Game Theory, Scrabble, and Poisons

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Outline

We describe two unexpected applications of game theory to recreational mathematics.

1. A Scrabble puzzle, composed jointly with Scrabble expert Nick Ballard.
   (This puzzle was presented at last year's MOVES Conference. A version of it was published in NASPA News and was the subject of a Will Anderson video.)

2. Michael Rabin's poison puzzle; see the Dec. 2025 issue of *Mathematics Magazine*.

Links to everything may be found at https://timothychow.net/cv.html

Experimental mathematics will make an appearance at the very end of the talk.

Quick Scrabble Refresher

There is a publicly known set of 100 tiles in a bag, each with a letter and a point value, except for two blanks, which are wildcards scoring zero points.

Each player draws a rack of seven random tiles, hidden from the opponent.

Players take turns forming words on the board, scoring points, possibly including bonus points if you play a tile on a bonus square. If you play all seven tiles at once, then you **bingo**, scoring 50 extra points.

Regardless of how many tiles you play, at the end of your turn, you draw tiles randomly from the bag so that you again have seven tiles on your rack, unless there are not enough tiles in the bag, in which case you draw as many tiles as you can.

If the bag is empty at the start of your turn and you play all your remaining tiles, then you "go out," ending the game, and collecting extra points from your opponent's unplayed tiles.

An UNUSUAL Scrabble Puzzle (Joint Work with Nick Ballard)

You just bingoed with GROGSHOP, but your opponent immediately bingoed back with UNUSUAL, taking a lead of 476 points to your 344 points.

You hold the tiles **MONKYSZ**.

The tiles unseen to you are **DEFJLLLQW**, two of which are in the bag, and seven of which are on your opponent's rack.

Can you possibly avert a loss if your opponent plays perfectly? What's your best play?

Three Preliminary Comments

1. For those who do not play tournament Scrabble: Yes, GROGSHOP is a word (and so are SETA, AMRIT, CABER, RICHWEED, EPIGEOUS, etc.), according to either the NASPA Word List 2023 or Collins Scrabble Words 2024, the two main lexicons currently in use. Our puzzle assumes the 2023 NASPA World List.

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3. We assume that a win is worth $+1$, a loss is worth -1, and a tie is worth 0. The margin of victory ("spread") is irrelevant.

Key Idea 1: Fish for MONKEYS

(Your rack: **MONKYSZ**. Unseen tiles: **DEFJLLLQW**.)

Trailing by 132 points, you need to bingo. You have no bingo, but you almost have **MONKEYS**. You can **set up** a winning play of **MONKEYS** down column O by playing **DITZ** on row 8, hoping to draw the **E**, which will win even if your opponent plays **JAB**.

Key Idea 2: Phantom Threats

(Your rack: **MONKYSZ**. Unseen: **DEFJLLLQW**.)

Skeptic: Upon seeing **DITZ***, wouldn't your opponent sense what you're up to, and block your play of* **MONKEYS***, e.g., by playing* **JELL** *at N2?*

Ah, but if you not only draw **E** but also leave **D** as the last tile in the bag, then your opponent might also worry that you have **DONKEYS**. And **JELL** followed by **DONKEYS** leads to 508–508 tie!

DONKEYS is a "phantom" threat; you cannot execute it, but your opponent doesn't know that.

Key Idea 3: Bluffing

(Your rack: **MONKYSZ**. Unseen: **DEFJLLLQW**.)

Skeptic: I'm still not convinced. **MONKEYS** *is a bigger threat than* **DONKEYS** *so your opponent will surely block* **MONKEYS**.

Okay, so what if you play **DITZ** down column H? By the skeptic's logic, your opponent will block the "bigger threat" of **DONKEYS** with **JELL** at 14B, allowing you to play **MONKEYS** for a 511–511 tie!

Playing **DITZ** at H11 is a **bluff**; no matter what you draw, you will not be able to execute your phantom threat of **DONKEYS**.

Is Bluffing with DITZ at H11 the Best Play?

What does "best play" even mean?

If you play **DITZ** at H11, is **JELL** at 14B (blocking the "bigger threat") your opponent's "best play"? But it only ties, whereas if your opponent suspects a bluff and **second-guesses** you by playing **JELL** at N2, then you lose.

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The **minimax theorem** for two-person zero-sum games says that there always exists a **mixed** (i.e., randomized) strategy for you and a mixed strategy for your opponent that together form a **Nash equilibrium**. By definition, in an equilibrium, neither player can profit by deviating unilaterally.

The equilibrium is not necessarily unique, but in a two-person zero-sum game, all equilibria yield the same expected payoffs for both players.

The Equilibrium Strategies

With a rack of MONKYSZ (or MNKEYSZ, MOKEYSZ, MONEYSZ, MONKESZ, MONKEYZ), play **DITZ** at H11 with probability $\frac{2}{3}$ and at 8K with probability $\frac{1}{3}$.

With a rack of DONKYSZ (or DNKEYSZ, DOKEYSZ, DONEYSZ, DONKESZ, DONKEYZ), play **DITZ** at 8K with probability $\frac{2}{3}$ and at H11 with probability $\frac{1}{3}$.

In other words, bluff with probability $\frac{2}{3}$ and set up with probability $\frac{1}{3}$.

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If your opponent sees **DITZ** at 8K, she should play **JELL** at N2 with probability $\frac{2}{3}$ and at 14B with probability $\frac{1}{3}$.

If your opponent sees **DITZ** at H11, she should play **JELL** at 14B with probability $\frac{2}{3}$ and at N2 with probability $\frac{1}{3}$.

In other words, she should block with probability $\frac{2}{3}$ and second-guess with probability $\frac{2}{3}$.

This is the first ever explicit Scrabble position where the best strategy is mixed.

Constructing the Puzzle was Harder Than It Looks!

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- Arranging for the bluff/block variations to be exactly tied was delicate.
- To calculate the opponent's equilibrium probabilities, all of your possible racks need to be analyzed; e.g., if, from one of those racks, you would have had a play better than **DITZ**, then your opponent can infer that you didn't have that rack.
  - With a rack of DONKESZ, you can bingo immediately with **ZEDONKS** at 15A. But it turns out that this never ties or wins.

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  - With a rack of ZONKEYS, both **DITZ** plays would be equally attractive, spoiling the $(\frac{2}{3}, \frac{1}{3})$ ratio. However, **ZONKEYS** is a valid word, so we ensured that playing **ZONKEYS** at N8 immediately would be better than either **DITZ**.

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  - The possible bingos **DOLMENS**, **DONZELS**, **ENFOLDS**, **FONDLES**, **KNOLLED**, and **MENFOLK** threaten *cooks* (unintended alternative lines of play). We needed several spots (3E, 9I, B10) where the Q could be played.

And now for something (almost) completely different...

Michael Rabin's Lateral Thinking Puzzle: The Setup

1. In Poisonworld, a healthy person who ingests a poison will die within an hour unless he or she ingests a stronger poison; the stronger poison restores complete health.
2. There are two distinct types of poisons: magical and medical, which are dispensed by the Royal Magician and the Royal Physician respectively. No poison is both magical and medical.
3. All poisons in this world are strictly linearly ordered in strength. In particular, no magical poison has exactly the same strength as a medical poison.
4. All these facts are public knowledge.

Michael Rabin's Lateral Thinking Puzzle: The Crisis

The king of Poisonworld wants to obtain the strongest poison, so he summons the Royal Magician and the Royal Physician and says:

> *Each of you must return here to my royal chambers at noon one week from now. Bring a vial of your strongest poison.*
>
> *Each of you must drink from the other's vial first, and then drink from your own vial. The person who has the stronger poison will survive, and the other will die.*
>
> *If I detect any attempt to circumvent these rules, then you will both be executed. You may go now, but you must return at the specified time.*

Michael Rabin's Lateral Thinking Puzzle: The Denouement

The Royal Servants desperately rack their brains all week. Neither wants to die. Each knows that the other has some very strong poisons.

They return at the appointed time and follow the protocol exactly.

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They return at the appointed time and follow the protocol exactly.

Within an hour, they both keel over and die from poisoning.

What happened?

Michael Rabin's Lateral Thinking Puzzle: The Intended Solution

Each Royal Servant drank a weak poison just before arriving at the showdown, and, instead of bringing a strong poison as requested, brought water. They drank the other Servant's water, then drank their own water, and died of their own poison.

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But why? The Magician was hoping the Physician would not think of the same trick and would bring a strong poison.

Then the Physician's strong poison would cure the Magician of the Magician's own weak poison, and the Magician would live, whereas the Physician would die from drinking the Physician's own poison.

The Physician thought similarly, so both died.

Cooking the Puzzle: Sicilian Reasoning

Strategy C (for Conventional): Drink nothing in advance; bring a strong poison.

Strategy A (for Advanced): Drink a weak poison in advance; bring water.

If you suspect your antagonist will adopt Strategy A, can you still win?

Cooking the Puzzle: Sicilian Reasoning

Strategy C (for Conventional): Drink nothing in advance; bring a strong poison.

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If you suspect your antagonist will adopt Strategy A, can you still win? **Yes!**

Strategy B (for Blank): Drink nothing in advance; bring water.

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Strategy A (for Advanced): Drink a weak poison in advance; bring water.

If you suspect your antagonist will adopt Strategy A, can you still win? **Yes!**

Strategy B (for Blank): Drink nothing in advance; bring water.

If you suspect your antagonist will adopt Strategy B, can you still win?

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Strategy A (for Advanced): Drink a weak poison in advance; bring water.

If you suspect your antagonist will adopt Strategy A, can you still win? **Yes!**

Strategy B (for Blank): Drink nothing in advance; bring water.

If you suspect your antagonist will adopt Strategy B, can you still win? **Yes!**

Strategy D (for Double Dose): Drink a weak poison in advance; bring a strong poison.

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Strategy B (for Blank): Drink nothing in advance; bring water.

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Strategy D (for Double Dose): Drink a weak poison in advance; bring a strong poison.

Alternative ways both could die:

1. B versus C
2. C versus D, where the C poison has intermediate strength
3. D versus D, where each weak poison is weaker than both strong poisons

Does Game Theory Tell Us What the Royal Servants *Should* Do?

If one Servant survives and the other dies, score 1 for the former and 0 for the latter.

If neither Servant dies of poisoning, they both score 0 (capital punishment!).

This is no longer a zero-sum game, but a Nash equilibrium must still exist.
However, there may be multiple equilibria with different expected payoffs.

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A Special Case: Assume that both Servants have exactly two poisons, and the strength rankings are MPMP or MPPM or PMPM or PMMP (but not PPMM or MMPP).

Assume also that each Servant is equally likely to have the strongest poison.

A Special Case, continued

The payoff matrices are ("Row" player on left, "Column" player on right):

	A	B	C	D
A	0	0	1	0
B	1	0	0	0
C	0	0	1/2	1/2
D	0	1	0	0

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Nash equilibria:

1. Both Servants choose a strategy uniformly at random.
2. One Servant chooses B or C with probability 1/2; the other chooses A with probability 1/3 and D with probability 2/3.

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   In the first case, each Servant has a survival probability of 1/4.

   In the second case, the B/C Servant has a survival probability of 1/3 and the A/D Servant has a survival probability of 1/2.

Take a Cue from Axelrod?

In 1980, Robert Axelrod set up a multi-player computer tournament for the **iterated prisoner's dilemma**.

Higher payoffs yielded higher "reproductive fitness," and the famous **tit-for-tat** algorithm unexpectedly emerged as a highly successful competitor.

Axelrod's work spawned a hugely fruitful line of research. What insights might a "Choose Your Poison" tournament yield? The situation is ripe for experimental exploration!