A Mathematician Reads the Kalam Cosmological Argument

Timothy Y. Chow

When I was an undergraduate, I had an interest in Christian apologetics, and one of the books I studied was by William Lane Craig [6]. Reading through the early chapters, I felt that it was one of the better books of the genre—until I came to the section on something Craig called the *kalam cosmological argument* for the existence of God. When I read the argument, I cringed. Not only did the argument strike me as fallacious; it seemed *obviously* fallacious, or at least obviously fallacious to anyone with a certain amount of background in mathematics and physics. I finished reading the book anyway, but was left feeling that the section on the kalam cosmological argument was a huge flaw in an otherwise decent book.

Recently, the kalam cosmological argument came to my attention again, and out of curiosity, I searched the literature for critiques. I found many, but to my surprise, nobody seemed to have articulated the objection that had seemed so obvious to my younger self. I decided to write out my objection in some detail, and I shared the draft with about a dozen people with some knowledge of the subject. What I found striking was that the best predictor of whether someone would agree with me was not their religious beliefs, but their academic background. Physicists and mathematicians generally agreed with me and did not think I was saying anything particularly controversial, whereas philosophers would balk and raise what I considered to be peculiar counterobjections. There seems to be some kind of culture gap or language gap; the hard-earned wisdom acquired by the mathematical and scientific communities through decades if not centuries of experience is failing to be transmitted to the philosophical community. I find this culture gap troubling, so I am publishing this essay with the hope that I can help bridge it.

But I am getting ahead of myself. Just what is the kalam cosmological argument anyway?

The Kalam Cosmological Argument

In apologetics, the term *cosmological argument*, or argument for the existence of God from first cause, refers to an argument with something like the following structure:

1. Whatever begins to exist has a cause of its beginning.
2. The universe began to exist.
3. Therefore, the universe has a cause of its beginning.

Since 1979, William Lane Craig (as well as other Christian apologists, but for brevity I will use Craig's name synecdochically throughout this article) has vigorously championed a version of the cosmological argument called the *kalam cosmological argument*. The literature on the kalam cosmological argument is remarkably large and continues to grow, with several entire books, some quite recent, devoted to it [4, 5, 11, 14–16].

The kalam cosmological argument itself has many subvariants, but in this paper, I will focus on a specific version, which argues for the second assertion above (the universe began to exist) as follows.

(A) An actual infinite cannot exist (in the physical world).
(B) An infinite temporal regress is an actual infinite.
(C) Therefore, an infinite temporal regress of events cannot exist.

Even more specifically, I want to criticize Craig's argument for assertion A, that *an actual infinite cannot exist* in the physical world. This claim is perhaps the feature that most clearly distinguishes the kalam cosmological argument from other cosmological arguments.

We will examine some of Craig's arguments in detail later, but for now, it is important to notice that Craig does not just argue that an actual infinite *does not* exist; he argues that an actual infinite *cannot* exist. In other words, its existence is *impossible*. Furthermore, the sense of impossibility that Craig intends here is *metaphysical impossibility*. That is, according to Craig, the problem with the existence of an actual infinite is not that it is *logically* absurd or that it contradicts experimental observations; the problem is that it violates some metaphysical principle that Craig considers to be inviolable.

The astute reader may have already guessed where I am headed. Scientists know from bitter experience that insisting that the physical world must conform to some metaphysical principle that our inner voice tells us must be true, even when no logical argument or empirical evidence requires it, has been shown time and time again to be a rash and highly unreliable approach to physics. This, in a nutshell, is why I find Craig's argument that an actual infinite *cannot* exist to be completely unconvincing.

The plan of this essay is as follows. First, as a preliminary step, I will clarify different senses of impossibility—logical, physical, and metaphysical. I will then recall some famous historical examples of failed attempts to argue on the basis of metaphysical impossibility; the scientific

https://doi.org/10.1007/s00283-022-10224-8

community has learned the hard way to shun this type of metaphysical argument. After laying this groundwork, I will examine some of Craig's arguments specifically. It will, I hope, become apparent that his supposedly inviolable metaphysical principles are even more flimsy and unconvincing than the principles that great thinkers of the past, including Einstein, mistakenly thought were mandatory.

Logical and Physical Impossibility

As mentioned already, Craig is insistent that the existence of an actual infinite is not just *false* but is *impossible*. But what does it mean to say that something is "impossible"?

One way that something can be impossible is for it to be *logically* impossible or *mathematically* impossible.[1] For example, one could claim that the very concept of an actual infinite—that is, a completed infinite totality of objects—is incoherent or logically self-contradictory. In particular, one could claim that Zermelo–Fraenkel set theory (ZF for short, or ZFC if one includes the axiom of choice), a widely accepted axiomatization of most of modern mathematics that among other things postulates the existence of an infinite set, is inconsistent.

Craig has made it clear that he is not claiming that an actual infinite is logically impossible, and in particular, he accepts that ZF is consistent. In this regard, Craig is more generous than some other critics of the actual infinite. Until the work of Georg Cantor in the nineteenth century, most thinkers accepted only something they called *potential infinity* and rejected actual infinities as being conceptually suspect and maybe even logically incoherent.[2] Even today, there are skeptics who do not take for granted that ZF is consistent. Such doubts are interesting in their own right, but we will set them aside, because Craig does not raise them. He accepts that actual infinities are conceptually coherent, and is concerned only with their physical existence. After all, even if something is logically possible, it might be *physically* impossible.

It is harder to define physical impossibility precisely than it is to define mathematical impossibility precisely, but here is a definition that should suffice for our present purposes. Let us use the term *physical theory* for a mathematical model of the physical world that makes empirically testable predictions about the physical world. Let us then say that a physically observable event or circumstance X is physically impossible if our best physical theories predict that X will never happen. For example, let X be the transmission of information faster than the speed of light. Then X is logically possible, and there is a clear sense in which we could observe X happening. For example, in 2011, there was a high-profile experiment that at first (before flaws were discovered in the experimental setup) seemed to show that neutrinos traveled faster than light from CERN to the Gran Sasso National Laboratory in Italy. However, our best mathematical models of the physical world predict that information cannot travel faster than the speed of light. Similarly, our best physical theories of statistical mechanics and thermodynamics predict that any machine we build will never exhibit perpetual motion—or at least that the probability of its doing so is absurdly small.

Notice carefully a couple of features of this definition of physical impossibility. First, a clear distinction is made between our mathematical models of the physical world and the physical world itself. This distinction is a hallmark of all modern science. In particular, physical impossibility as we have defined it is not directly predicated of the physical world itself. Arguably, the real world just is the way it is, and if so, it is not totally clear what it would mean for something that is *not true* in the real world to be *possible*. On the other hand, in a mathematical model of the physical world, we can hypothesize a variety of circumstances and give a clear meaning to possibility and impossibility.

Second, a clear distinction is made between the mathematical model and its physical predictions. A mathematical model might have all sorts of exotic mathematical features—including the presence of infinite sets—but not all such features necessarily yield observable predictions, and they may or may not correspond directly to features of the physical world. It is only the model's predictions about the physical world that necessarily have physical meaning. In particular, I would defend Craig against one of his critics, Quentin Smith [19], who says that "Big bang cosmology implies that there is an actually infinite manifold, topology, and metrication." The usual *mathematical model* of big bang cosmology does indeed contain such actually infinite things, but even if we accept the model, we are not automatically committing ourselves to asserting that the *physical world* contains actual infinities. Craig's arguments against an actual infinite cannot be refuted so cheaply.

Metaphysical Impossibility?

Craig certainly believes that an actual infinite is physically impossible. However, as already mentioned, Craig argues for something stronger—namely that an actual infinite is *metaphysically* impossible, and it is this aspect of Craig's arguments that I take issue with. But what does it mean for something to be metaphysically impossible?

The exact meaning of metaphysical impossibility is a notoriously controversial philosophical issue, but roughly speaking, it means a kind of impossibility that is not sensitively dependent on accidental or contingent facts about our current mathematical models or the physical universe we happen to live in, but holds for more general reasons.

[1]There is a philosophy of mathematics known as logicism that equates mathematics with logic. I do not necessarily endorse logicism, but I believe that the distinction between logical possibility and mathematical possibility does not matter for the present discussion.

[2]By contrast, Cantor himself believed not only that actual infinities exist abstractly, but "in the natura creata an Actual Infinite must be confirmed, as for example with respect to, in my strong conviction, the actual infinite number of created individual beings, not only in the universe but also already on our earth" [2].

At minimum, if something is metaphysically impossible, then not only must it be impossible according to the physical theories that we currently deem best; it must also be impossible according to any possible physical theory that we might someday deem best, and it must also be physically impossible in conceptually coherent hypothetical universes other than the actual universe.

Suppose we grant that metaphysical impossibility is a meaningful concept. Then the following strategy for arguing that X is metaphysically impossible—and a fortiori physically impossible—is tempting and, as we shall see later, is exactly how Craig argues.

1. Decide a priori on some metaphysical principle P that one feels must be obeyed by any satisfactory physical theory.
2. Show that X is incompatible with P, using abstract argumentation only and not the results of any physical observation or experiment.
3. Conclude that X is absurd or impossible.

Unfortunately, as many historical examples illustrate, the above strategy has an embarrassingly bad track record.

For our first example, let us recall Giovanni Girolamo Saccheri, a Jesuit priest who, in the eighteenth century, published what is now regarded as a mathematical investigation into non-Euclidean geometry [18]. Saccheri considered what would happen if we were to reject the fifth postulate of Euclidean geometry and instead were to suppose that the sum of the angles of a triangle were less than 180 degrees. Saccheri failed to deduce any mathematical contradiction, yet he famously declared in Proposition XXXIII, "*Hypothesis anguli acuti est absoluta falsa; quia repugnans naturae lineae rectae*." (The acute angle hypothesis is absolutely false, because it is repugnant to the nature of the straight line.)

It is important to remember that at the time, Euclidean geometry was regarded not just as a purely logical construction, but as a mathematical model of physical space. Therefore, Saccheri was in effect declaring that the phenomena of non-Euclidean geometry not only *were* not, but *could not be*, part of a valid physical theory. There is some debate about whether Saccheri, in his heart of hearts, truly believed that non-Euclidean geometry was absurd, or whether he was saying such things to protect himself from anticipated criticism, but either way, we have here a clear example of a declaration of impossibility based purely on a metaphysical prejudice about what "the nature of a straight line" must be, and not on any logical inconsistency or experimental tests. Today, of course, not only do we know that non-Euclidean geometry is perfectly consistent mathematically, but we have good reasons to believe that the actual geometry of spacetime is non-Euclidean.

Our second example concerns an old and (at one time) widely believed metaphysical principle that if something, such as light, exhibits wave behavior, then it must be supported by some kind of medium. Indeed, it seemed part of the very concept of a wave that it was a disturbance in an underlying medium. In the case of light, it was hypothesized, on the basis of this metaphysical principle and not on any direct experimental evidence, that there existed such a medium, called the *luminiferous aether*. But after the famous Michelson–Morley experiment and the development of Einstein's theory of relativity, the aether hypothesis was eventually discarded, and with it, the claim that it was impossible for a wave to propagate through a vacuum.

For our third example, let us recall that Albert Einstein [9] famously rejected quantum mechanics, saying:

> Quantum mechanics is very impressive. But an inner voice tells me that it is not yet the real McCoy. The theory yields much, but it hardly brings us closer to the Ancient One's secrets. I, at any rate, am convinced that he does not throw dice.[3]

Along with Podolsky and Rosen [10], Einstein proposed a thought experiment that we would nowadays describe as exhibiting quantum entanglement. For example, one can prepare an entangled electron–positron pair[4] such that (1) each particle's spin when measured along (say) the x-axis is equally likely to be "spin up" or "spin down," and (2) when measured along the same axis, the positron's spin is always the opposite of the electron's spin, even if the two particles are separated so far apart that they cannot communicate their "decisions" to each other without violating the speed-of-light barrier. This state of affairs is unsurprising if there is a "hidden variable" that determines the state of the system before the particles are separated and measured, but it seems absurd if the spins are indeterminate. Einstein, being committed to a metaphysical principle of realism that required quantum-mechanical observables such as spin to have a definite value at all times, felt that this thought experiment showed that quantum mechanics had to be incomplete. However, later work by Bell, Aspect, Kochen, Specker, and others has shown that quantum entanglement is real, and that the kind of local hidden variable theory that Einstein hoped for is inconsistent with known experimental facts.[5] It is Einstein's metaphysical principle that has been shown to be dubious.

It would be easy to give more examples from modern physics that might seem metaphysically impossible—light behaving like a wave as well as a particle, identical twins aging at different rates simply because one of the twins has made a very speedy round trip to a distant location (the twin paradox of relativity theory), the universe possibly having ten dimensions, and so on. Physicists have learned the hard way to obey what we might dub a "prudence principle":

[3] "Die Quantenmechanik ist sehr achtunggebietend. Aber eine innere Stimme sagt mir, daß das noch nicht der wahre Jakob ist. Die Theorie liefert viel, aber dem Geheimnis des Alten bringt sie uns kaum näher. Jedenfalls bin ich überzeugt, daß der nicht würfelt." I have borrowed the colorful translation "the real McCoy" from Ryckman [17, p. 157], who also suggests that *der wahre Jakob*, in addition to being a biblical reference, may be an allusion to a satirical German newspaper of that name.

[4] The observables in the original Einstein–Podolsky–Rosen paper were position and momentum, but the phenomenon is easier to describe using discrete observables such as spin.

[5] For a thorough discussion of this point, see Wiseman and Cavalcanti [21], and for experimental evidence, see Hensen et al. [12].

> If a physical theory is consistent with known experimental and observational facts, then do not reject it out of hand purely because it contradicts some intuitively plausible metaphysical principle.

After all, if Einstein's intuition was fallible, who dares claim to be immune?

If you propose an unorthodox new theory, scientists will naturally react skeptically, but if the only thing your theory contradicts is some metaphysical principle and not any mathematical calculation or experimental fact, then they will probably not dismiss the theory as impossible a priori. Instead, their instinct will be to ask whether you can design an experiment to test your theory and mathematically calculate the predicted outcome. Whenever possible, scientists gravitate toward mathematics and/or experiment and not metaphysics.

The prudence principle embodies a certain kind of skepticism of metaphysical dogmatism, and may remind the reader of logical positivism or of Karl Popper's famous concept of *falsifiability*, but there is a crucial difference. For example, suppose you have a favorite metaphysical principle P. Popper would reject P as unscientific unless you could articulate some conceivable experimental outcome that would refute it. On the other hand, the prudence principle makes the weaker claim that you should not insist on P to the point where you declare the alternative to be absurd. While falsifiability is widely accepted by scientists, it is not entirely uncontroversial; for example, the physicist Sean Carroll has criticized it. However, even Carroll accepts (a version of) the prudence principle. Regarding certain theories that are arguably unfalsifiable, Carroll [3] has written:

> Refusing to contemplate their possible existence on the grounds of some a priori principle, even though they might play a crucial role in how the world works, is as non-scientific as it gets.

Far from a blanket rejection of metaphysics, the prudence principle is a call for metaphysical tolerance, and not a call to positivism or scientism. It should of course not be thought of as an ironclad rule, but it represents hard-won wisdom, and I maintain that a heavy burden of proof rests on anyone who wants to violate it. A metaphysical principle had better be extremely compelling if we are going to use it to reject physical theories a priori. As we shall soon see, none of Craig's metaphysical principles comes remotely close to qualifying.

Hilbert's Hotel

Let us now turn to one of Craig's favorite arguments against the existence of an actual infinite, namely Hilbert's hotel, a thought experiment due to the German mathematician David Hilbert. Most readers of the *Intelligencer* will be familiar with Hilbert's Hotel, but for the sake of completeness, let us review the setup. We imagine a hotel with a countably infinite number of rooms, labeled 1, 2, 3, etc. Suppose that every room is occupied. Now a countably infinite number of new guests show up. The proprietor accommodates them all by shifting the person in room n to room $2n$ for all n, thereby vacating infinitely many rooms for the new guests. Craig and Sinclair [7] continue the narrative as follows.

> But Hilbert's Hotel is even stranger than the German mathematician made it out to be. For suppose some of the guests start to check out. Suppose the guest in room #1 departs. Is there not now one fewer person in the hotel? Not according to infinite set theory! Suppose the guests in rooms #1, 3, 5, … check out. In this case an infinite number of people has left the hotel, but by Hume's Principle, there are no fewer people in the hotel. In fact, we could have every other guest check out of the hotel and repeat this process infinitely many times, and yet there would never be any fewer people in the hotel. Now suppose the proprietor does not like having a half-empty hotel (it looks bad for business). No matter! By shifting guests in even-numbered rooms into rooms with numbers half their respective room numbers, he transforms his half-vacant hotel into one that is completely full. In fact, if the manager wanted double occupancy in each room, he would have no need of additional guests at all. Just carry out the dividing procedure when there is one guest in every room of the hotel, then do it again, and finally have one of the guests in each odd-numbered room walk next door to the higher even-numbered room, and one winds up with two people in every room!
>
> One might think that by means of these maneuvers the proprietor could always keep this strange hotel fully occupied. But one would be wrong. For suppose that the persons in rooms #4, 5, 6, … checked out. At a single stroke the hotel would be virtually emptied, the guest register reduced to three names, and the infinite converted to finitude. And yet it would remain true that as many guests checked out this time as when the guests in rooms #1, 3, 5, … checked out! Can anyone believe that such a hotel could exist in reality?
>
> Hilbert's Hotel is absurd. But if an actual infinite were metaphysically possible, then such a hotel would be metaphysically possible. It follows that the real existence of an actual infinite is not metaphysically possible.

Again, most readers of the *Intelligencer* probably do not need me to point out the flaws in the above reasoning, but let us go ahead and spell some of them out. First of all, as other critics have noted, the claim that "if an actual infinite were metaphysically possible, then such a hotel would be metaphysically possible" is obscure. By analogy with the concept of NP-completeness in computational complexity theory, we might rephrase Craig and Sinclair's claim as a claim that Hilbert's hotel is *AI-complete* (where "AI" stands for "Actual Infinite"), i.e., that the impossibility of an actual infinite reduces to the impossibility of Hilbert's hotel. But why is Hilbert's hotel AI-complete? Craig and Sinclair do not explain this point. In fact, it looks as though they commit an error that students of complexity theory often make, which is to get the direction of the reduction backward. If all actual infinities are impossible, then in particular, Hilbert's hotel is impossible; that much is obvious. But the converse is not at all obvious.

But let us not dwell on this objection, since a more important question is, what justifies the conclusion that Hilbert's hotel is absurd?

There does seem to be something absurd about having one person leave and yet not having one fewer person. But this absurdity comes not from infinite set theory, as Craig and Sinclair claim, but from a linguistic quibble. If by *fewer* we mean *strictly lower cardinality*, then indeed, removing a single element from an infinite set does not lower its cardinality, but there is nothing absurd about that. On the other hand, if A and B are sets and $A \subseteq B$, then we could choose to define "A has one fewer member than B" by $|B \setminus A| = 1$, where $\setminus$ denotes set difference. With that definition, the set of guests after the departure does indeed have one fewer member than the set of guests before the departure. Any appearance of absurdity evaporates as soon as one clearly defines one's terms.

The other observations that Craig and Sinclair make about Hilbert's hotel may seem strange to some, but where is the absurdity? To those who work with infinite sets on a daily basis, everything in the account seems normal, and nothing in sight seems absurd.

As mentioned earlier, Craig and Sinclair try to state more explicitly what they think is absurd by stating metaphysical principles that they regard as inviolate. For example, one metaphysical principle P that Craig and Sinclair propose is that physical quantities (i.e., numbers of physically existing entities) must obey the law that equal quantities can always be subtracted from equal quantities and the resulting quantities must always be equal. There are different ways to formulate P precisely; one possibility is this:

> P: If $A_1 \subseteq B_1$ and $A_2 \subseteq B_2$ and $|A_1| = |A_2|$ and $|B_1| = |B_2|$, then $|B_1| - |A_1|$ and $|B_2| - |A_2|$ always exist and are equal to each other and to $|B_1 \setminus A_1|$ and $|B_2 \setminus A_2|$.

It is indeed true that P does not necessarily hold for infinite sets. Craig and Sinclair not only maintain that P is plausibly true for physical quantities; they make the far stronger claim that $\neg P$, the negation of P, is absurd. But why? Why should we believe in P so strongly for physical quantities that we should rule out a priori any physical theory that violates it, even when no logical principle or experimental fact is contradicted?

My objection here is not entirely new. Other critics have pointed out that $\neg P$ just comes with the territory of infinite sets and is not the absurdity that Craig claims it is. Craig's reaction has been to say that his critics do nothing to address the apparent absurdity of $\neg P$ for physical quantities. But Craig misplaces the burden of proof. The burden of proof is rather on Craig to demonstrate that $\neg P$ is so repugnant to the nature of the universe that it justifies throwing caution, in the form of the prudence principle, to the winds.

Another metaphysical principle suggested by Craig and Sinclair is that "it is ontologically absurd that a hotel exist which is completely full and yet can accommodate untold infinities of new guests just by moving people around." Once again we are led to ask, what exactly is so absurd about it? It seems far easier to swallow than quantum entanglement or the twin paradox. The only argument that Craig and Sinclair present is an argument from incredulity.

To make matters worse, it is not even clear that Hilbert's hotel is *physically* impossible, let alone metaphysically impossible. Of course, we human beings are finite creatures and could not possibly build Hilbert's hotel, nor could we engineer infinite evacuations and reassignments. However, the question is not whether we humans have the capacity to carry out infinite operations, but whether Hilbert's hotel could exist. To put it another way, if we were to send out a space probe and it were to stumble upon what looked like the beginning of Hilbert's hotel floating in interstellar space, is there anything in our current best physical theories that would demand that the hotel be finitely long?

The answer seems to be no. Now, if it were the case that our current best physical theories predicted that the universe were spatially finite (e.g., that its spatial topology were like that of a three-dimensional torus), then they would also predict that any sufficiently long (non-self-intersecting) hotel would fill up the universe. In particular, Hilbert's hotel would be physically impossible. However, the current scientific consensus is that the spatial finitude of the universe is an open question,[6] so the physical impossibility of Hilbert's hotel cannot be deduced in this manner. Let us note also that if a spatially infinite universe were really as absurd as Craig and Sinclair claim, it is curious that astrophysicists seem not to have noticed this absurdity and continue to regard it as a viable theory.

Benardete's Book

Here is a thought experiment by José Benardete [1] that Craig likes to cite as an argument against an actual infinite.

> Here is a book lying on the table. Open it. Look at the first page. Measure its thickness. It is very thick indeed for a sheet of paper—1/2 inch thick. Now turn to the second page of the book. How thick is this second sheet of paper? 1/4 inch thick. And the third page of the book, how thick is this third sheet of paper? 1/8 inch thick, &c. ad infinitum. We are to posit not only that each page of the book is followed by an immediate successor the thickness of which is one-half that of the immediately preceding page but also (and this is not unimportant) that each page is separated from page 1 by a finite number of pages. These two conditions are logically compatible: there is no certifiable contradiction in their joint asser-

[6]See, for example, Levin et al. [13]. More recent calculations in a similar vein have yielded the same qualitative conclusion.

> tion. But they mutually entail that there is no last page in the book. Close the book. Turn it over so that the front cover of the book is now lying face down upon the table. Now—slowly—lift the back cover of the book with the aim of exposing to view the stack of pages lying beneath it. There is nothing to see. For there is no last page in the book to meet our gaze. [Emphasis in original.]

Benardete's book has a feature that Hilbert's hotel lacks. Namely, there is an obvious reason why Benardete's book is *physically* impossible: according to our best physical theories, in particular the so-called standard model of particle physics, a sheet of paper cannot be arbitrarily thin. However, remember that Craig wishes to argue for more than the physical impossibility of Benardete's book; he wants to argue that its impossibility does not depend on accidental features of our current best physical theories but is a general fact that must hold of all possible physical theories that we might someday deem best.

But it is totally unclear why Benardete's book could not exist in some hypothetical physical universe that is similar to ours in many respects but in which pages of a book can have arbitrarily small finite thickness. At first glance, it does seem absurd that there would be "nothing to see" if we were to open the back of the book. But let us think more carefully about the situation. In the mathematical model of Benardete's book, there is indeed no last page, but in order to make a physical prediction, we must flesh out some further details. Vision, in our physical universe, is based on electromagnetic radiation in the visible part of the spectrum. Light scatters off a page and some of it enters our eyes. In order to predict what we would see when we lift the back cover of Benardete's book, we need some model of vision in this hypothetical universe. Many possibilities suggest themselves. Perhaps in this universe, there are waves that are somewhat similar in character to light waves, but there is some finite thickness d such that the light waves pass through pages with thickness less than d without interacting, so that what we would see would be the last page whose thickness is at least d. There is nothing absurd in this scenario; there would be infinitely many invisible pages, with no last page, but it would not be the case that there would be "nothing to see."

To get a contradiction, we have to postulate some metaphysical principle—perhaps a claim that the only thing that can cause a light wave to scatter is a specific page, and it cannot scatter off a specific page if there is another page in front of it. But what forces us to adopt this metaphysical principle? Is the argument just that the alternative is repugnant to the nature of a page? Benardete's book is no more bizarre than non-Euclidean geometry or a wave in a vacuum, and we are offered no credible reason to reject it a priori. Ironically, in his essay, Benardete himself argues strongly *against* Craig's belief that we know a priori, apart from all empirical evidence, that something like Hilbert's hotel cannot exist.

Nowacki's Substance-Based Metaphysics

There are various other thought experiments that Craig has cited, but they all have a similar flavor, and in every case the weakness is the same—a contradiction is claimed, but the only thing being contradicted is some metaphysical postulate that Craig finds intuitively obvious, e.g., that the number of orbits that a planet makes must be either even or odd (and therefore cannot be infinite). Insisting on such metaphysical postulates simply because the alternative seems weird exhibits dogmatism of a type that the entire scientific community knows from long experience to avoid.

I said earlier that when I have presented my argument to philosophers, they often balk. I have found that appealing to principles (such as what I have been calling the prudence principle) that are backed primarily by the collective experience of subject-matter experts, rather than by deductive reasoning, is typically not recognized by philosophers as a convincing mode of argumentation. More acceptable to them is to propose a metaphysical principle that seems plausible to philosophers, no matter how unfounded it may seem to expert practitioners of the subject in question (in this case, physicists). This seems to be a cultural gap between the two academic communities that is difficult to bridge.

To be fair, however, not every philosopher who argues for the impossibility of the actual infinite pulls arbitrary metaphysical principles out of thin air and expects others to accept them as obvious. For example, Nowacki [15] is sympathetic to the kalam cosmological argument, but he correctly recognizes that one must carefully articulate and defend the metaphysical assumptions that are being contradicted. I applaud Nowacki for recognizing this point. Unfortunately, I have serious doubts about his suggested metaphysical framework. Nowacki proposes what he calls a *substance-based metaphysics*. It is not entirely clear exactly what Nowacki means by a *substance*, but he does seem to agree with what he calls the commonsense position of Aristotle, "who defined individual substance as what exists without either being predicated of or existing in anything else." Moreover, Nowacki intends substances to be medium-sized objects, such as a lump of clay, and he explicitly rejects reducing substances to subatomic particle physics. Already this is a red flag, since it is far from clear that his substances are even *physically* possible; commonsense experience with medium-sized objects is notoriously unreliable when it comes to the peculiar features of the quantum world such as the Heisenberg uncertainty principle. But let us give Nowacki the benefit of the doubt on this point and consider the thought experiment that he offers as an analogue of Craig's infinite library (which we have not discussed, but which is similar to Hilbert's hotel):

> A *hyperlump* is an actually infinite lump of clay that is composed of a denumerably infinite quantity of different colored handfuls of clay that have been firmly pressed together. … The same operations Craig performs with his actually infinite library have analogs in the hyperlump thought experiment. Thus, instead of a library visitor removing books from the shelves, an artist might approach the hyperlump and remove handfuls of clay. The same difficulties apply as well. For instance, it would not be possible to add a new, numbered handful of clay to the surface of the hyperlump: All available numbers have already been used up in numbering the various handfuls of clay that

constitute the hyperlump. Again, removing handfuls of clay from the hyperlump will result in counterintuitive absurdities. Removing a denumerably infinite number of handfuls of clay from the hyperlump yields varying results: Employing one method results in an infinite quantity of clay remaining; employing another method wipes out the supply of clay almost entirely.

What the hyperlump example does allow us to do, however, is bring out a family of features implicit in Craig's thought experiment that have not been fully explained thus far. The nub of what is at issue is this: since the hyperlump is a substantial body, it must have a surface and hence possess some particular shape. This is because all substantial bodies have surfaces and the surface of a body determines its shape. However, no particular shape we could mention is consistent with the hyperlump. Insofar as all substantially possible substantial bodies necessarily have a different shape, it follows that the hyperlump is substantially impossible.

Confronted with Nowacki's claims that a hyperlump is metaphysically impossible, we can raise exactly the same questions that we raised before: why are we compelled to uphold these claims with such firmness that we must declare any alternative theory impossible? To this, Nowacki does have a possible rejoinder that was not available to Craig; namely, if challenged to explain why a substance must have a particular shape, he can insist that *that is just how substances are*. But now Nowacki has set himself an even more challenging task, which is not only to explain everything we know about the physical world—all the phenomena that we currently explain using quantum field theory, general relativity, etc.—on the basis of substances, but to show, on top of all that, that it is *impossible* to explain the physical universe without adopting the metaphysical premises of substances. Needless to say, Nowacki has not put in anywhere near the amount of effort required to accomplish such a gargantuan task.

Even if we bend over backward to cut Nowacki some slack and grant his theory of substances, there is still a giant loophole in his argument. To arrive at the conclusion that an actual infinite cannot exist, Nowacki must give an AI-completeness proof for hyperlumps. That is, he has to deduce the impossibility of an actual infinite from the impossibility of a hyperlump. Since a hyperlump has lots of special properties, this seems even more challenging than proving AI-completeness for Hilbert's hotel. How would one show that (as an example) the possible existence of an actual infinity of electrons implies the possible existence of a hyperlump? Nowacki gives us no hint as to how such a reduction might go.

An Argument from Occam's Razor

At this point in the discussion, the reader might be persuaded that we cannot rule out a priori the physical existence of an actual infinite, but might still feel uneasy about actively postulating an actual infinite in the physical world. Can we not argue as follows? Even if it is convenient to introduce infinities into our mathematical models of the physical world, when it comes to making testable predictions, we have to make *finitely* testable predictions, for the mundane reason that we human beings are finite creatures with finite resources at our disposal. For example, suppose for a moment that Hilbert's hotel exists out there and suppose that we stumble upon it. Even if Hilbert's hotel is actually infinite, any prediction about Hibert's hotel that we can test can involve only a finite portion of it. In fact, something more is true; if current physical theories are correct, then there is a limit to the size of the observable universe and so there is only a finite portion of Hilbert's hotel that we could ever observe, even in principle. A finite number of observations can always be explained by a finitary theory. So why hypothesize an infinite wing of Hilbert's hotel, or infinitely many invisible pages of Benardete's book, when we can perfectly well explain all our observations with a finitary theory? Occam's razor, which in one popular form states that entities should not be multiplied without necessity, would seem to tell us to avoid postulating the existence of an actual infinite. Yes, maybe postulating an actual infinite is not as absurd and verboten as Craig would have us believe, but if we would never postulate it anyway, what difference does it make?

Formulating physical theories that avoid infinities is something that some physicists do attempt from time to time. For example, Max Tegmark [20] has publicly argued that infinity is not needed in physics, and that we should seek to get rid of it:

> Our best computer simulations, accurately describing everything from the formation of galaxies to tomorrow's weather to the masses of elementary particles, use only finite computer resources by treating everything as finite. So if we can do without infinity to figure out what happens next, surely nature can, too—in a way that's more deep and elegant than the hacks we use for our computer simulations. Our challenge as physicists is to discover this elegant way and the infinity-free equations describing it—the true laws of physics.

Just to be clear, I certainly do not insist that actual infinities be postulated in physical theories. Perhaps one day, Tegmark's dream will be realized, and our best physical theories will avoid actual infinities of all kinds. However, there are two points I would like to emphasize.

First of all, it is by no means clear that Occam's razor requires us to excise the infinite from our theories. Another version of Occam's razor appeals to *simplicity* rather than multiplication of entities. Sometimes, introducing an actual infinity into our mathematical model yields a simpler theory than a theory with some explicit finite bound. Indeed, most of our best physical theories today employ actual infinities inside the mathematical model. As we alluded to earlier, in general relativity, spacetime is modeled as a manifold, which is traditionally thought of as an infinite set of points. Now in principle, we could construct some finitary approximation to a manifold and reconstruct all the calculations needed for experimental predictions without appealing to infinity. However, in practice, physicists and mathematicians often introduce infinities because doing so *simplifies the theory and the calculations*. Finite models are not always simpler, so even if we accept Occam's razor, it remains an open question whether the best and simplest physical theories will involve postulating actual infinities. It is therefore important to keep an open mind, and not rule out infinities a priori, as Craig wants us to do.

Secondly, any argument for finitary physical theories on the grounds of our own finitude is at best an *epistemological* argument and not a *metaphysical* one. That is, such an argument, at best, is that we have no epistemological warrant for postulating an actual infinite, not that we have a positive argument for the metaphysical impossibility of an actual infinite. But for the kalam cosmological argument, Craig needs the actual infinite to be metaphysically impossible; he does not just want to claim that *even if* the universe truly had no beginning then we would not be able to know it (or at least, we would never be able to have confidence in a physical theory that said that the universe had no beginning). So even if Occam's razor drives us to finitary theories, it does not salvage the conclusion that Craig wants.

Concluding Remarks

As far as I have been able to tell, all the "arguments" proposed by supporters of the kalam cosmological argument that an actual infinite cannot exist in the physical world amount to nothing more than an instinctive repugnance that, at bottom, is simply a metaphysical prejudice that is best abandoned, along with all the other failed metaphysical prejudices of the past. But even if I am right about this, I would like to end this essay not on a negative note, but with two positive suggestions.

The first suggestion is that working scientists and mathematicians take time to write down the philosophical principles that they use to guide their own research. These principles often go unstated, but they are valuable and important. Writing them down helps disseminate these principles to more than the few privileged students and colleagues who have firsthand contact with successful senior researchers. What I have been calling the prudence principle is, I believe, widely accepted, but it is the sort of thing that is usually not written down formally. As I said earlier, it is not the kind of principle that most philosophers have been trained to accept as valid, so they are not going to appreciate the hundreds of years of experience lying behind it unless scientists take the time to communicate that distilled wisdom.

My second suggestion is more speculative, and is directed primarily to theists, especially those who are tempted to try to salvage the kalam cosmological argument by rebutting the argument I have given in this essay. All mathematicians have had the experience of trying unsuccessfully to prove a conjecture and then finally coming to the realization that the reason for the elusiveness of the proof is that the conjecture is false. Perhaps the reason all attempts to show that an actual infinite cannot exist have failed is that an actual infinite *can* exist (not necessarily that it *does* exist, merely that it can). In fact, perhaps theists could try to argue for the existence of God not from the *impossibility* of the actual infinite, but from its *possibility*.

The argument that our ability to conceive of infinity is evidence for the existence of God goes back at least to René Descartes's *Meditations on First Philosophy*. Descartes's argument is usually categorized as an *ontological* argument; it does not rely on the physical existence of an actual infinite. Theists could consider going further, and using the possible existence of an actual infinite as an *argument from design* of the universe. Though speculative, such an argument for God would seem to be more promising than desperate attempts to rescue the kalam cosmological argument (at least in the form that I have presented it in this article).

References

[1] J. A. Benardete. *Infinity: An Essay in Metaphysics*. Oxford University Press, 1964.

[2] G. Cantor. On the theory of the transfinite: correspondence of Georg Cantor and J. B. Cardinal Franzelin. *Fidelio* 3:3 (1994), 97–110.

[3] S. Carroll. 2014: What scientific idea is ready for retirement? falsifiability. Available at https://www.edge.org/response-detail/25322.

[4] C. Copan and W. L. Craig, editors. *The Kalām Cosmological Argument. Volume 1: Philosophical Arguments for the Finitude of the Past*. Bloomsbury Academic, 2018.

[5] C. Copan and W. L. Craig, editors. *The Kalām Cosmological Argument. Volume 2: Scientific Evidence for the Beginning of the Universe*. Bloomsbury Academic, 2018.

[6] W. L. Craig. *Apologetics: An Introduction*. Moody Publishers, 1984.

[7] W. L. Craig and J. D. Sinclair. The kalam cosmological argument. In *The Blackwell Companion to Natural Theology*, edited by W. L. Craig and J. P. Moreland. Blackwell, 2009.

[8] J. W. Dauben. Georg Cantor and Pope Leo XIII: mathematics, theology, and the infinite. *Journal of the History of Ideas* 38 (1977), 85–108.

[9] A. Einstein. Letter to Max Born. In *The Born–Einstein Letters*, translated by I. Born. Walker, 1971.

[10] A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete? *Physical Review* 47 (1935), 777–780.

[11] J. Erasmus. *The Kalām Cosmological Argument: A Reassessment*. Springer, 2018.

[12] B. Hensen et al. Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* 526 (2015), 682–686.

[13] J. Levin, E. Scannapieco, and J. Silk. Is the universe infinite or is it just really big? *Physical Review D* 58 (1998), 103516.

[14] A. T. E. Loke. *God and Ultimate Origins: A Novel Cosmological Argument*. Palgrave Macmillan, 2017.

[15] M. R. Nowacki. *The Kalam Cosmological Argument for God*. Prometheus, 2007.

[16] J. M. S. Pearce, J. J. Lowder, and J. East. *Did God Create the Universe from Nothing?: Countering William Lane Craig's Kalam Cosmological Argument*. Onus, 2016.

[17] T. Ryckman. *Einstein*. Routledge, 2017.

[18] G. Saccheri. *Euclides Vindicatus*, 2nd ed., edited and translated by G. B. Halsted. Chelsea, 1986.

[19] Q. Smith. Kalam cosmological arguments for atheism. In *The Cambridge Companion to Atheism*, edited by M. Martin. Cambridge University Press, 2006.

[20] M. Tegmark. Infinity. In *This Idea Must Die*, edited by J. Brockman, pp. 48–51. Harper Perennial, 2015.

[21] H. M. Wiseman and E. G. Cavalcanti. *Causarum Investigatio* and the two Bell's theorems of John Bell. In *Quantum [Un]Speakables II: Half a Century of Bell's Theorem*, edited by R. Bertlman and A. Zeilinger, pp. 119–142. Springer, 2017.

Timothy Y. Chow, Center for Communications Research, 805 Bunn Drive, Princeton, NJ 08540, USA.
E-mail: tchow@alum.mit.edu