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Solutions to "Ein Kleines Schach"

Problems on page 49

TIMOTHY CHOW

Solution to Problem 1

One first verifies that White is not in check from the Black queen, and so is not constrained by having to parry a check. 1.Ba5 is checkmate; all the flight squares of the king are covered, and the queen cannot capture the bishop. Trying to interpose the queen also does not work, because the bishop is checking from two opposite directions.

Closer analysis reveals that the bishop's moves enjoy a symmetry about the horizontal axis of the board; that is, barring obstructions, if the bishop can move to a certain square X on the board, then it can also move to the mirror image of X in the horizontal axis. It follows from this symmetry that 1.Ba4 is also checkmate.

Solution to Problem 2

Black plays Ka6, White plays Nf2, Black plays Kb7, and White checkmates with Nh1.

On an ordinary chess board, knight moves have parity; that is, if a knight can reach a certain square in an even number of moves, then it cannot also

reach the same square in an odd number of moves. Chess players say that a knight "cannot lose a tempo." However, on a Klein bottle, this is not true, because there are odd cycles (in the graph whose edges connect squares that are a knight's move apart), as this problem illustrates.

Note that pawns "reverse direction" when they cross the left or right edge of the board, and that the standard starting position in chess is illegal on a Klein bottle. This raises some questions about the proper rules for pawn promotion, double moves, and *en passant*, which we have glossed over in this problem, even though pawns appear. Fortunately, almost any reasonable way of settling these questions leaves the solution unchanged, as long as one stipulates (as we did) that Black's pawns are moving down the board and White's pawn is moving up the board in the set position.

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