come to the main insight of the present paper. *Although a ring or a p-cycle has no branch points, the converse is not true.* That is, if there are no branch points, this does not automatically imply that the protection bandwidth is arranged into a set of rings. As an example, suppose we modify Fig. 1 slightly by choosing a different route for *P1*, as shown in Fig. 2.

On both *AE* and *ED, P1* and *P2* share bandwidth. We pre-cross-connect *CA* to *AE, AE* to *ED*, and *ED* to *DB*.

The crucial observation is that the branch point at node *E* has now been removed, so that pre-cross-connecting *AE* to *ED* works correctly for all failure scenarios. Furthermore, despite appearances, *D* is *not* a branch point either. To see this, note first that if *W1* fails, then the pre-cross-connection between *ED* and *DB* is exactly what we want, because it connects up *P1* and allows the intermediate node *D* to pass through protection traffic without making a real-time switch. On the other hand, if *W2* fails, then the *ED-DB* pre-cross-connection is admittedly improper and must be broken in order to terminate *P2* at *D*, but the key fact is that for *this* failure, node *D* is not an *intermediate* node on the protection path but an *end node*. (Recall the proviso about intermediate nodes in the definition of a branch point.) The idea is that *D* must perform a real-time switch in any case, and no *additional* delay is incurred by requiring it to break the cross-connection between *ED* and *DB*. Similarly, *A* is not a branch point. In Fig. 2, just as in a ring, regardless of the failure, only the end nodes need to switch.

We argue, therefore, that as long as we avoid branch points, we can approach the "speed of rings." Moreover, since avoidance of branch points can be achieved without necessarily committing to rings or p-cycles, we open up the possibility of overcoming some of the limitations of the latter. For example, we are no longer committed to a link-based protection scheme, and even if we do choose a link-based scheme, we can use our newfound flexibility to improve the bandwidth efficiency or the robustness to dynamic traffic. These points will be explored further in the following sections.

Before plunging into technical details, we make one final general remark. We do not claim that pre-cross-connection is the *only* way to close the performance gap between ring and mesh. For example, if optical switch technology improves to the point where the time taken to cross-connect drops by an order of magnitude, then avoiding branch points may no longer be necessary. However, even if such an alternative solution is used, our scheme may still be employed to further boost performance.

B. Graph-Theoretic Terminology

Before we proceed further, it is helpful to give a formal statement of the problem we are trying to solve. To do this, we must first briefly review some graph-theoretic terminology.

By a *graph* we mean an *undirected multigraph*, i.e., an undirected graph that may have multiple edges. An edge is also called a *link connection*. A *link* in a graph G is the set of all edges between a given pair of nodes. Two edges that share exactly one end node v in common are said to be *incident to each other at* v. The *degree* of a node v is the number of edges that have v as an end node. A graph is *regular* if every node has the same degree.

In this paper, the distinction between an *edge* and a *link* is more important than is usually the case, so we explain it in slightly more detail. The reader familiar with ITU terminology may find it helpful to think of the terms "link" and "link connection" as short for "regenerator section link" and "regenerator section link connection" (although this correspondence should not be taken *too* literally since we describe our algorithms in terms of abstract graphs and not in terms of actual optical networks). Alternatively, the reader can think of an edge as the smallest unit of bandwidth that can be switched, while a link comprises the totality of bandwidth between two adjacent nodes. For example, in a WDM network (with no sub-wavelength muxing), an edge would be a wavelength, while a link would comprise all the fibers in the conduit between two nodes.

In the graph in Fig. 3, there are two edges or link connections on the link between nodes *D* and *E*, namely *e* and *f*. The degree of node *D* is 4.

A *walk* in G is an alternating sequence of nodes and edges $(v_0, e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n)$ in G such that for all i, the end nodes of e_i are v_{i-1} and v_i. A *trail* is a walk whose edges are all distinct and a *path* is a walk whose nodes are all distinct. A walk is *closed* if $v_0 = v_n$. Note that a path cannot be closed unless $n = 0$. A walk is said to *connect* v_0 and v_n.

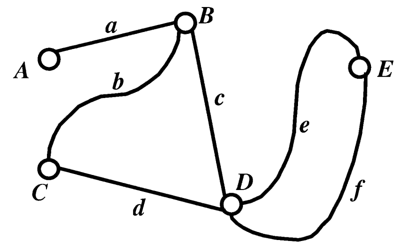

Fig. 3. Example of a graph.

In Fig. 3, $(C, d, D, f, E, e, D, d, C)$ is a walk but not a trail because the edge d is repeated. On the other hand, $(C, d, D, f, E, e, D, c, B, b, C)$ is a trail—in fact a closed trail—but not a path, because the node D is repeated.

Two walks in G are *edge-disjoint* (respectively, *link-disjoint*) if there is no edge (respectively, link) that both of them traverse. The *interior* of a walk is the set $\{v_1, v_2, \ldots, v_{n-1}\}$, i.e., the set of all of its nodes other than its end nodes. Two walks are *node-disjoint* if they are link-disjoint and no node in the interior of one walk is a node in the other walk.

For example, in Fig. 3, the paths (D, e, E) and (D, f, E) are edge-disjoint but not link-disjoint, and hence they are not node-disjoint. The paths (A, a, B, c, D) and (C, b, B) are not node-disjoint, because *B* is an interior node of the first path that is also a node of the second path. On the other hand, the paths (A, a, B, b, C) and (E, e, D, d, C) *are* node-disjoint, because even though node *C* is on both paths, C is not an *interior* node of either path.

C. Problem Statement

For brevity, we describe only a path-based protection scheme and omit the details of the analogous link-based version. We also assume that all traffic and all links are bidirectional.

INPUT: A graph G and a list *D* of *demands*. A demand *d* is an unordered pair of distinct nodes—the *terminals* of *d*—of G ; the same demand may appear multiple times in the list *D*. For simplicity we assume that each edge of G has unit cost and unit capacity and that each demand in *D* requires unit capacity to route, although allowing arbitrary integer costs and capacities would not change the essence of the problem.

OUTPUT: An *allocation plan*, i.e., a list that contains, for each demand *d* in *D*, two paths (note: paths, not arbitrary trails or walks) connecting the terminals of *d*—a *working path* and a *protection path*—and that satisfies the following conditions:

1) For each *d*, the working and protection paths are node-disjoint. (Note: here and in what follows, "node-disjoint" may be changed to "link-disjoint" if survivability against node failures is not required.)
2) No edge (note: edge, not link) that appears in the working path of some demand appears in either the working or protection path of any other demand.
3) If the working paths of two demands d_1 and d_2 are not node-disjoint, then their protection paths are edge-disjoint.
4) There are no branch points. That is, if v is a node and e_1, e_2, and e_3 are three distinct edges that each have v as an end node, and the protection path of some demand

contains both e_1 and e_2, then no protection path of any demand contains both e_1 and e_3.

The number of edges that appear in some working (respectively, protection) path is called the *working* (respectively, *protection*) *bandwidth* of the allocation plan. Note that some protection edges are *shared*, i.e., they appear in the protection paths of more than one demand; such edges are counted only once when computing the protection bandwidth. The *total bandwidth* is the sum of the working and protection bandwidths. The smaller the total bandwidth, the better the allocation plan. The objective is to find as good an allocation plan as possible.

This completes the problem statement. Condition 4, on branch points, is what distinguishes our shared mesh protection problem statement from others in the literature.

We are now able to explain the "PXT" terminology alluded to earlier. Given an allocation plan, we say that two edges e_1 and e_2 are *pre-cross-connected* if they appear consecutively in some protection path. Pre-cross-connection should be thought of as "linking up" the protection edges. The crucial fact, easily proven, is that although condition 4 does not force the protection edges to be arranged into a disjoint union of cycles, *the absence of branch points does imply that the protection edges must be pre-cross-connected into a disjoint union of trails* (some of which may be closed trails). We call each such trail of protection edges a *pre-cross-connected trail* or PXT of the allocation plan. Note that each protection path is part of some PXT, but that a PXT may be much longer than any single protection path, since protection paths may overlap (e.g., in Fig. 2 the two protection paths combine to form a PXT of length four). In fact, a PXT may even form a closed trail in which the last edge comes back and is pre-cross-connected to the first edge. Such a PXT is called a *closed PXT*. A closed PXT is similar to a ring, except that a closed PXT may "self-intersect" in the sense of visiting the same node or link more than once.

As we argued above, the absence of branch points allows us to achieve ring-like protection speeds, but two important questions remain. 1) Given a graph and a list of demands, how does one compute a good allocation plan in a reasonable amount of time? 2) Is it still possible to achieve low total bandwidth when the no-branch-point constraint is imposed? These questions are addressed in the next sections.

IV. Computing an Allocation Plan

Now that we have a formal statement of our PXT protection scheme as a combinatorial optimization problem, our first inclination might be to apply a standard mathematical programming or local search algorithm to try to find solutions to given instances. But although we hope to pursue such approaches in the future, we do not discuss them further in the present paper. The reason is that most such algorithms are best suited to so-called *offline* computation, by which we mean that the list of demands is completely known in advance and does not change with time. While this is a reasonable assumption in some networks, most real networks experience *dynamic traffic*, i.e., the list of demands arrives over time. Each demand must be satisfied when it arrives, without knowing what the future may bring. Moreover, network operators often desire a *cap-and-grow* routing method, i.e., once a demand is routed, its routing should if possible remain undisturbed when later demands arrive and are routed. In other words, re-routing and re-optimizing existing demands when a new demand arrives is strongly discouraged if not forbidden. Under these circumstances, what is needed is an *online algorithm* that routes one demand at a time. In spite of their practical importance, online algorithms and their *competitive ratios*—i.e., their performance relative to offline algorithms—have not received much attention in the literature on shared mesh protection. We hope that our work here will be a first step toward remedying this deficiency.

Suppose that we have a graph G and a list D of demands for which an allocation plan has already been calculated. Now suppose a new demand $d = \{u, v\}$ arrives and we need to find a working path $w(d)$ and a protection path $p(d)$ for it, without disturbing the existing allocation plan. The idea is to find the working path first, and then to find the protection path that re-uses existing protection bandwidth as much as possible, subject to the constraint that the protection path be a path, i.e., that it never revisit the same node more than once. (This requirement that the protection path be a path complicates the algorithm more than one might initially expect, as the reader will see shortly. We say a few words at the end of this section about relaxing this requirement.) More precisely, we proceed as follows.

1) Find $w(d)$ by applying a shortest-path algorithm such as Dijkstra's algorithm to the unused part of G (i.e., the edges that are not already used in the allocation plan).
2) The existing protection edges are, as explained above, arranged into PXTs. Form a list L_1 of all PXTs.
3) Discard all closed PXTs from L_1 except those that contain at least one occurrence of u and at least one occurrence of v. (The reader should convince himself that unless a PXT T contains both u and $v, p(d)$ cannot possibly use any edge from T without introducing a branch point.)
4) Pick a PXT T and find all occurrences of u and v on it. These occurrences subdivide T into *subtrails*, i.e., contiguous segments of T that run from one occurrence of u or v to the next occurrence of u or v.
5) Discard any such subtrails that are not paths, and append the remaining subtrails, if any, to a list L_2. (If any edge from a subtrail is used in $p(d)$, then the entire subtrail must be used, and since $p(d)$ is required to be a path, it cannot contain a subtrail that is not a path.)
6) Delete T from L_1. If L_1 is empty, go to step 7; otherwise go to step 4.
7) Discard every path from L_2 with a *prohibited edge:* A prohibited edge is an edge that either has an end node in the interior of $w(d)$ or that is contained in the protection path of an existing demand $d' \neq d$ whose working path $w(d')$ is not node-disjoint from $w(d)$.

The paths that remain in L_2 are called *shortcut paths*. These shortcut paths are used to help create an auxiliary graph H as follows. The nodes of H are the same as the nodes of G. If v_1 and v_2 are nodes in G and there exist one or more edges e' in H between v_1 and v_2 such that (1) e' does not appear in any existing working or protection path or in $w(d)$ and (2) e' is not a prohibited edge, then we create an edge *e* in H between v_1 and v_2. (Only one such edge is created in H between v_1 and v_2 even if there are many edges e' in G between v_1 and v_2 with the necessary properties.) We call *e* an *unused edge*. It has a cost of one unit. Additionally, for each shortcut path P, we create an edge in H of zero cost between the end nodes of P. Such edges are called *shortcut edges*.

If we now run Dijkstra's algorithm on H to find the minimum-cost path between u and v, and then "expand" each shortcut edge into the shortcut path in G that it came from, then this produces the protection route $p(d)$ that uses the least amount of new bandwidth. Note that the idea behind the shortcut paths and edges is that we want $p(d)$ to consist of existing protection bandwidth as much as possible, since using such edges does not add to the overall cost of the final allocation plan. However, since branch points are forbidden, a shortcut path must be used either in its entirety or not at all. The effect of using a shortcut path is to "jump" from one end to the other at zero cost; this is modeled by the shortcut edges.

However, there is one slight problem with using Dijkstra on H. It is possible that when the shortest path in H is "expanded" into a walk in G, the result will not be a path in G. For example, two shortcut paths may cross each other in G but their corresponding shortcut edges in H may not. Therefore, in order to ensure that $p(d)$ is a path, one must mark each edge e in H with a list of its "rivals," i.e., edges e' with the property that any path containing both e and e' expands into a nonpath in G. Then one must run a constrained version of Dijkstra that ensures that rival edges never appear in the same path. For details of this constrained version of Dijkstra, see the Appendix.

(**Note:** The only reason to run constrained Dijkstra rather than ordinary Dijkstra is to ensure that the protection paths are indeed paths. If the operator of a network finds it acceptable to have protection "paths" that may revisit the same node or link more than once, then ordinary Dijkstra may be used. Not only will this speed up the algorithm, but the added flexibility of allowing such "self-intersections" can potentially increase the overall bandwidth efficiency. However, in spite of these considerations, our experimental results below cleave to tradition and enforce the constraint that protection paths must indeed be paths.)

It is possible that the constrained Dijkstra algorithm will not be able to find *any* protection path, e.g., because the working path, which is chosen first, is positioned in such a way that it is not possible to find a disjoint protection path. In this case it is recommended that one find the shortest cycle C containing both the source and the destination, and re-run the algorithm with one half of C as the new working path. If this also fails then one could try still more sophisticated searches, but in our software implementation we simply give up trying at this point.

We remark in passing that in contrast to p-cycles, PXTs are typically not closed, and therefore, can be extended incrementally at either end when new demands arrive. They also shrink incrementally when old demands disappear. This makes them well suited to online algorithms.

V. Experimental Results

The problem of evaluating the bandwidth efficiency of a shared mesh protection scheme is still largely unsolved. Ideally one would like analytical results that prove that: 1) the optimal total bandwidth is never more than (say) twice the bandwidth that would be needed if no protection were required; and 2) there are polynomial time offline and online algorithms that are guaranteed to get within a certain percentage of the optimal total bandwidth. Unfortunately, these analytical results seem to be currently out of reach. Therefore, it seems that the only thing to do is to obtain experimental results on specific instances. Although this is a reasonable approach, it raises the question, which instances should one choose?

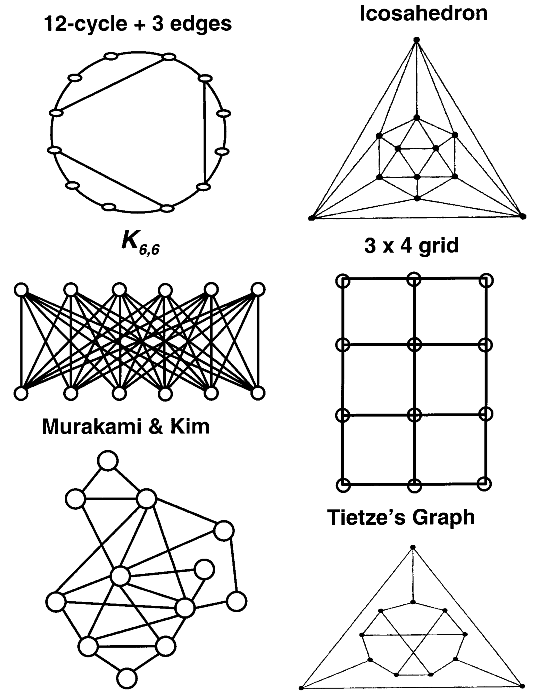

Fig. 4. Experimental results: Selected twelve-node networks.

There is no standard test suite for shared mesh protection studies. Most researchers, therefore, pick a few small networks more or less arbitrarily. This leaves one with the nagging worry that the instances chosen may not be truly representative of typical networks.

We do not have a fully satisfactory solution to this problem, but we suggest that some concepts from experimental design theory may be helpful. The idea is to identify certain parameters that are likely to be important and to determine the range of values that these parameters will take in practice. This defines a parameter space of possible instances. The parameter space will be too large to study exhaustively, so the goal is to sample it judiciously in order to obtain as much information as possible for the least possible effort.

For example, for shared mesh protection, three factors that seem to be important are: 1) the average degree of the graph (this relates to how well-connected the graph is); 2) the girth (i.e., the length of the smallest cycle; since a working path and a protection path together form a cycle, a large girth means that short working paths will have long protection paths); and 3) the extent to which the demands are localized (i.e., whether each node wants to talk to everyone else or just to certain nodes). With these factors in mind, we have chosen six 12-node graphs, shown in Fig. 4, to cover a range of possibilities in the parameter space. Tietze's graph, for example, is a small modification of the famous *Petersen graph*, which in turn is a *(3, 5)-cage*, i.e., the smallest regular graph of degree 3 and girth 5. The graph labeled "Murakami and Kim" is a slight modification of a network from [10]. The icosahedron has the maximum degree (five) attainable by a regular planar graph. Of course, our selection of graphs is still somewhat arbitrary; it remains an open question how to apply experimental design methodology more rigorously in the selection process.

For each of the six graphs we have considered three different lists of demands: *uniform, nearest-neighbor*, and *unbalanced*. In uniform traffic, every pair of nodes appears exactly five times in the list of demands. In nearest-neighbor traffic, every pair of *adjacent* nodes appears exactly ten times in the list, and there are no other demands. In unbalanced traffic, three nodes are chosen to be "large" nodes and the remainder are deemed "small" nodes. The number of times a demand appears in the list depends on the "sizes" of the terminals: If u and v are respectively small/small, small/large, and large/large, then $\{u, v\}$ appears respectively 2, 8, and 14 times.

For each of the 18 instances thus obtained, the demands were fed one at a time in a random order to our online algorithm and routed accordingly. For comparison, we also routed the demands using (1) a $1 + 1$ path protection algorithm and (2) a simple shared path protection scheme that finds a node-disjoint pair of paths between every pair of terminals and routes all the demands on those paths, sharing when possible without worrying about branch points.

The results are shown in Table I. The column labeled "Working" indicates the working bandwidth, which is the same for all three schemes—$1 + 1$, simple shared path, and PXT. The remaining three columns report the protection bandwidth for each of the three schemes; the column labeled "Path" refers to the simple shared path protection scheme. We see that our results confirm the conventional wisdom that shared mesh protection saves a lot of (total) bandwidth compared to dedicated protection. The percentage savings in our table varies from about 20% to 60%, which is a wider variation than has typically been reported in the literature; this may be due to our deliberate selection of a wide variety of networks. We also see that the bandwidth efficiency of the PXT algorithm is comparable to and often better than that of the conventional path protection algorithm. Therefore, the answer to our second question at the end of Section III is that the no-branch-point constraint does not negate the bandwidth efficiency. This result is consonant with what Grover and Stamatelakis report for p-cycles.

There are further questions that are natural to ask, e.g., how does the bandwidth efficiency of our online PXT algorithm compare to an offline PXT algorithm? How do the online/offline PXT algorithms compare to the best online/offline algorithms that do not necessarily avoid branch points? These are interesting and important questions, but they are not as easy to answer as one might initially think. Formulating the offline PXT problem in such a way that good solutions can be obtained in a reasonable amount of time is a research project in itself, as is the problem of devising an optimal online algorithm. We hope to investigate these problems, but they lie beyond the scope of the present paper. However, we did attempt to compute a loose lower bound for the bandwidth usage by feeding to CPLEX an integer linear program (similar to that described in [7]) that not only had no pre-cross-connection constraint but also allowed backup routes to depend on the location of the failure and also allowed stub release. The results of these (offline algorithm) computations are shown in the last column of Table I (the asterisks are there because the computations for the densest graph exceeded our available computer capacity). If we think of the difference between the $1 + 1$ column and the lower bound column as the "maximum potential savings due to sharing," then we see that even the online PXT algorithm typically achieves a significant percentage of the maximum savings.

TABLE I
EXPERIMENTAL RESULTS: BANDWIDTH USAGE

	Working	1+1	Path	PXT	L. Bound
UNIFORM					
12-cycle + 3 edges	840	1440	905	894	780
3 x 4 grid	770	1070	495	587	363
Tietze's graph	645	1125	340	362	208
Murakami & Kim	600	820	560	533	396
Icosahedron	540	690	280	178	112
K6,6	480	840	365	139	*
NEIGHBOR					
12-cycle + 3 edges	150	510	150	189	120
3 x 4 grid	170	510	170	236	115
Tietze's graph	180	690	170	206	90
Murakami & Kim	240	500	220	233	122
Icosahedron	300	600	290	205	80
K6,6	360	1080	200	188	*
UNBALANCED					
12-cycle + 3 edges	768	1368	824	794	708
3 x 4 grid	704	1004	594	476	320
Tietze's graph	636	1152	436	395	192
Murakami & Kim	516	742	450	399	252
Icosahedron	540	690	356	210	204
K6,6	480	840	378	154	*

Finally, we remark that we have run the PXT protection scheme on several much larger networks, including one with over two hundred nodes and over three hundred links, and with thousands of demands. The bandwidth efficiency was similar to that exhibited in Table I, but unfortunately we cannot give further details, for intellectual property reasons.

VI. CONCLUSION

The Achilles heel of shared mesh protection is its relatively slow restoration speed. This problem can be surmounted by forbidding branch points and thereby allowing protection paths to be pre-cross-connected. Grover and Stamatelakis took a first step in this direction with their p-cycle protection scheme, but we have gone further and have allowed a more general structure called a "pre-cross-connected trail" or PXT, whose flexibility allows it to be used in both link-based and path-based protection schemes and in both offline and online algorithms. Experimental results demonstrate that forbidding branch points does not destroy the main advantage of shared mesh protection, namely its high bandwidth efficiency.

APPENDIX
CONSTRAINED DIJKSTRA ALGORITHM

A key subroutine of our algorithm for computing allocation plans is a variant of Dijkstra's shortest-path algorithm that we call the *constrained Dijkstra algorithm*. Constrained Dijkstra is of some interest in its own right so we give a self-contained description of it in this section.

INPUT: A directed graph G, each of whose edges e has a nonnegative weight (its *length*) and a (possibly empty) list of edges of G (called the *rival edges* of e), and a distinguished node v of G (called the *source node*).

OUTPUT: For each node u of G, the shortest *admissible* path from v to u. A path p is *admissible* if, for all edges e in p, no rival edge of e is in p.

Definition: A *partial path* P in G is an ordered quadruple (p, l, F, s), where p is a directed path in G, l is the length of the path (i.e., the sum of the lengths of its edges), F is a set of edges of G (called the *forbidden edges* of P), and s is the *state* of the path (which takes one of two values: *penciled in* or *inked in*). We use the letters p, l and F to denote "coordinate functions," i.e., $F(P)$ is the set of forbidden edges of P, and so on.

Definition: A partial path P_1 is said to *dominate* a partial path P_2 if $l(P_1) \leq l(P_2)$ and $F(P_1) \subseteq F(P_2)$. Intuitively, this means that P_1 is at least as good as P_2. Note the use of $\leq$ rather than $<$ and $\subseteq$ rather than $\subset$. This is important.

Preliminary Remarks on the Algorithm: During the course of the algorithm, each node u maintains a list of partial paths from v to u. We say that a node is *black* if there exists an inked-in partial path in its list and we say that it is *white* otherwise. Initially only v is black; as the algorithm runs, more and more white nodes become black. Once a node becomes black it stays black permanently.

If a node u is black, it has at most one inked-in partial path; this represents the shortest admissible path from v to u. If u is white, its penciled-in partial paths represent paths that are potential shortest paths to u. If u is black, its penciled-in partial paths represent initial segments of potential shortest paths to other nodes.

Like Dijkstra, constrained Dijkstra is a breadth-first search algorithm. At each step, one of the nodes u is designated to be the *active node* and one of the partial paths of u is designated to be the *active partial path*. Partial paths are extended one node at a time at the active node. Again like Dijkstra, constrained Dijkstra keeps the partial paths in a heap, so that it can quickly find the shortest partial path when it needs to.

Initialization: As a pre-processing step, we examine each edge e of G in turn; for each rival edge f of e, we add e to the list of rival edges of f if e is not already on that list. We are free to do this since it does not change the admissibility or length of any path in G, and it is convenient for our purposes.

The source node's list of partial paths is initialized to contain a single entry $P : p(P)$ is the path consisting solely of the source node v itself, $l(P) = 0, F(P)$ is the empty set, and $s(P)$ has the value "inked in." Thus, v is black. We also designate v to be the active node and its (unique) partial path to be the active partial path. At every other node the list of partial paths is empty, so they are all white. The partial path P is put on a heap.

Main Loop: We "probe forward" from the active node. That is, suppose that u is the active node and that P is the active partial path. We consider in turn each edge e that emanates from u. If e is forbidden, i.e., if $e \in F(P)$, then we ignore it and move on to the next edge. Otherwise, let w be the node that e points

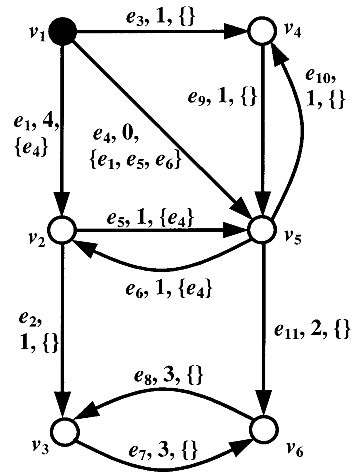

Fig. 5. Sample input for constrained Dijkstra algorithm.

to. We let P' be the partial path obtained from P by appending w to $p(P)$, adding the length of e to $l(P)$, and adding the rival edges of e to $F(P)$. If P' is dominated by some partial path in w's list, then we forget about it and move on to the next edge emanating from u. Otherwise, we add P' to the list of partial paths at w, penciling it in. We also add it to the heap. We then delete any penciled-in partial paths in the list at w that are dominated by P'. These partial paths are also deleted from the heap. We repeat this process until all the edges emanating from u have been exhausted. We then remove P from the heap, but do not delete it from the list of partial paths at u.

Next, we extract the shortest partial path Q from the heap and designate it to be the new active partial path. We also designate the node x where we found Q to be the new active node. If x is white, we ink in Q (thereby making x black). Otherwise, we simply leave Q penciled in.

Termination: The algorithm terminates when we try to extract a partial path from the heap but find that it is empty, or when all nodes become black, whichever occurs first.

Example: Each edge in the directed graph in Fig. 5 has three labels: the name of the edge, the length of the edge, and the set of rival edges. The distinguished node is v_1, which is the first black node.

Observe that if we ignore the constraints given by the rival edges, then the shortest path from v_1 to v_3 is $(v_1, e_4, v_5, e_6, v_2, e_2, v_3)$. However, this path is not admissible because it contains both e_4 and e_6, which are rivals of each other.

Initially v_1 is the active node. If we probe forward then we obtain three partial paths. The partial path at v_5 is the shortest so we ink it in, making v_5 black. These become the new active partial path and active node. (See Fig. 6. To avoid clutter, we have suppressed edge labels. Shaded entries are penciled in and unshaded entries are inked in.)

We now probe forward from v_5. At v_4, the new partial path is dominated by the existing partial path so it is not added. We cannot probe forward to v_2 because e_6 is forbidden. Probing forward to v_6 is all right and we add a new partial path there: $((v_1, e_4, v_5, e_{11}, v_6), 2, \{e_1, e_5, e_6\})$. This, however, does *not* become the new active partial path, because the penciled-in

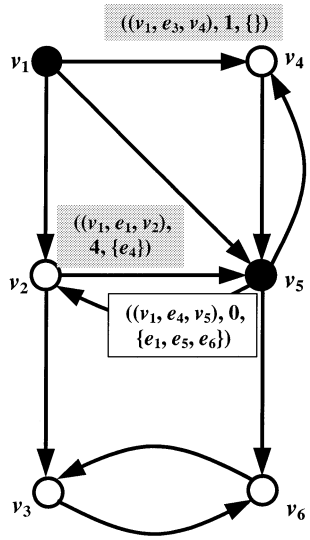

Fig. 6. Intermediate stage in constrained Dijkstra algorithm.

partial path at v_4 is shorter. We ink in the partial path at v_4, make v_4 black, and probe forward from v_4. The only new partial path created at this stage is at $v_5 : ((v_1, e_3, v_4, e_9, v_5), 1, \{\})$. Even though v_5 is black, we retain this new partial path because it is not dominated by the existing partial path at v_5. (The existing partial path is shorter but has forbidden edges that are not forbidden in the new partial path.) In fact this becomes the new active partial path, although we do not ink it in because v_5 is already black.

Continuing in this way, we find that the remaining shortest admissible paths are $(v_1, e_3, v_4, e_9, v_5, e_6, v_2)$, $(v_1, e_3, v_4, e_9, v_5, e_6, v_2, e_2, v_3)$, and $(v_1, e_4, v_5, e_{11}, v_6)$. Notice that these paths do not arrange themselves into a tree; this is one difference from the usual Dijkstra algorithm.

Final Remarks: We omit a detailed proof of the correctness of the constrained Dijkstra algorithm; the basic idea is that constrained Dijkstra is equivalent to ordinary Dijkstra on an auxiliary graph that can be built out of the partial paths.

Although we have described constrained Dijkstra for directed graphs, it can be applied to undirected graphs using the usual trick of replacing an undirected edge with two directed edges.

The running time of the constrained Dijkstra algorithm is exponential in the worst case. As an example of this, consider the "grid graph" G_n whose nodes are the points in the plane whose coordinates are integers with absolute value at most n, and whose edges point from each vertex to its immediate southern neighbor and to its immediate western neighbor. Give each edge of G_n one rival edge, namely its image under reflection in the line $x + y = 0$. It is not hard to show that if we take the node with coordinates (n, n) as the source node, then by the time the algorithm first reaches the line $x + y = 0$ it will be keeping track of about 2^n partial paths. It should be pointed out, however, that this example relies on an artificial assignment of rival edges that does not arise in the PXT algorithm, which generates only limited sets of rival edges that arise out of the geometric properties of the working and protection paths. So the practical performance of the PXT algorithm is much better than this worst-case analysis might suggest. Nevertheless, because of this potentially exponential consumption of resources, it is important that the actual implementation of the algorithm contain parameters that allow it to exit gracefully and report failure if it exceeds a certain amount of time or memory.

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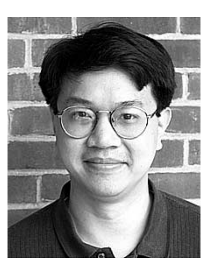

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