The Metric Formula

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Quantifying distribution in long races by estimating the sum of its parts in terms of efficiency; is a solution-based approach to the ever present, often problematic issue of anomalous activity saturating distribution efficiency.

The Metric Formula

By Nigel Merrigan

Distribution Efficiency is something we all equate with the bear-off. Distribution shape as in the case of the "magic triangle" (Kit Woolsey) is consistent with an efficient bear-off. The same however, cannot be said of long races. There is very little in terms of a relationship between shape and efficiency because distribution is loosely strung out across 2/3 quadrants. Distribution anomalies may be the result of a ripple effect in distribution efficiency; it's point of origin being the ace-pt expanding as far as the 24-pt. Kleinman's "bull-in" method in Vision Laughs at Counting acknowledges the value in the 10-pt for its efficiency. More recently on Stick Rice's BGonline, Nack Ballard makes reference to the 'poorly placed mid and 8-pts to handle large numbers' (In response to: 123 vs. 137 with mid-pt contact, Neil Kazaross, 4 Feb 2010, 8.41 pm). If indeed the entire outfield suffers from similar efficiency hang-ups as in the bear-off (yet, significantly less perhaps), then, there may be a way of finding a better solution in adjusting for extra crossovers and pip differences in long races.

An ideal starting point is the "Keith Count" (devised by Tom Keith) for the way it adjusts for wastage on the ace, deuce and three-pts. A 2 pip penalty is added for each checker more than 1 on the ace-pt or the 200% equivalent. Extra checkers on the deuce and three carry penalties of a pip or 100% and 66.6% respectively. Gaps on the 4, 5 and 6 are treated equally, although, given this initial finding, it's probably safe to assume as the distance from the ace-pt increases so too will the percentage for each further point continue to decline. Thus, as the ripples expand the less significant the wastage will be. Another feature of the Keith Count that could prove useful is the 1/7 added to the leaders count. Why 1/7? Is it to account for distribution efficiency or some other wastage anomaly present within the positional configuration? Is it to align the double/take criteria? Maybe it's all of these or something completely different altogether!

An area in need of redress is the issue concerning crossovers, although, the debate tends to focus more on the harmfulness of crossovers than how they are accounted for in LRB's. Yet, the delicate interaction between long races and the 753 model (the late Walter Trice) where, smoothing and outfield efficiency is often subtle, can stir up anomalies in distribution that the simple tallying of crossovers is unable to detect. 0.5 seems to be the widely accepted value of a crossover, although, miraculously it is unaffected by distribution efficiency or any other anomaly that could extraneously impinge on it. Given the dynamics of LRB's, it probably does to some extent where, in reality the crossover value is somewhere between 0.4 and 0.5.

One other topic of interest is the difference in opinion as to the number of quadrants there are. The "Ward" and "Thorp Count" along with the Lamford/Gasquoine Formula factor in the state of having fewer checkers off whereas; the Keith Count makes no direct reference of this. The need for a fifth quadrant in LRB's seems unnecessary; unless of course you wanted to know the number of crossovers required and the estimated weight of each crossover in terms of efficiency. This approach would allow for greater sensitivity where, the entire outfield can be mapped out to function analogously to a sensor grid designed to detect the slightest fluctuation in distribution efficiency. A more in depth discussion follows.

Table 1: The Distance Effect on Distribution Efficiency

Quadrant 5						
Distance	**24-pt**	**23-pt**	**22-pt**	**21-pt**	**20-pt**	**19-pt**
Efficiency	**21%**	**22%**	**23%**	**24%**	**25%**	**26%**
Crossovers	**3.75**	**3.70**	**3.64**	**3.57**	**3.50**	**3.42**
Quadrant 4						
Distance	**18-pt**	**17-pt**	**16-pt**	**15-pt**	**14-pt**	**13-pt**
Efficiency	**22%**	**24%**	**25%**	**27%**	**29%**	**31%**
Crossovers	**2.67**	**2.59**	**2.50**	**2.40**	**2.29**	**2.15**
Quadrant 3						
Distance	**12-pt**	**11-pt**	**10-pt**	**9-pt**	**8-pt**	**7-pt**
Efficiency	**25%**	**27%**	**30%**	**33%**	**38%**	**43%**
Crossovers	**1.50**	**1.36**	**1.20**	**1.00**	**0.75**	**0.43**
Quadrant 2						
Distance	**6-pt**	**5-pt**	**4-pt**	**3-pt**	**2-pt**	**1-pt**
Efficiency	**33.3%**	**40%**	**50%**	**66.6%**	**100%**	**200%**
Crossovers	**2.00**	**2.00**	**2.00**	**2.00**	**2.00**	**2.00**
Quadrant 1						
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The data presented in Table 1 was compiled by exploring various mathematical ideas/methods and techniques sourced from the "Kleinman Count" (Danny Kleinman); the "Ultimate Pip Count" (Kit Woolsey); the "Keith Count" (Tom Keith) and the "Effective Pip Count" (the late Walter Trice) The primary argument for the existence of such a table is wastage; from the perspective of a single outfield point the wastage is negligible but taken together from the perspective of the entire outfield, wastage can have a significant effect on distribution efficiency across a wide expanse of long race positions.

To grasp how the table works, consider the distance a checker on the 24pt has to travel to the first point of bear-in i.e. the 6pt. For each pip the imaginary checker travels the higher the efficiency weighting and less crossovers required. Hence, as the distance shortens between the 24pt and 6pt the less efficient (i.e. higher efficiency weighting) each point is. There is a logical premise here, for in the bear-off, the 6pt is significantly distributional efficient compared to the 1pt; a fact that is supported by Woolsey's "Magic Triangle" and Trice's "753 Model".

Efficiency

Danny Kleinman may have been the first to acknowledge the importance of the 10pt. In terms of efficiency, the 10pt is considered to be the most efficient point on the backgammon board. Consider for a moment a situation where one has half a roll to play from the 10pt to the 6pt. There are on average 4.083 pips per half roll and the product of half a roll and the 10pt's efficiency weighting (quad 3) is 4.083*30%, 1.224 crossovers compared to 1.200 crossovers.

To put this into perspective, consider the product of half a roll and the 8pt i.e. 4.083*38%, 1.551 crossovers compared to 0.750 crossovers; a massive inefficiency of 107%.

The Starting Position

Of many long races with minimum contact, there is often residual "left-over's" from the starting position i.e. the 13 and 8pts. The starting position is by its very nature inefficient, although, efficiency is relative with the 8pt getting star billing. The 13pt is next in line, followed by the 6pt and then the 24pt. It is not surprising in the least that in "Backgammon Openings" written by Nack Ballard & Paul Weaver (2007) that the mid and 8pts are given special treatment. Why on earth, would one want to preserve/strengthen such inefficiencies? One explanation could be where there is contact the mid and 8pts require a higher "decay tolerance" than any other point due to their weakness in the chain of communication between offence/defence. However, once contact has been broken, the 8pt has outlived its usefulness; decay sets in and should be avoided whenever one can when bearing in efficiently.

Distribution

In minimal contact races; or where non-contact is imminent distribution is considered normal. In these classes of 100+ pips distribution efficiency clusters close to the *mean* suggesting a stable environment with moderate sensitivity. The mean (see below) is an excellent benchmark in these types of races as it simplifies the task of estimating distribution efficiency without having to refer to the table.

Efficiency	Ratio
24	0.787
23	0.814
22	0.837
21	0.856
20	0.875
19	0.889
18	0.587
17	0.621
16	0.625
15	0.648
14	0.664
13	0.666
12	0.375
11	0.367
10	0.360
9	0.330
8	0.285
7	0.184
6	0.666
5	0.800
4	1.000
3	1.332
2	2.000
1	4.000

E-Pips & Crossovers

The efficiency ratio for a checker on the 10-pt is 1.20*30% = 0.360. When calculated for all points show the mean to be 0.856 with a Standard Deviation of 0.767.

The accumulative mean is therefore 0.856 * 24 = 20.56.

We can use the *mean* to reflect the cost of a positions' distribution efficiency by computing the number of outfield pips to the first point of bear-in i.e. the 6pt. Each efficiency pip or "E-Pip" is worth 20.56%.

The crossover value is the mean average of crossovers as given in the table divided into the number of quadrants. Therefore, 2.267/5 = 0.453. The expected loss is (0.5-0.453)*24 = 1.128 in percent. Rounding 20.56 to 21% takes up some of the slack where the overall expected loss is 1.128-0.44 = 0.688 of a percent. Thus:

The sum of E-Pips is 21% of the Trailers "pips to the 6" minus the Leaders "pips to the 6" plus 0.5 for each extra crossover.

“Rule 62”

The culmination of knowledge spanning many years of experience and arguably the best statistician in the modern game; the late Walter Trice, continues to inspire all through his masterpiece known to many as *Backgammon Boot Camp*.

The nifty and compact “rule 62” (p135-138) is designed for races of 20 pips and above. Below this threshold, go checkout the section on EPC. Building on the popularity of Rule 62 is the “Nack 57 Rule” developed by Nack Ballard. It is believed both “rule 62 and the “Nack 57 Rule” were derived from the “Gold Standard Table” (GST), although, the rollout data collated is somewhat of a mystery; maybe Walter compiled it many moons ago or some other mystery soul(s). According to Ballard, Walter derived a set of “Leader Counts” that did a pretty good job of matching the GST data i.e. “Points of Last Take” (PLTs) and from there developed “Rule 62”.

“Nack 57 Rule”

Following Walter’s research, Ballard arrived at a more precise model that matched the GST data perfectly. He coined it the “Nack 57 Rule” and works for leader counts as low as 48 and well into and above the 100+ range. Like “Rule 62”, the 57 part of Ballard’s formula is the “dividing line between two formulas that yield the same result i.e. -5/7d and 57-33*2 to the nearest square root” (Ballard).

“Nack 58 Rule”

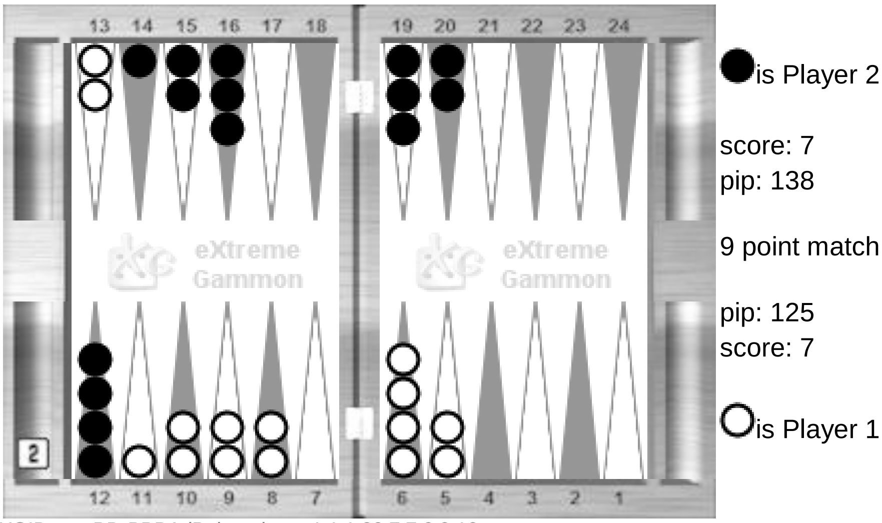
The “Nack 57 Rule” and Nack 58 Rule” are one of the same as both in general arrive at the nearest square root whether, you use the subtract -33 (57) or -32 (58). However, there are differences between them such as, the “Nack 58 Rule” matches the lower part of the GST with greater ranger; i.e. 48-60, compared to 55-60 for the “Nack 57 Rule”. Rollout data seems to suggest for Leader Counts of 61, 78, 88, 89, 111 and 123+, “the trailer can take a pip more aggressively” under Nack58 (Ballard). Whether Nack58 matches many more Leader Counts is a question for future research. For the time being, Nack57 is the only model that matches the GST; whereas “Nack58 presumably matches a table that hasn’t yet been invented” (Ballard).

“Metric Formula”

Perhaps not entirely an alternative approach to “Rule 62” but rather a complimentary addition is the “Metric Formula” (**Tric**e/**Me**rrigan). The simple formula below converts the *point of last take* to raw unadjusted winning chances in percent. As a matter of *appropriateness,* it is important to mention here in cases where the pip-count exceeds 48 pips the “Nack 58 Rule” is adopted.

$$50 + \frac{62 - PLT}{PLT + L} * L$$

PLT: Point of Last Take L: Lead in Pips

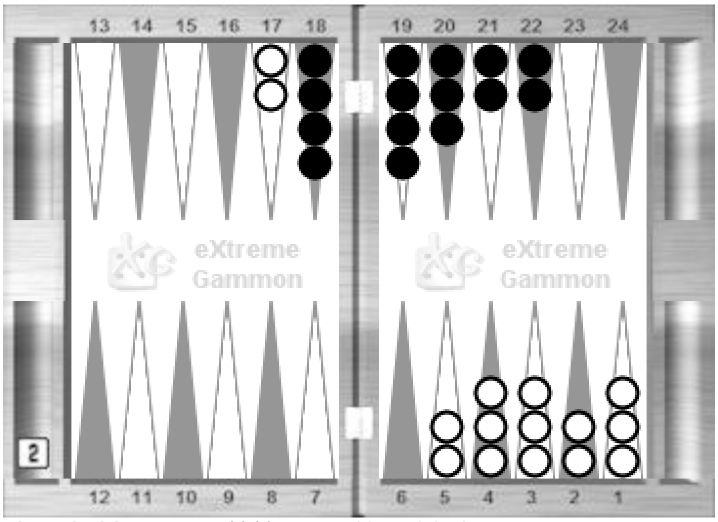


XGID=-----BD-BBBAdBabc--cb-----:1:1:1:00:7:7:0:9:10

○ on roll

Analyzed in Rollout	No redouble	Redouble/Take
Player Winning Chances:	76.81% (G:0.10% B:0.00%)	76.81% (G:0.10% B:0.00%)
Opponent Winning Chances:	23.19% (G:0.00% B:0.00%)	23.19% (G:0.00% B:0.00%)
Cubeless Equities	+0.536	+0.536
Cubeful Equities		
No redouble:	+0.536 (0.000)	±0.000 (+0.536..+0.536)
Redouble/Take:	+0.536	±0.000 (+0.536..+0.536)
Redouble/Pass:	+1.000 (+0.464)	
Best Cube action: Redouble / Take		
Rollout details		
5184 Games rolled with Variance Reduction.		
Dice Seed: 75829917		
Moves and cube decisions: 3-ply		
Double Decision confidence:		50.7%
Take Decision confidence:		100.0%
Duration: 11 minutes 47 seconds		

According to the "Nack 58 Rule" the PLT is **14**+125 = 139 suggesting double/take (that is, at any score other than DMP). The Metric Formula converts this to 50+ (62-14) / (14+13), ***rounding down to 1 decimal place*** 1.7 *13 = 72.1%. You're not done yet! Black's surplus E-Pips cost him 13*21%+1.5 = 4.23%. White's adjusted winning chances are 72.1+4.23 = 76.33%. An Extreme Gammon (XG2) rollout indicates white wins 76.81%. The Kleinman Count estimates a tad over 76% with Lamford/Gasquoine giving 75.83%. Given the simplicity of normal distribution all three approaches are in the ball park.

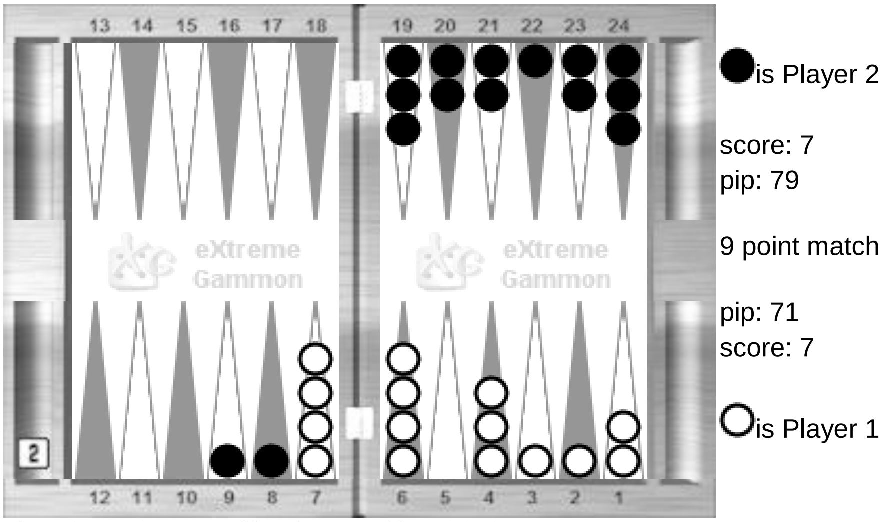


XGID=-CBCCB----------Bddcbb---:1:1:1:00:7:7:0:9:10

on roll

Analyzed in Rollout	No redouble	Redouble/Take
Player Winning Chances:	69.90% (G:0.00% B:0.00%)	69.90% (G:0.00% B:0.00%)
Opponent Winning Chances:	30.10% (G:0.00% B:0.00%)	30.10% (G:0.00% B:0.00%)
Cubeless Equities	+0.398	+0.398
Cubeful Equities		
No redouble:	+0.398 (0.000)	±0.000 (+0.398..+0.398)
Redouble/Take:	+0.398	±0.000 (+0.398..+0.398)
Redouble/Pass:	+1.000 (+0.602)	
Best Cube action: Redouble / Take		
Rollout details		
5184 Games rolled with Variance Reduction.		
Dice Seed: 75829917		
Moves and cube decisions: 3-ply		
Double Decision confidence:		50.1%
Take Decision confidence:		100.0%
Duration: 3 minutes 33 seconds		

The position above is representative of a class of positions in which, one side burdened with wastage is ahead in the stakes to bear-off but way behind in the race to bear-in. They are characteristically latent in nature which, is probably the best way to describe them – "Latent Bear-offs". How one should adjust for wastage in these types of positions, can often be tricky. The Keith Count estimates 6 pips of wastage for white, although, according to Bob Koca 5.5 is a more reasonable approximation. With a ratio of 0.36, the Kleinman Count estimates white's winning chances around 68.6%. Alternatively, the Ward Count estimates 2 pips of wastage for white and with E-Pips thrown into the mix; the Kleinman Count predicts an estimated 69.2% with Lamford/Gasquoine indicating 71.57%. The Metric Formula performs best of all; estimating white's winning chances a tad more than 69.3%.

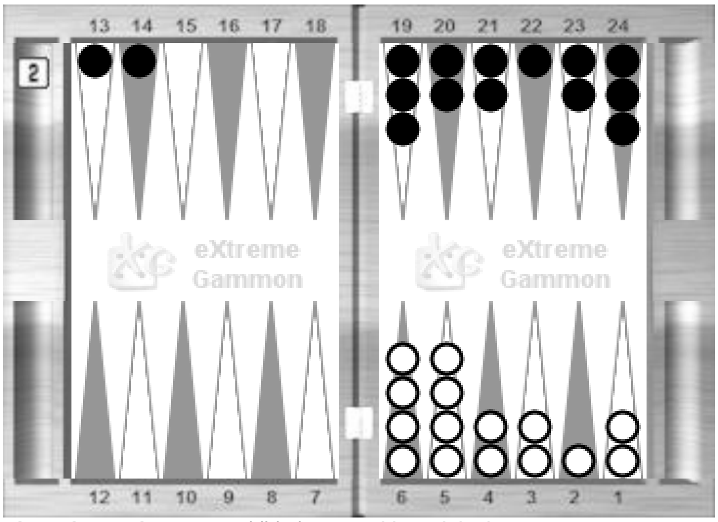


XGID=-CBABBC---------AAdd-caab-:1:-1:-1:00:7:7:0:9:10

on roll

Analyzed in Rollout	No redouble	Redouble/Take
Player Winning Chances:	77.76% (G:0.26% B:0.00%)	77.76% (G:0.26% B:0.00%)
Opponent Winning Chances:	22.24% (G:0.00% B:0.00%)	22.24% (G:0.00% B:0.00%)
Cubeless Equities	+0.555	+0.555
Cubeful Equities		
No redouble:	+0.555	±0.000 (+0.555..+0.555)
Redouble/Take:	+0.555 (0.000)	±0.000 (+0.555..+0.555)
Redouble/Pass:	+1.000 (+0.445)	
Best Cube action: No redouble / Take		
Percentage of wrong pass needed to make the double decision right: 0.0%		
Rollout details		
5184 Games rolled with Variance Reduction.		
Dice Seed: 75829917		
Moves and cube decisions: 3-ply		
Double Decision confidence:		50.1%
Take Decision confidence:		100.0%
Duration: 7 minutes 10 seconds		

There are tendencies to over-cook these types of race positions by complicating the degree of wastage one really needs to adjust for when, more often than not a succinct approach will suffice. For instance, a simple comparison using the Keith Count indicate a single pip adjustment for the gap on white's 5pt and 3 pips for the extra checkers on black's ace and deuce-pts Plugging the adjusted pip counts (72-82) into the Kleinman Count yield a ratio of 1.3 with white winning 78% of the time. The Metric Formula and Lamford/Gasquoine Formula also fair pretty well, with winning chances estimated at 77.89 and 77.9 respectively. The latter estimate is interesting, in that, a gap on the 5pt carries a 3 pip penalty; whereas a 2 pip penalty is advocated by the Ward Count. Given the wall of checkers on the 6pt, one could intuitively approximate a 2.5 pip penalty for the gap on the 5pt and 2 pips for the extra on black's ace-pt. Adjusted pip counts of 73.5-81 suggest that a Kleinman/Ward/E-Pip application reveals an estimate of 77.37 with the Metric Formula (E-Pips included) reporting 77.57.



is Player 2

score: 7
pip: 69

9 point match

pip: 62
score: 7

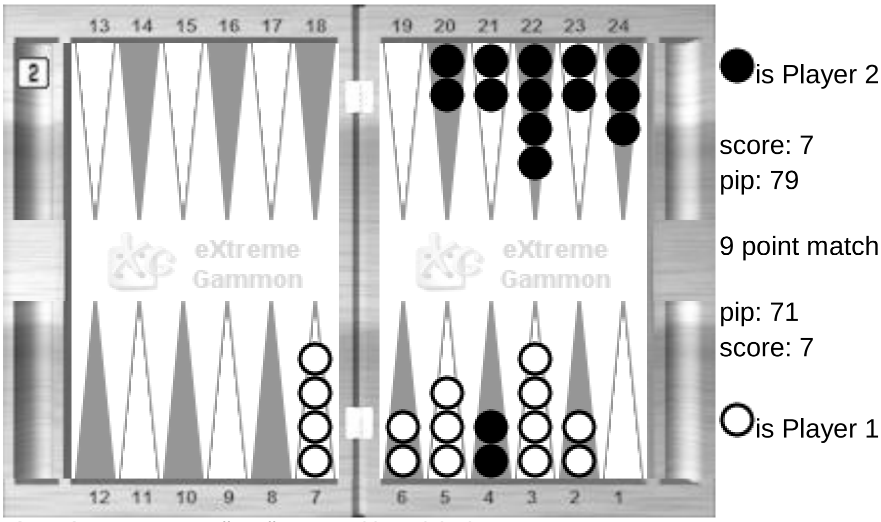
is Player 1

XGID=-CBABBC----AA------ddbbab-:1:1:-1:00:7:7:0:9:10

on roll

Analyzed in Rollout	No redouble	Redouble/Take
Player Winning Chances:	83.16% (G:0.11% B:0.00%)	83.16% (G:0.11% B:0.00%)
Opponent Winning Chances:	16.84% (G:0.00% B:0.00%)	16.84% (G:0.00% B:0.00%)
Cubeless Equities	+0.663	+0.663
Cubeful Equities		
No redouble:	+0.663	±0.000 (+0.663..+0.663)
Redouble/Take:	+0.663 (0.000)	±0.000 (+0.663..+0.663)
Redouble/Pass:	+1.000 (+0.337)	
Best Cube action: No redouble / Take		
Percentage of wrong pass needed to make the double decision right: 0.0%		
Rollout details		
5184 Games rolled with Variance Reduction.		
Dice Seed: 75829917		
Moves and cube decisions: 3-ply		
Double Decision confidence:		50.2%
Take Decision confidence:		100.0%
Duration: 1 minute 19 seconds		

The significant difference in this position is, in contrasts to black, white has completed the bear-in; whereas black on the other hand, has two remaining checkers in the outfield. The sum of E-Pips in these kinds of positions is most revealing and can often very costly. In this case, black's outfield stragglers will cost him 3.3% (11*21%+2*0.5) in winning chances. As for the position, black has three additional pips of wastage increasing his count to 62-72 in white's favour. Both the Kleinman Count and the Metric Formula estimate winning chances of 80%; with the E-Pips in tow; 83.3% - very accurate!



XGID=-CBDBB-----------dbcBdb--:1:1:-1:00:7:7:0:9:10

on roll

Analyzed in Rollout	No redouble	Redouble/Take
Player Winning Chances:	78.69% (G:0.81% B:0.01%)	78.69% (G:0.81% B:0.01%)
Opponent Winning Chances:	21.31% (G:0.13% B:0.00%)	21.31% (G:0.13% B:0.00%)
Cubeless Equities	+0.574	+0.574
Cubeful Equities		
No redouble:	+0.574 (0.000)	±0.001 (+0.573..+0.575)
Redouble/Take:	+0.574	±0.001 (+0.573..+0.575)
Redouble/Pass:	+1.000 (+0.426)	
Best Cube action: Redouble / Take		
Rollout details		
5184 Games rolled with Variance Reduction.		
Dice Seed: 21854500		
Moves and cube decisions: 3-ply		
Double Decision confidence:		50.0%
Take Decision confidence:		100.0%
Duration: 5 minutes 20 seconds		

The one exception deemed critical in contact races and contact race bear-offs such as the case in the one above is the leaders' wastage which, must be factored in before any other consideration. When black rolls a 3 he'll incur wastage behind white's anchor with a 6, 5 or 4. We can approximate three pips of contact wastage and then, with the Ward count, add an extra pip for the fourth man on the 3pt and half a pip for a future gap on the 4pt. White's total adjusted pip count is 75.5 with black incurring 3 pips of wastage for the extras on the ace and deuce-pts; increasing his count to 82 pips. A Kleinman Count with E-pips estimates winning chances of 78.7% and the Metric Formula, 78.6%. A Keith Count suggests an adjustment of 74-86 with winning chances of (1.64, Kleinman) 80.5% and Metric 80%. A Keith Count comparison with contact wastage however, indicates adjustments of 74.5-84 which, reports 76.7 and 76.6% respectively.

Summary

Accuracy

Given the small but qualitative sample, the Metric Formula performs equally well; if not slightly better than its competitors in all of the positions tested.

Reliability

The authenticity of the Kleinman Count makes it "one of a kind"; a remarkable feat of mathematics in the pre-bot era. Top players have come to trust it and rely on it, in providing sound approximations across a wide expanse of normal distribution. In the search of ever increasing accuracy, one can only hope that the Metric Formula performs equally well. Thus far, the data looks promising!

Ease of Use

This being a personable/relative argument, on face value, the Lamford/Gasquoine is the least easiest to use. There are 8 steps to negotiate plus a chunky formula to compute. Apart from the complexity, the upside is it caters for the entire race including the bear-off. The Kleinman Count is the easiest to compute with the only minor concern being memorizing the table and interpolating between ratios. The downside is its applicability beyond long race normal distribution. The Metric Formula is the middle ground for ease of use. Given the level of objectivity, there are no clear downsides at this time.

Adaptability

The Metric Formula works exceptionally well with the Keith/Ward Counts in a variety of positional types including the bear-off. In positions where distribution in long races is awkward as a result of wastage, gaps or any other anomaly the Kleinman Count does not fair too well with the Keith Count but improves dramatically when E-Pips are added. The Lamford/Gasquoine Formula pretty much stands alone, although, there is some evidence of E-Pip improvement.

Additional Notes

The sample above are positions where the pip lead is greater than 4 pips but where the race is even or where the pip lead is less than 4 pips some additional adjustments are necessary. The following adjustments only apply in positions where one or both sides have checkers remaining in the outfield.

Even Pip Races

Add 2 pips to the Trailers pip count and then add 21% of the Metric excluding the 50% CPW.

1 Pip Lead/2 Pip Lead

Add 2 pips to the Trailers pip count.

3 Pip Lead

Decrease Leaders pip count by 1 pip.

The Metric Formula

$$50 + \frac{62 - PLT}{PLT + L} * L$$

The Sum of E-Pips is 21% of the Trailers "pips to the 6" minus the Leaders "pips to the 6" plus 0.5 for each extra crossover.

Acknowledgements

This is by far been the most comprehensively satisfying piece of work ever undertaken. Its development acknowledges the writers/authors, backgammon players and the backgammon community in general for their studious approach to backgammon; for their ideas and mathematical/statistical know-how has undoubtedly been a major influence. To Kit Woolsey for the Ultimate Pip Count; his ideas and insight marked the beginning. To Tom Keith and his thorough examination of a variety of racing formulae from his article "Cube Handling in Noncontact Positions". To Stick Rice and BGonline, for whom I'm sure, I would have never stumbled upon Nack Ballard's "Nack 57 Rule" and "Nack 58 Rule". Finally, a special acknowledgment to the late Walter Trice who paved the way in so many respects that it is only fitting here his name bears witness.