Mathematics Of Various Entertaining Subjects

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Quick Scrabble Refresher

There is a publicly known set of 100 tiles in a bag, each with a letter and a point value, except for two blanks, which are wildcards scoring zero points.

Each player draws a rack of seven random tiles, hidden from the opponent.

Players take turns forming words on the board, scoring points, possibly including bonus points if you play a tile on a bonus square. If you play all seven tiles at once, then you **bingo**, scoring 50 extra points.

Regardless of how many tiles you play, at the end of your turn, you draw tiles randomly from the bag so that you again have seven tiles on your rack, unless there are not enough tiles in the bag, in which case you draw as many tiles as you can.

If the bag is empty at the start of your turn and you play all your remaining tiles, then you "go out," ending the game, and collecting extra points from your opponent's unplayed tiles.

An UNUSUAL Scrabble Puzzle

You just bingoed with GROGSHOP, but your opponent immediately bingoed back with UNUSUAL, taking a lead of 476 points to your 344 points.

You hold the tiles **MONKYSZ**.

The tiles unseen to you are **DEFJLLLQW**, two of which are in the bag, and seven of which are on your opponent's rack.

Can you possibly avert a loss if your opponent plays perfectly? What's your best play?

Three Preliminary Comments

1. For those who do not play tournament Scrabble: Yes, GROGSHOP is a word, and so are SETA, AMRIT, CABER, RICHWEED, EPIGEOUS, and all the other words on the board. They are all listed in the 2023 NASPA Word List and the 2024 Collins Scrabble Words, the two main lexicons currently in use.

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3. We assume that a win is worth $+1$, a loss is worth -1, and a tie is worth 0. The margin of victory ("spread") is irrelevant.

Key Idea 1: Fish for MONKEYS

Trailing by 132 points, you need to bingo. You have no playable bingo on your rack, but you almost have **MONKEYS**. You can **set up** a winning play of **MONKEYS** down column O by playing **DITZ** on row 8, hoping to draw the **E**.

Key Idea 2: Phantom Threats

*Skeptic: Upon seeing **DITZ**, wouldn't your opponent sense what you're up to, and block your play of **MONKEYS**, e.g., by playing **JELL** at N2?*

Ah, but if you not only draw **E** but also leave **D** as the last tile in the bag, then your opponent might also worry that you have **DONKEYS**. And **JELL** followed by **DONKEYS** leads to 508–508 tie!

DONKEYS is a "phantom" threat; you cannot execute it, but your opponent doesn't know that.

Key Idea 3: Bluffing

Skeptic: I'm still not convinced. **MONKEYS** *is a bigger threat than* **DONKEYS** *so your opponent will surely block* **MONKEYS**.

Okay, so what if you play **DITZ** down column H? By the skeptic's logic, your opponent will block the "bigger threat" of **DONKEYS** with **JELL** at 14B, allowing you to play **MONKEYS** for a 511–511 tie!

Playing **DITZ** at H11 is a **bluff**; no matter what you draw, you will not be able to execute your phantom threat of **DONKEYS**.

Is Bluffing with DITZ at H11 the Best Play?

What does "best play" even mean?

If you play **DITZ** at H11, is **JELL** at 14B (blocking the "bigger threat") your opponent's "best play"? But it only ties, whereas if your opponent suspects a bluff and **second-guesses** you by playing **JELL** at N2, then you lose.

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The **minimax theorem** for two-person zero-sum games says that there always exists a **mixed** (i.e., randomized) strategy for you and a mixed strategy for your opponent that together form a **Nash equilibrium**. By definition, in an equilibrium, neither player can profit by deviating unilaterally.

The equilibrium is not necessarily unique, but in a two-person zero-sum game, all equilibria yield the same expected payoffs for both players.

The Equilibrium Strategies

With a rack of MONKYSZ (or MNKEYSZ, MOKEYSZ, MONEYSZ, MONKESZ, MONKEYZ), play **DITZ** at H11 with probability $\frac{2}{3}$ and at 8K with probability $\frac{1}{3}$.

With a rack of DONKYSZ (or DNKEYSZ, DOKEYSZ, DONEYSZ, DONKESZ, DONKEYZ), play **DITZ** at 8K with probability $\frac{2}{3}$ and at H11 with probability $\frac{1}{3}$.

In other words, bluff with probability $\frac{2}{3}$ and set up with probability $\frac{1}{3}$.

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If your opponent sees **DITZ** at 8K, she should play **JELL** at N2 with probability $\frac{2}{3}$ and at 14B with probability $\frac{1}{3}$.

If your opponent sees **DITZ** at H11, she should play **JELL** at 14B with probability $\frac{2}{3}$ and at N2 with probability $\frac{1}{3}$.

In other words, she should block with probability $\frac{2}{3}$ and second-guess with probability $\frac{1}{3}$.

This is the first ever explicit Scrabble position where the best strategy is randomized.

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▶ To calculate the opponent's equilibrium probabilities, all of your possible racks need to be analyzed; e.g., if, from one of those racks, you would have had a play better than **DITZ**, then your opponent can infer that you didn't have that rack.

   ▶ With a rack of DONKESZ, you can bingo immediately with **ZEDONKS** at 15A. But it turns out that this never ties or wins.

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  ▶ The possible bingos **DOLMENS**, **DONZELS**, **ENFOLDS**, **FONDLES**, **KNOLLED**, **MENFOLK**, plus several more Collins-only words, threaten *cooks* (unintended alternative lines of play). We needed several spots (3E, 9I, B10) where the Q could be played.