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ISSUE NO. 11
OCTOBER 1962

RECREATIONAL MATHEMATICS magazine

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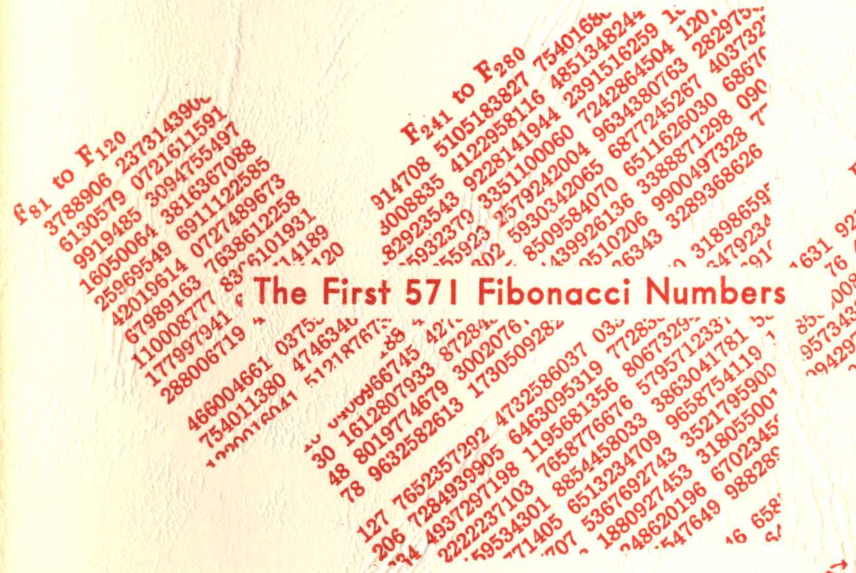

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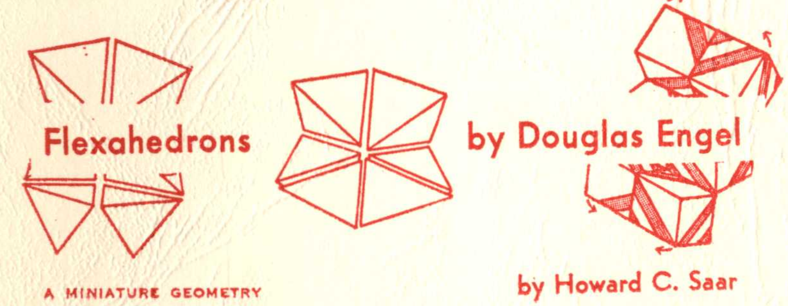

Flexahedrons

by Douglas Engel

A MINIATURE GEOMETRY

by Howard C. Saar

RECREATIONAL MATHEMATICS magazine

BOX 35 KENT, OHIO

OCTOBER 1962 ISSUE NUMBER 11

PUBLISHED AND EDITED BY JOSEPH S. MADACHY

ASSOCIATE EDITOR J. A. H. HUNTER

JUNIOR DEPARTMENT EDITOR HOWARD C. SAAR

ARTICLES

DEPARTMENTS

MISCELLANEOUS

ILLUSTRATIONS

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From the Editor

We humbly apologize for not being able to meet our planned schedule. This October issue is being completed long after we had hoped to have the December issue in your hands.

However, a small note of consolation (?), even while this issue is reaching you the December issue is in the process of being published and it should be mailed out very soon. A vast improvement, considering that the August issue was mailed in November and the October issue in December!

As noted in the *Letters to the Editor* department, we've decided to drop, temporarily, the Cross-Number Puzzles. They are popular with readers, but no more will appear until a different proofreading method has been tried out for this type of puzzle.

The names of puzzle-solvers for the August issue will be published in the December issue and the October issue puzzle-solvers will be published in the February 1963 issue along with the December 1962 solvers. Such short times have elapsed between issues recently that too few persons were able to submit answers. The Editor will establish his own deadline for answer acceptance based on the actual mailing dates of issues.

We are also postponing the circulation tabulation (see *From the Editor* in the August 1962 issue) until the February 1963 issue. We are anxious to catch up on publication and the compilation of such a list, up-to-date, would only delay publication of the December issue even further.

We have had a nice response in answer to the *Opinion Please* request in the August issue. Keep sending in your ideas about RMM — what you want or don't want, suggestions for improvement, criticism of any aspect of RMM (we *do* know we're late!), *your* opinion on *any* phase of RMM. We need your letters to know how to improve RMM. So, don't hesitate—write a letter or even a postcard—remember, *your* opinion counts.

Coming up for word enthusiasts (though we've dropped the *Word Games,* enough requests have come in for *something* in the way of words) is a neat article on *Word Shifts* by J. A. Lindon, to appear in the December 1962 issue. It should offer many hours of pleasurable recreation. The December issue will also carry an article on *Hexaflexagon Constructions* by S. H. Scott; *Chinese Arithmetic* by G. E. Ashley; and more. The yearly *Index* will also be published.

Coming up for next year (just a *few* of the many articles): *Problem in a Goldfish Bowl* by Alan Sutcliffe; *Tessellations of Polyhedra* by Sidney Kravitz; an article by Francis L. Miksa on magic Square transformations by a linear method; another collection of unusual magic squares; *Sun Dial* (construction) by Maxey Brooke; and much more in the regular departments. We hope to expand the *Puzzles and Problems* Department to at least 12 puzzles in each issue.

20 December 1962 J.S.M.

Flexahedrons

by Douglas Engel

If anyone has three hands and a little patience he can manufacture the structures I will describe here which I discovered in the Fall of 1961. I was playing with the possibilities of flexagon, or Jacob's Ladder, hinges. After much fumbling and bumbling I managed to get six tetrahedrons connected as shown by the process of weaving in Figure 3. The finished product looked like Figure 7. I used thin aluminum cut into strips to make the tetrahedrons weaving along the dotted line shown in Figure 1 and Figure 2. String was first used, but I found that this was the hard way of going about it. Paper bands worked better, giving much less trouble in connecting the tetrahedrons. Cardboard will work as well as aluminum to make the solid, but aluminum is more durable. I call this chain, unmercifully, a hexaflexatetrahedron. All of its movements seem to be regular and symmetrical. Besides the flexing from a rectangular shape to a hexagonal shape, shown in Figures 4, 5, 6, and 7, it can be got into other shapes where it does not flex but stops short and the only way out is to go "backwards".

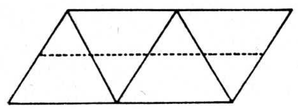

Figure 1

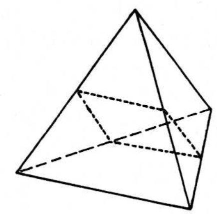

Figure 2

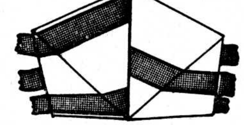

Figure 3

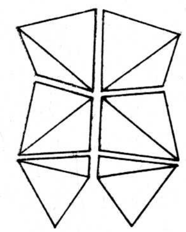

Figure 4

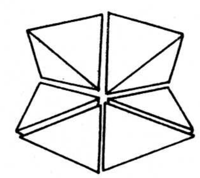

Figure 5

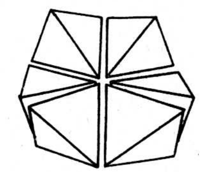

Figure 6

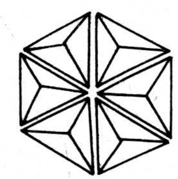

Figure 7

I also connected four, seven, and eight tetrahedrons into chains. The four tetrahedrons did nothing but roll around each other. Since there was no possibility of a space between them they could not possibly have had more than one definite movement. The eight tetrahedrons were irregular in movement, though quite fascinating. The seven-tetrahedron chain was formed and then given a half-twist before connecting the ends. Its movement was very limited and compared to the others it resembled an old Model T Ford beside a new Edsel. I suppose had I connected enough of an odd number of them by the above method they would have flexed but the only advantage I could see was that the combination of faces would constantly change.

Next I connected four cubes by following the dotted lines shown in Figures 8 and 9. The right triangles cut off by the two dotted lines in Figure 8 are not needed in the structure as they do not enter into the active movement. I then connected six of them and, lo and behold! I had a whole new fascinating set of movements to play with. In Figure 10 the six cubes are in the position where you fold them over and over to flex them until it becomes quite boring and you want something different.

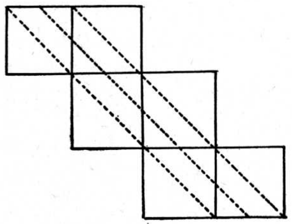

Figure 8

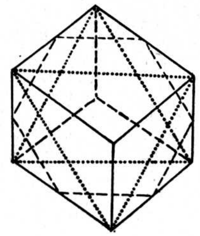

Figure 9

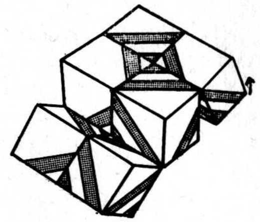

Figure 10

Figures 11 to 15 show how to get the apparatus into a position where it has a wholly new way of flexing. By flexing I mean that each object is made to roll around and the bands to move over them. There is a similarity in the symmetry and flexing property here resembling the tetrahedron chain of six. There are at least two locked positions it can achieve and this is also similar to the tetrahedron chain of six. I observed none of this in the octahedron chains which were constructed. It should be noted that only in the tetrahedron and the hexahedron can the bands be made to pass over all the fences on the solid. The other three Platonic solids can be connected in similar fashion, but no such fascinating flexing properties are observed - they only roll around and around.

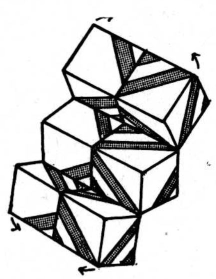

Figure 11

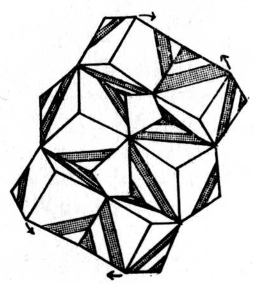

Figure 12

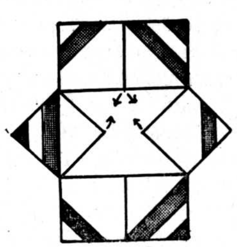

Figure 13

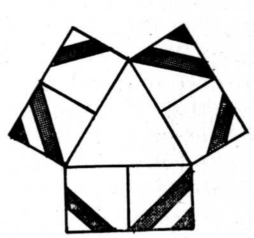

Figure 14

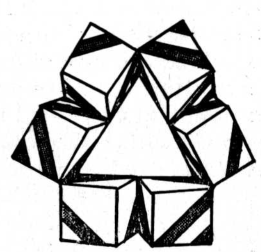

Figure 15

Since I have arrived at these flexing solids from a study of flat flexagons I tried to flatten the tetrahedrons and connect them. By using a strip of four isosceles right triangles to construct it, instead of four equilateral triangles, the tetrahedron becomes a flat square. Following the same pattern of con-

nection as in the regular tetrahedrons, I connected six of these flattened tetrahedrons, or squares.

The only similarity I could discern between the squares and the tetrahedrons was the rolling around - otherwise they differed considerably. Getting nowhere with flattened-out tetrahedrons forced me onto another track. I decided that since the triangles in the flattened-out tetrahedrons were the same as the triangles used in the "strip part" of the cube and since the cube has two more faces that the tetrahedron I should flatten a cube out to a hexagon and use the resulting 120° isosceles triangles. Figure 16 depicts this strip. Figure 17 shows the completed figure.

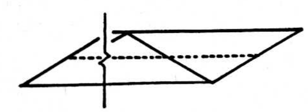

Figure 16

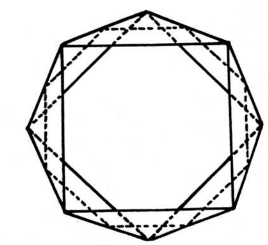

Figure 17

If the plane of each face were extended a kind of octahedron-like solid would result. I connected four of these eight-sided figures and found that it had a new way of flexing but only one definite way as in all the other connections of four. There is one locked position obtained by giving opposite figures in the chain a twist in opposite directions. When this is done they come to the closest possible contact with each other.

My next task was to connect six of these eight-sided figures. It took monstrous patience but I managed to do it. I advise anyone trying to do so for himself to have help when it comes to the process of connecting it. However, it is well worth the effort - the fascinating movements are never to be forgotten. I call it a hexaflexaoctahedron. While the chain was intact I was able to observe numerous locked positions. In fact it was locking more than it was flexing. There was a distinct similarity between it and the cubes and tetrahedrons but because of the size of the angles involved and the resulting forces, it had a tendency to come apart and I soon found myself amidst a pile of sight-sided figures whose only remaining property was to exasperate!

What must follow is some mathematical formulation of the properties of these flexing solids so that the properties of each succeeding one can be predicted. The next type solid would give ten sides and 144° in each triangle composing the strip since flattening out the eight-sided figure produces the 144° in the interior angles of the resulting octagon. I do not know the truth of this since I have not been able to reduce any of it to mathematics - my brainchildren have surpassed my methods. Perhaps the future will produce results.

Geometry and Men

by Hugo W. Traub

It is written: In the Beginning was THE WORD. A WORD. The Word was to become a favorite target for the dialectic prowess of theologians, philologians and metaphysicians. Indeed, the transliterative and evaluative content of one little Greek work appears inexhaustible. The Giant of Weimar once stubbed his toe on it: *"Im Anfang war Das Wort; hier stock' ich schon"*. Time and matter, then, are inadequate; hence away with the logos to begin anew. But where and with what? Well, there remains Space! But without geometry there is no Space.

Geometry, then, shall be our start. There being numerous of these, let it be Euclid's, Lobatchefsky's, Bolyay's, Rieman's - anybody's. It doesn't really matter. For in the beginning of them all, there was the Point. What Point? Elementary! A Cosmic Point, a certain Point, any old Point. Two of these Points, nomads roaming about Space met, and soon there were several. Being pure-bred, orderly, orthodox, they obeyed the rectilinear commandment — thus there was born a Line. Creation of the First Dimension and much rejoicing. The prototype of one subclass of the species Man. Long fellows of indefinite length, any length, with invariably one-track minds. The track is narrow, the narrowest conceivable and straight as a taut string. There can be no deviation to right or left, up or down does not exist, nor forward or back. Inflexibility and rigidity rule. A completely humorless breed; militarists, bureaucrats, hardshell clerics, officialdom. A handpicked few, strategically placed, are conditionally useful; still fewer may on occasion be even indispensable in an imperfect world.

But it was not good for a Line to be alone. Roaming about Space two met; soon there were more, any number. Geometric progression is now at work. A Plane surface came to be and with two-dimensional creation more rejoicing. Another prototype for existing Man. There are more, many more, of this subclass. They need more room and on occasion may, too, justify their existence. For even idiots are not purposeless when they indicate to so-called normals the smile on the latter of Lady Luck. But their appetite is always vast and they serve chiefly as supports for Points and Lines.

The mental track is nearly always standard-gauge and the equipment stream-lined, designed to get to places fast. Heaven, Hell or Purgatory is immaterial. They must get there fast, never mind the risks. Go-getters, uplifters, forward-lookers, manipulators, promoters, investment bankers, hack-politicians, "Big Shots", ever with an eye to the main chance: Babbitt. They often have attractive labels and pretty wrappers. Many are on the level of purpose, but far more askew. The type is really flat as stale beer. And bad accidents happen on that track, witness '29 and since. Look at the thing!

Meandering about Space two Surfaces met. Quickly there was a Solid, there appeared many Solids. Tridimensional Creation, the Third Cosmic Day with a roll of drums and mighty fanfare of trumpets. The process re-

mains forever the same: addition, conjugation, subtraction, division. Infinite numbers of Solid figures arose; cylinders, cones, pyramids, cubes - of all sizes.

The mental track is still standard-gauge but curves have appeared; globoids, spheroids. The prototype values vary widely; Falstaff, pater familias vulgaris, the Court Jester, some American Presidents. In the sunlight some refract light and become prismatic: political orators, high-powered evangelists, demagogues.

Evolution continues and several Solids unite for better or worse. Athirst, they drink and behold - a Crystal emerges. Again union occurs among these and the building of a shelter wall. Flicked with the tiniest bit of the aurora borealis, hormones and endocrines enter the now brilliant spectacle - and we have the prototype of the First Real Man. Homunculus is the creature's name but the sturdy and imposing little thing remains a neuter until it has outgrown the diminutive. Dirt pioneers, sailors before the mast, old-time, almost extinct country doctors, a scattered handful of architects and engineers, iconoclasts, warranted blown-in-bottle brand of humanitarians, Jane Addams, Ghandi. Less than a handful of American Presidents or other statesmen, alas.

Four, five and *N*-dimensional bodies, whence and how? The process remains forever the same while natural selection is getting in its ruthless work. The air has become more rarified and numbers are much fewer. The prototype is for that of a highly restricted, very exclusive company. The ancient Greeks had the original copyright only to lose it after a mere century and a half. Here only poets, pure mathematicians, creative artists, seers and saints can belong. The qualifications are highly complex and the properties peculiar. Talk about them is still endless. Nobody understands, neither did or do they themselves, Euclid, Archimedes, Sophocles, Aristophanes.

But plague take it all! My own infirmities are overtaking me. That Point, that primeval Ur-Point. Whence came it, and what does it represent among living men or dead? A sense preception, a wisp of the imagination? No matter! In the Beginning was THE WORD.

* * * *

Alphametics

This department appears to be the most popular feature in RMM. The *Puzzles and Problems* are indeed very popular, too. However, we believe alphametics have greater appeal since no more than a knowledge of the basic principles of arithmetic is required to solve the problems—plus the ability to apply these principles logically.

Perhaps other readers can supply their own reasons why such an essentially simple type of puzzle can appeal to—and stump—solvers from grade school level through post graduate college students.

* * * *

```
 NOEL
 NOEL
-----
BELLS
```

We all know what this means. *(H. S. Tribe;* Sutton, England.)

* * * *

Simple Simon's pseudo-summing certainly seems silly. Still, certain substitution solves Simple Simon's summing.

(Robert Ruderman; New Hyde Park, New, York)

```
 THREE
 SEVEN
  NINE
------
TWELVE
```

* * * *

```
SQUARE
 DANCE
------
DANCER
```

Square dancing, as we all know, soon has us going in *circles.* This alphametic may keep *you* going in circles if you don't solve the hidden problem first.

(A. G. Bradbury; North Bay, Ontario)

* * * *

A colorful alphametic, most assuredly! One that may turn some of you purple trying to solve it and make you green with envy seeing others solve it with ease.

(A. G. Bradbury; North Bay, Ontario)

```
 GREEN
  GREY
  FAWN
------
YELLOW
```

* * * *

```
CANADA
UNITED
------
STATES
```

North is north, and south is south, so there can be no argument about the relative positions here.

(R. Robinson Rowe; Sacramento, California)

* * * *

Here's wishing all our readers a very happy New Year.

(J. A. H. Hunter)

```
HAPPY
HAPPY
HAPPY
 DAYS
-----
AHEAD
```


Four Thousand Years of Easter

by Richard K. Allen

Many reference books state that Easter falls on the first Sunday after the first full moon after spring begins. In 1962 the vernal equinox occurred on March 20 at 9:30 P.M., EST. The moon was full at approximately three o'clock the following morning. It would seem logical that in 1962 Easter should have fallen on March 25 instead of April 22 as it did. Why does the date of Easter behave as it does?

The Crucifixion took place on a Friday after the Jewish Passover. The first Easter came on the Sunday after the Passover. In ancient times the Passover was celebrated on the fourteenth day of the Jewish month of Nisan. This month began at the time of the new moon such that the full moon should be the first to follow the vernal equinox.

We can see that the first Easter was the first Sunday after the first full moon after spring began. When the Council of Nicaea was held in 325 A.D., the date of Easter was defined as we have seen above. In a day of poor communication and slow transportation it was only natural that various sections of the Christian world should celebrate this holiday at different times in widely separated locations.

When Pope Gregory XIII revised the calendar effective in 1582 he at the same time laid down definite rules for the date of Easter which entirely divorced the holiday from all connection with astronomy. The tables which he set forth usually give the results which we have quoted, but as we have seen, 1962 was an exception.

The tables as set forth in the Encyclopaedia Britannica and elsewhere are relatively simple in their application but their derivation and the reasoning behind them is somewhat involved. It would serve no useful purpose to reproduce them here, although the writer would be glad to discuss them with any interested reader.

In this day of automation when all knowledge seems to be springing from computers, these tables were submitted to a Remington Rand Univac STEP computer, instructions given the machine (228 steps), and the dates of Easter were printed for each year from 1 B.C. to 3999 A.D. Admittedly it is ridiculous to compute the date of Easter before the death of Jesus, and nearly as foolish to derive them for the years before 1583, but the job was done partly as an exercise in programming. The computer is being used by the National Life Insurance Company, Montpelier, Vt., and since the date of Easter is not related to insurance all programming was done outside of office hours. As a hobby the programmer worked during the evenings and required a total elasped time of six days to complete the project. After the computation actually began the results were obtained in about six minutes, but this time could have been shortened by more sophisticated techniques of programming. Assuming that a person became expert enough to be able to derive one date each 15 seconds (which would require great care to take into account the change of leap years and centuries) this job would have required about seventeen hours of continuous labor, assuming no errors, etc. His work would have to be checked, of course.

Easter can fall on any one of 35 different days, from March 22 to April 25, inclusive. The earliest date took place in 1818 and will not occur again 2285. April 25 was Easter in 1943 and will fall on that date again in 2038.

The present Gregorian calendar is not perfect, having an average annual error of 26 seconds. Eventually this error would accumulate to an entire day and in view of that some authorities advocate that leap day be omitted in the year 4000 A.D. The usual Gregorian rule causes leap year to fall every four years except that leap year is omitted in those centennial years not evenly divisible by 400. It was only this doubt about the future existence of February 29, 4000, that caused the above computation to terminate at 3999.

Among the results obtained were the following:

April 10, 1583*	March 22, 1818	April 22, 1962	March 22, 2285
April 19, 1620*	April 16, 1865	April 14, 1963	April 18, 2962
April 13, 1732*	April 15, 1900	March 29, 1964	March 24, 2999
April 7, 1776	April 25, 1943	April 25, 2038	April 18, 3999

*These dates are Gregorian dates. The calendar was not adopted in England (and the colonies) until 1752. The corresponding dates in England were March 31, 1583; April 9, 1620; April 2, 1732.

* * * *

Permutacrostic . . . Blowing Our Own Horn

by Remy Landau

Each of the following phrases is a permutation of the letters of a mathematical term. Our idea is expressed by the first letters of the mathematical terms.

(1) NOT A LIAR
(2) AS IN MAST
(3) AD MINE
(4) SO? LESS ICE!
(5) ERASED TIGHT
(6) DIP AT ZERO
(7) MOB RUSH
(8) A QUIET NO
(9) LOUD SUM
(10) I MEAN LINT
(11) TAME OR RUN
(12) FIT IT ON! FREE DINA!
(13) EAT IN ROD
(14) NUN, KNOW!
(15) SCOUT RAT BIN
(16) SAM LENT A FUND
(17) TUNIS
(18) A PIER NINA!

Solution is on page 39

Numbo-Carrean

by J. A. Lindon

Speijdba yiu Dve Podda?

No part of Numerica has been more thoroughly explored than the Pays des Carres, which lies between the surrounding plains of Linea and the Teritary upjuts of the Cubian Mountains. Yet its current language is quite unknown. Readers may recall that when the Great Square assumed full second-powers a year ago, he dissolved the rival Hip-and-Hep Party, condemned Jitta Bug, its leader, to life equations in Quadratica, and set about reorganizing the country's institutions. The old language was jettisoned and replaced by Numbo-Carrean *(Dve Podda,* 'Two Power'). This, though still imperfectly understood, even by the natives, I propose to discuss here.

Word formation is simple. The letters of the alphabet are numbered A=1, B=2, etc., and from a table of squares those numbers are picked out which form convenient groups of letters. Thus 238144 (the square of 488) gives us *whadd* (paper-money), and 16581184, *pehard* (horse). (I apologize for this latter example, but Numbo-Carrean is very expressive and so, I fear, in that country are the horses.) Other immediately comprehensive words are *zudd* (soapy water), *lyddi* (affected female), *kabby* (taxi driver — his vehicle is a *kabbbbe),* and *dogedadda* (male pooch with pups). Some words indeed are identical with their Yankanglian equivalents. *"Have side-of ship up,"* I add is perfectly good Numbo-Carrean, though the native terms *zbif, uzbifa* would be more commonly met with for ocean-going vessels. (These float sideways.) Plurals are formed by preceding the noun by the appropriate numeral *(au, dve, teti, pha, dfive,* etc.) or by a quantitative adjective like *bothi* (both), *sazbi* (several, some), or *mili* (many). A few simple sentences will give the idea.

Dve y teti maccaf dfive.
Two and three make five.

Ceijy ryta be a odbird, botcu cepdi kidbard de sgifdi.
This lady novelist is a queer creature, but that juvenile poet is gifted.

"Iive fibbbd aniu," szedd Aniba. "Yiu awta be a gagwifa."
"I've told another whopper," said Aniba. "You should stop me from talking."

D nubi waichef wastbe a ituhard, eebbe oddeerd y gebeefy.
The colored waiter intended to be a boxer, his hat-rests aren't a pair and he's sure tough.

It should be noted that a single square number may lead to more than one Numbo-Carrean word. Thus 55225 gives *eebbe,* which as we have just seen means 'he is'. But it can equally well give *eeby,* which is sometimes used in the expression *dby eby* (sleep), or *eeve* (a bikiniless beauty). On the other hand, some squares give no words at all, e.g. 2704 or 10609 *(bg*d* or *jf*i)*.

Most Numbo-Carrean words just happen; you find them while browsing through your table of squares, idly converting these into their lettered equivalents. You cannot search for a word like *cicbedda* (invalid), only delight in having found it. Some organized searching is possible. There will always be two separate infinities of squares *starting* with a given sequence of digits, and some at least of the smaller ones may prove useful. Thus EGG——requires a square starting with the digits 577, which means having roots starting with 24 or 76. *Eggf* (chicken-fruit) is easily found. But others occur too: *eggibaf* (omelette), *egggeta* (opphagite), etc. (An opphagite is something which eats eggs, such as a weasel or a dictator having breakfast.) We cannot always find squares *ending* with a given sequence of digits. But often, by some alteration in spelling, we can obviate this difficulty. Thus, although we cannot have squares ending with 414 (to give ——DAD), we can find separate series ending with 4144 (DADD), 41441 (DADDA), and 41449 (DADDI), and very amusing some of these are:

a-guy-dadd, poor old pop with whiskers,
tif-dadda, father who keeps losing his temper,
tik-dadda, father who likes holding his watch to his infant's ear,
hobbb-dadda, a sit-by-the-fire-and-nurse-it type,
eeerdd-daddi, father with intriguingly prominent otic appendages,
ah-daddi, father who is always warning Junior,
and so on.

Let us consider a bit of genuine Numbo-Carrean dialogue, which I overheard in an *edcafi* in the little square town of *Qudta.* (An *edcafi* is an important restaurant or teashop.) Two Carrean ladies are scandalmongering.

Wed-bilifi	Married double-life
"Huiwef ha wyphd ciebadd Gehardi?"	"Who's spliced up with cock-eyed Gerard?"
"Wyfbe nneau d leufa Nety."	"His wife's that old sponge Netty now."
"Gichha! Wyfbe dwidha Di Figby. Zbadd! Eebbe a diwifa, maccaf wedd-hobi. Ealbe jayld."	"Get out! His wife is widow Di Figby. Shocking! He's a bigamist, makes a hobby of marriage. He'll do time for this."
"Zgodlove! Botcu huiwef be dwidha Di Figby?"	"For heaven's sake! But who is widow Di Figby?"
"Efy wed H. Figbi —— hhca-dedd."	"She married H. Figby —— died of a kick."
"Ahszadd!"	"Very sad!"
"Iuf."	"Yes."

Here is a more descriptive passage.

Cicli	Ill
Aniba ly abedd, efy be cicli, be a cicbedda. Botcu liif netbe up.	Aniba lies abed, she's poorly, she's an invalid. But life is not over.
Aniba maccaf d iccof. "Scu!" szedd Aniba. I jese-give a lahhaf—oxlaf. "Aahha!"	Aniba hiccups. "Pardon!" says Aniba. I just give a laugh — a guffaw. "Aahha!"
Aniba netbe rheli cicli, netbe a dii-eve, botcu a gaii-eve! Efy be qwifi, be DT-hapi. Staix!	Aniba is not really ill, she's not *in extremis,* she's a gay-bird, a lady drunk. She's squiffy, merry with DTs. Here's how!

Aniba is the beatnik daughter of Di Figby by her former husband, H. Figby, who died, you remember, of a kick *(hhac-dedd).* Not a horse but an *uzmu* was responsible, one of the hump-backed mules whose normal habitat is the neighboring Catenoid uplands. However, the *waacati* (the many irritable feline denizens of those regions, which give the latter their name) often drive the peace-loving *uzmu* down into Carrean country. The only thing they will not tolerate is being milked, and poor H. Figby learned this too late. But let us see how his widow is faring in her second marriage with cock-eyed Gerard.

Debibi	The Baby
Di Figby have a biibi. Dfahta be Gehardi, huiwef wyphd dwidha. Debibi be drty, a caciolove, be washicacive. Debibi be doxi, botcu goefy, y dadly marhhd. Be a ceehave, a megive, a fixgive.	Di Figby has a baby. The father is Gerard, who married the widow. The baby is dirty, fond of mess, dreadful to wash. The baby is sweet, but goofy, and paternally spoiled. It wants everything it sees, demands to be given things, expects to keep them.
"Webbe debibi, debebebi, ddebbibi?" szedd dwidha.	"Where's that boofy-woofy baby?" said the widow.
Dwidha ha ffild d baaf vidda zudd.	The widow has filled the bath with soapy water.
Botcu debibi netbe andi. Eebbe nd dby eeby, hushdd, brushd, y abedd widaddi, side-of ddadta. Bizha!	But junior's not around. He's in bye-bye, quiet, tidy, and in bed with Daddy, alongside the old man. How peculiar!

And there, for the present, we must leave him, and Numbo-Carrean too. We hope, in some later issue of RMM, to give further examples of this entertaining language.

Three Old Chestnuts: A Semantic Approach

by Norman M. Lang

Many of us can remember how, as school-children, we belabored one another with certain questions which we thought of as deeply philosophical. One of these had to do with the mythical trees in a lonely forest. "If a tree fell in the middle of a great forest which was completely empty of all animal life, would there be any sound?" we asked. Now that we are grown up and in touch with semantics the answer comes quite easily: It all depends. If by "sound" you mean vibrations in the air, the answer is Yes; but if you mean the effect of vibrations on a living nervous system, it is No.

Another question of which we were fond—one somewhat more abstruse—asked whether or not time would exist within a volume of interstellar space completely devoid of all matter. To this a semanticist might reply that "time" is a four-letter word used by English-speaking people to denote a concept that has proved very useful in connection with physical happenings. This concept is, however, not the same everywhere, by any means; the Hopi Indians, for example, have ideas about it that are totally different from most other dwellers in North America.[1] Modern physicists, moreover, have cast doubts upon the possibility of simultaneity for widely separated events, for one thing, and this has done considerable damage to our previous notion of time. Aside from all this, though, it does seem that time, as we understand it, requires something physical to measure it by and therefore can *not* exist in completely empty space. In other words, we must abandon with regret that beautiful Newtonian abstraction, absolute time.

A third question requires a longer explanation, though essentially it is a simpler one. This is the so-called Paradox of Zeno. The Greek philosopher asked his pupils to imagine a race between Achilles and a tortoise in which the former could run faster and the latter was given a head start. He then undertook to demonstrate that Achilles would never catch the tortoise. Let us say that the tortoise started a hundred yards ahead, that Achilles could run this distance in ten seconds, and that the tortoise's speed was one-tenth as great. Then, as Zeno described the race, when Achilles had reached the tortoise's starting point the latter had advanced ten yards. When Achilles had run the ten yards, the tortoise had moved ahead one yard. When Achilles had covered the one yard, the tortoise was still one-tenth of a yard ahead, and so on. Thus, said the philosopher, Achilles never caught the tortoise.

Suppose we imagine that the race starts at high noon. Then, knowing the speed of each runner, we can, of course, predict where each will be at any moment in the afternoon. We can also say, with the help of a little algebra, that Achilles will pass the tortoise at 11-1/9 seconds PM. So where is the paradox? To answer this, we resort to a fundamental rule of semantics, "words are not things". What Zeno has done, in saying "when" repeatedly, is to *talk* about certain moments of time, all later than noon but never quite as late as the moment when Achilles catches up to the tortoise. We can, if we like, say that Zeno has mentioned a series of diminishing time increments

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[1]See "Science and Linguistics" by Benjamin Lee Whorf, *The Technology Review,* April 1940. Reprinted by permission in *Language in Action* by S. I. Hayakawa, 1941, Harcourt, Brace and Co., Inc., New York, page 302.

the sum of which approaches a limit, but never reaches it.[2] However, there is no need to go that far. We simply point out that all Zeno's talking has no effect whatever on the outcome of the race. The paradox dissolves into nothing. The only puzzle that remains is why, philosophers have bothered to continue the discussion for the past 2,500 years.

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[2]The same thing happens when we undertake to discharge a debt by making each payment one-half of the previous one. Owing $100, we pay $50, $25, and so on. We can get as close as we like to paying off the debt, but we will never reach the limit of $100.

* * * *

A Miniature Geometry

by Howard C. Saar

So often when a person thinks of a geometry which is non-Euclidean, he considers only those which originate by modifying the Euclidean parallel postulate. In order to develop our understanding of geometry as a logical system, while having some fun, we turn instead to a finite or miniature geometry. By considering a few definitions and postulates which form a consistent set, we hope to be able not only to construct an accurate model of the geometry in question, but also to set down and prove a few theorems concerning it. For the reader inexperienced in such deviations from the Euclidean structure, we recommend that he keep in mind that his past experiences with measurement, intuitive notions of the behavior of points and lines, etc., will be of no value to him here.

Beginning with but one set of undefined elements, *points,* we can logically consider sets of points as lines. We can assume that when we talk about a line, we mean a straight line in the intuitive sense, although this is really just a matter of accommodating our model. Likewise, we shall adopt the intuitive notions of a point on a line and a line through a point, again to facilitate the construction of our model. It is important to note that these intuitive notions are not the least bit essential to the geometry in question, but are merely matters of functional convenience.

In addition to our undefined elements, *points,* we need a few definitions:

D-1 A line is a set of points
D-2 A point is on a line if it belongs to the set of points which is the line
D-3 A line goes through a point if the point lies on the line
D-4 A set of three distinct points is known as a triangle
D-5 If two lines have no point in common, they are parallel

Just three postulates are necessary:

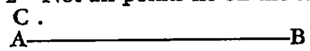
P-1 Two distinct points determine a unique line
P-2 A line goes through exactly two points
P-3 There exists five distinct points

For those people who prefer their gifts wrapped up quickly in small packages, an acceptable model can now be drawn. If, however, the game intrigues you, continue on as we develop the theorems which will themselves construct the model of our miniature geometry.

T-1 There exists at least one line

(Proof: Use *P-3* and *P-1)*

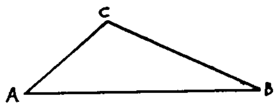
T-2 Not all points lie on the same line



(Proof: Use *P-3* and *P-2)*

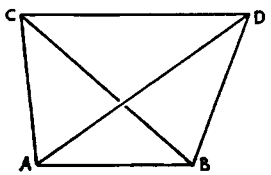
T-3 For any given line, there exists a point not on this line (Proof: Use *T-2)*

T-4 There exists at least three distinct lines



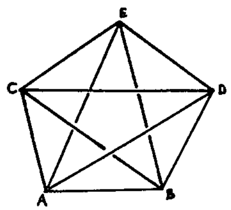
(Proof: Use *T-1* and *T-3* together with *P-2)*

T-5 There exists at least six distinct lines



(Proof: From *T-4* we know there are three distinct lines utilizing three distinct points. There is a fourth point, not on any of these lines, which when paired with the three in use, provides three more distinct lines)

T-6 There exists at least ten distinct lines



(Proof: Utilize and expand the argument of the proof of *T-5)*

T-7 There exists exactly ten lines

Our model is now complete and consistent! For those who would like further discussion of this system, the following theorems are proposed for proof by the reader:

T-8 Each point lines on four distinct lines

T-9 Through a point not on a line, there exists at least one line parallel to the given line

T-10 Through a point not on a line, there exists exactly two parallels to the given line

T-11 Each line has exactly three lines parallel to it

T-12 Two lines parallel to a third line are not parallel to each other

T-13 A triangle has three lines

T-14 There exists exactly ten triangles

T-15 Two distinct triangles cannot have more than one line in common

Having utilized the undefined elements, some definitions, and a consistent set of postulates, we have built *a* model of our system. It is important to note that our model is neither unique nor has it any particular merit in explaining the physical universe. It has significant value, however, in the study of the geometry herein presented.

Letters to the Editor

New Journal

Dear Mr. Madachy:

This is to announce that the *Pro Mathematica Fibonacci Group,* with headquarters at San Jose State College, is now accepting subscriptions to its new magazine THE FIBONACCI QUARTERLY JOURNAL. Plans are underway to have the first issue available by February 1963. The subscription rates per year of four issues are:

Regular Subscription	$4.00
Libraries	$5.00
Sustaining	$20.00 or more

Subscription inquiries should be addressed to Brother U. Alfred, St. Mary's College, St. Mary's, California. The General Editor is now accepting manuscripts at the Mathematics Department of San Jose State College.

We are not officially affiliated with San Jose State College.

Pro Mathematica Fibonacci Group
San Jose State College
San Jose, California

The Fibonacci Quarterly Journal
Verner E. Hoggatt, Jr.
General Editor

* * * *

Will Roulette Beat the Reader?

Dear Mr. Madachy:

If Morrow Mayo thinks his system (see "Will This System Beat Roulette?", RMM No. 9, June 1962, pages 32 to 38) can beat the game of roulette he should be advised to read the chapter on Roulette in John Scarne's *Complete Guide to Gambling* (Simon & Schuster). Scarne is recognized as the world's foremost authority on gambling, and in his book he discusses various betting systems and why they fail.

One should invest ten bucks in buying this book before he embarks on M.M.'s roulette system or any other system.

Atlantic City, N. J. Rudolf Ondrejka

* * * *

Dear Mr. Madachy:

Mr. Mayo, at the end of his article, commented that his system was valid only insofar as the mathematics was correct; and he asked for comments by mathematicians.

These comments are available in any standard book on probability; e.g. William Feller, *Introduction to Probability Theory* (John Wiley & Sons). In essence, no system can change an unfavorable situation into a more favorable one.

The category of circle squarers, inventors of perpetual motion, and angle trisectors includes inventors of systems to beat the game.

I am disappointed to see such an article published. It encourages unknowledgeable individuals to risk money they probably cannot afford to lose; and lose it they will.

Idaho Falls, Idaho W. B. Lewis

* * * *

Dear Mr. Madachy:

A basic fallacy of this type of system is expressed at the bottom of page 33 ("Backed by sufficient capital, and without the imposition of a limit, it would always win."). There is *always* a limit beyond which you cannot bet

—either because the rules of the bank won't allow it, or because you don't have the capital available. This limit may be very large, but it is not infinite.

Mathematically, the only way to beat roulette would be to watch wheels, use statistics to determine which ones may be slightly off-balance, and then utilize such information to bet accordingly. However, the casinos are doing this very thing to catch irregularities in the wheels, and they adjust them promptly. So you have to beat them to it if you want to succeed. Or you need just plain luck—which will last just so long.

Benidorm (Alicante) Spain — Hayo Ahlburg

* * * *

Cross-Number Puzzles

Dear Mr. Madachy:

Although there were errors in the clues, I found it possible to solve the June 1962 Cross-Number Puzzle.

However, thanks to these errors, the problem became a real beauty. Possibly, you have created a new problem-type: the solvable Cross-Number with errors in the clues.

Antwerp, Belgium — Edward Joris

Well, that's what we're always looking for—new material!

Seriously, though, we've decided to drop the Cross-Number Puzzles *temporarily. The August 1962* Junior Department *cross-number puzzle was even drawn incorrectly! For some peculiar reason, the Editor has let errors slip by in all but two of the six cross-number puzzles which have appeared in* RMM. *It is due entirely to the Editor's proofreading, and only one author's error (which, indirectly, is also the Editor's error for not catching it before acceptance.)*

* * * *

Dear Mr. Hunter:

In the April 1962 RMM, page 33, H. W. Gould states that the properties of 143 have not come to light in connection with 1962. What about

$$143=(1+9+6/2)\ (19-6-2)?$$

Los Angeles, Calif. — Leon Bankoff

* * * *

Dear Sir:

A Note in the June 1962 RMM, pages 45-46, listed many cases of integer pairs, N and $N + k$, and such that

$$N + k = x^3 + y^3 \text{ and } N = z^3 + w^3$$

Professor Waclaw Sierpinski conjectured that every positive integer, say m, can be represented in an infinitude of ways in the form

$$m = x^3 + y^3 - (z^3 + w^3)$$

the parameters being positive integers. This conjecture has been proven true for *all* m less than 1000, and for many higher values.

From this conjecture it follows that, for any given k greater than zero, there must exist infinitely many N such that both N and $N + k$ are the sums of two cubes.

In a different vein, I wonder if any numbers (except 31 and 8191) can be written in two different bases only with ones (at least three ones must be used)? We have

$$31 = (111)_5 = (11111)_2$$
$$8191 = (111)_{90} = (1111111111111)_2$$

Warsaw, Poland — Andrezej Makowski

The First 571 Fibonacci Numbers

*by S. L. Basin and V. E. Hoggatt, Jr.**

The abundance of references in the literature is evidence that the subject of Fibonacci numbers has enjoyed constant interest by the amateur as well as professional mathematician for many years.

A very active effort in compiling all the past and present research on this subject has been undertaken by the newly formed organizations:

THE PRO MATHEMATICA FIBONACCI GROUP

and

THE FIBONACCI BIBLIOGRAPHICAL RESEARCH CENTER

located at San Jose State College, San Jose, California. The objectives of the above organizations include collecting and cataloguing past publications and stimulating continued research in the above subject.

* * * *

The following table of Fibonacci numbers is an example of a recent effort in extending past research. A history of such tables is given by Dov Jarden[1] along with the first 385 Fibonacci numbers and their prime factors. The first 571 Fibonacci numbers are listed in the following table.

We acknowledge the efforts of E. G. McNeil and C. Warden who were responsible for programming the IBM 7090 computer on which this table was computed. We are also indebted to E. O'Connell and M. Bichnell for proofreading the overwhelming mass of digits.

The above authors invite your inquiries about the above Fibonacci organizations and welcome any papers for the Fibonacci Quarterly Journal soon to appear.[2]

* * * *

The table following is set up in column form with a break after every tenth term to enable one to find, say, F_{379} quickly. Each Fibonacci term is the sum of the previous two terms, excepting the first two terms.

A separately bound reprint of this table can be obtained by remitting 25c in coin, stamps, or check for each copy. Send your orders, prepaid, to Editor—RMM, Box 35, Kent, Ohio. Sorry, no invoicing or billing on this unless orders of 10 or more are made to one address. No further reprints are anticipated after the initial supply runs out.

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*Members of the *Fibonacci Bibliographical Research Center,* San Jose State College.

[1]DOV JARDEN, *Recurring Sequences,* published by Riveon Lematematika, Jerusalem, Israel, 1958.

[2]See *Letters to the Editor* on page 17 of this issue.

F_1 to F_{40}
1
1
2
3
5
8
13
21
34
55
89
144
233
377
610
987
1597
2584
4181
6765
10946
17711
28657
46368
75025
121393
196418
317811
514229
832040
1346269
2178309
3524578
5702887
9227465
14930352
24157817
39088169
63245986
102334155

F_{41} to F_{80}
165580141
267914296
433494437
701408733
1134903170
1836311903
2971215073
4807526976
7778742049
1 2586269025
2 0365011074
3 2951280099
5 3316291173
8 6267571272
13 9583862445
22 5851433717
36 5435296162
59 1286729879
95 6722026041
154 8008755920
250 4730781961
405 2739537881
655 7470319842
1061 0209857723
1716 7680177565
2777 7890035288
4494 5570212853
7272 3460248141
11766 9030460994
19039 2490709135
30806 1521170129
49845 4011879264
80651 5533049393
130496 9544928657
211148 5077978050
341645 4622906707
552793 9700884757
894439 4323791464
1447233 4024676221
2341672 8348467685

F_{81} to F_{120}
3788906 2373143906
6130579 0721611591
9919485 3094755497
16050064 3816367088
25969549 6911122585
42019614 0727489673
67989163 7638612258
110008777 8366101931
177997941 6004714189
288006719 4370816120
466004661 0375530309
754011380 4746346429
1220016041 5121876738
1974027421 9868223167
3194043463 4990099905
5168070885 4858323072
8362114348 9848422977
1 3530185234 4706746049
2 1892299583 4555169026
3 5422484817 9261915075
5 7314784401 3817084101
9 2737269219 3078999176
15 0052053620 6896083277
24 2789322839 9975082453
39 2841376460 6871165730
63 5630699300 6846248183
102 8472075761 3717413913
166 4102775062 0563662096
269 2574850823 4281076009
435 6677625885 4844738105
704 9252476708 9125814114
1140 5930102594 3970552219
1845 5182579303 3096366333
2986 1112681897 7066918552
4831 6295261201 0163284885
7817 7407943098 7230203437
12649 3703204299 7393488322
20467 1111147398 4623691759
33116 4814351698 2017180081
53583 5925499096 6640871840

F_{121} to F_{160}
86700 0739850794 8658051921
140283 6665349891 5298923761
226983 7405200686 3956975682
367267 4070550577 9255899443
594251 1475751264 3212875125
961518 5546301842 2468774568
1555769 7022053106 5681649693
2517288 2568354948 8150424261
4073057 9590408055 3832073954
6590346 2158763004 1982498215
10663404 1749171059 5814572169
17253750 3907934063 7797070384
27917154 5657105123 3611642553
45170904 9565039187 1408712937
73088059 5222144310 5020355490
118258964 4787183497 6429068427
191347024 0009327808 1449423917
309605988 4796511305 7878492344
500953012 4805839113 9327916261
810559000 9602350419 7206408605
1311512013 4408189533 6534324866
2122071014 4010539953 3740733471
3433583027 8418729487 0275058337
5555654042 2429269440 4015791808
8989237070 0847998927 4290850145
1 4544891112 3277268367 8306641953
2 3534128182 4125267295 2597492098
3 8079019294 7402535663 0904134051
6 1613147477 1527802958 3501626149
9 9692166771 8930338621 4405760200
16 1305314249 0458141579 7907386349
26 0997481020 9388480201 2313146549
42 2302795269 9846621781 0220532898
68 3300276290 9235101982 2533679447
110 5603071560 9081723763 2754212345
178 8903347851 8316825745 5287891792
289 4506419412 7398549508 8042104137
468 3409767264 5715375254 3329995929
757 7916186677 3113924763 1372100066
1226 1325953941 8829300017 4702095995

F_{161} to F_{200}
1983 9242140619 1943224780 6074196061
3210 0568094561 0772524798 0776292056
5193 9810235180 2715749578 6850488117
8404 0378329741 3488274376 7626780173
13598 0188564921 6204023955 4477268290
22002 0566894662 9692298332 2104048463
35600 0755459584 5896322287 6581316753
57602 1322354247 5588620619 8685365216
93202 2077813832 1484942907 5266681969
150804 3400168079 7073563527 3952047185
244006 5477981911 8558506434 9218729154
394810 8878149991 5632069962 3170776339
638817 4356131903 4190576397 2389505493
1033628 3234281894 9822646359 5560281832
1672445 7590413798 4013222756 7949787325
2706074 0824695693 3835869116 3510069157
4378519 8415109491 7849091873 1459856482
7084593 9239805185 1684960989 4969925639
11463113 7654914676 9534052862 6429782121
18547707 6894719862 1219013852 1399707760
30010821 4549634539 0753066714 7829489881
48558529 1444354401 1972080566 9229197641
78569350 5993988940 2725147281 7058687522
127127879 7438343341 4697227848 6287885163
205697230 3432332281 7422375130 3346572685
332825110 0870675623 2119602978 9634457848
538522340 4303007904 9541978109 2981030533
871347450 5173683528 1661581088 2615488381
1409869790 9476691433 1203559197 5596518914
2281217241 4650374961 2865140285 8212007295
3691087032 4127066394 4068699483 3808526209
5972304273 8777441355 6933839769 2020533504
9663391306 2904507750 1002539252 5829059713
1 5635695580 1681949105 7936379021 7849593217
2 5299086886 4586456855 8938918274 3678652930
4 0934782466 6268405961 6875297296 1528246147
6 6233869353 0854862817 5814215570 5206899077
10 7168651819 7123268779 2689512866 6735145224
17 3402521172 7978131596 8503728437 1942044301
28 0571172992 5101400376 1193241303 8677189525

F_{201} to F_{240}
45 3973694165 3079531972 9696969741 0619233826
73 4544867157 8180932349 0890211044 9296423351
118 8518561323 1260464322 0587180785 9915657177
192 3063428480 9441396671 1477391830 9212080528
311 1581989804 0701860993 2064572616 9127737705
503 4645418285 0143257664 3541964447 8339818233
814 6227408089 0845118657 5606537064 7467555938
1318 0872826374 0988376321 9148501512 5807374171
2132 7100234463 1833494979 4755038577 3274930109
3450 7973060837 2821871301 3903540089 9082304280
5583 5073295300 4655366280 8658578667 2357234389
9034 3046356137 7477237582 2562118757 1439538669
14617 8119651438 2132603863 1220697424 3796773058
23652 1166007575 9609841445 3782816181 5236311727
38269 9285659014 1742445308 5003513605 9033084785
61922 0451666590 1352286753 8786329787 4269396512
100191 9737325604 3094732062 3789843393 3302481297
162114 0188992194 4447018816 2576173180 7571877809
262305 9926317798 7541750878 6366016574 0874359106
424420 0115309993 1988769694 8942189754 8446236915
686726 0041627791 9530520573 5308206328 9320596021
1111146 0156937785 1519290268 4250396083 7766832936
1797872 0198565577 1049810841 9558602412 7087428957
2909018 0355503362 2569101110 3808998496 4854261893
4706890 0554068939 3618911952 3367600909 1941690850
7615908 0909572301 6188013062 7176599405 6795952743
12322798 1463641240 9806925015 0544200314 8737643593
19938706 2373213542 5994938077 7720799720 5533596336
32261504 3836854783 5801863092 8265000035 4271239929
52200210 6210068326 1796801170 5985799755 9804836265
84461715 0046923109 7598664263 4250799791 4076076194
136661925 6256991435 9395465434 0236599547 3880912459
221123640 6303914545 6994129697 4487399338 7956988653
357785566 2560905981 6389595131 4723998886 1837901112
578909206 8864820527 3383724828 9211398224 9794889765
936694773 1425726508 9773319960 3935397111 1632790877
1515603980 0290547036 3157044789 3146795336 1427680642
2452298753 1716273545 2930364749 7082192447 3060471519
3967902733 2006820581 6087409539 0228987783 4488152161
6420201486 3723094126 9017774288 7311180230 7548623680

F_{241} to F_{280}

1	0388104219	5729914708	5105183827	7540168014	2036775841
1	6808305705	9453008835	4122958116	4851348244	9585399521
2	7196409925	5182923543	9228141944	2391516259	1622175362
4	4004715631	4635932379	3351100060	7242864504	1207574883
7	1201125556	9818855923	2579242004	9634380763	2829750245
11	5205841188	4454788302	5930342065	6877245267	4037325128
18	6406966745	4273644225	8509584070	6511626030	6867075373
30	1612807933	8728432528	4439926136	3388871298	0904400501
48	8019774679	3002076754	2949510206	9900497328	7771475874
78	9632582613	1730509282	7389436343	3289368626	8675876375
127	7652357292	4732586037	0338946550	3189865955	6447352249
206	7284939905	6463095319	7728382893	6479234582	5123228624
334	4937297198	1195681356	8067329443	9669100538	1570580873
541	2222237103	7658776676	5795712337	6148335120	6693809497
875	7159534301	8854458033	3863041781	5817435658	8264390370
1416	9381771405	6513234709	9658754119	1965770779	4958199867
2292	6541305707	5367692743	3521795900	7783206438	3222590237
3709	5923077113	1880927453	3180550019	9748977217	8180790104
6002	2464382820	7248620196	6702345920	7532183656	1403380341
9711	8387459933	9129547649	9882895940	7281160873	9584170445
15714	0851842754	6378167846	6585241861	4813344530	0987550786
25425	9239302688	5507715496	6468137802	2094505404	0571721231
41140	0091145443	1885883343	3053379663	6907849934	1559272017
66565	9330448131	7393598839	9521517465	9002355338	2130993248
107705	9421593574	9279482183	2574897129	5910205272	3690265265
174271	8752041706	6673081023	2096414595	4912560610	5821258513
281977	8173635281	5952563206	4671311725	0822765882	9511523778
456249	6925676988	2625644229	6767726320	5735326493	5332782291
738227	5099312269	8578207436	1439038045	6558092376	4844306069
1194477	2024989258	1203851665	8206764366	2293418870	0177088360
1932704	7124301527	9782059101	9645802411	8851511246	5021394429
3127181	9149290786	0985910767	7852566778	1144930116	5198482789
5059886	6273592314	0767969869	7498369189	9996441363	0219877218
8187068	5422883100	1753880637	5350935968	1141371479	5418360007
13246955	1696475414	2521850507	2849305158	1137812842	5638237225
21434023	7119358514	4275731144	8200241126	2279184322	1056597232
34680978	8815833928	6797581652	1049546284	3416997164	6694834457
56115002	5935192443	1073312796	9249787410	5696181486	7751431689
90795981	4751026371	7870894449	0299333694	9113178651	4446266146
146910984	0686218814	8944207245	9549121105	4809360138	2197697835

F_{281} to F_{320}

	237706965	5437245186	6815101694	9848454800	3922538789	6643963981
	384617949	6123464001	5759308940	9397575905	8731898927	8841661816
	622324915	1560709188	2574410635	9246030706	2654437717	5485625797
	1006942864	7684173189	8333719576	8643606612	1386336645	4327287613
	1629267779	9244882378	0908130212	7889637318	4040774362	9812913410
	2636210644	6929055567	9241849789	6533243930	5427111008	4140201023
	4265478424	6173937946	0149980002	4422881248	9467885371	3953114433
	6901689069	3102993513	9391829792	0956125179	4894996379	8093315456
1	1167167493	9276931459	9541809794	5379006428	4362881751	2046429889
1	8068856563	2379924973	8933639586	6335131607	9257878131	0139745345
2	9236024057	1656856433	8475449381	1714138036	3620759882	2186175234
4	7304880620	4036781407	7409088967	8049269644	2878638013	2325920579
7	6540904677	5693637841	5884538348	9763407680	6499397895	4512095813
12	3845785297	9730419249	3293627316	7812677324	9378035908	6838016392
20	0386689975	5424057090	9178165665	7576085005	5877433804	1350112205
32	4232475273	5154476340	2471792982	5388762330	5255469712	8188128597
52	4619165249	0578533431	1649958648	2964847336	1132903516	9538240802
84	8851640522	5733009771	4121751630	8353609666	6388373229	7726369399
137	3470805771	6311543202	5771710279	1318457002	7521276746	7264610201
222	2322446294	2044552973	9893461909	9672066669	3909649976	4990979600
359	5793252065	8356096176	5665172189	0990523672	1430926723	2255589801
581	8115698360	0400649150	5558634099	0662590341	5340576699	7246569401
941	3908950425	8756745327	1223806288	1653114013	6771503422	9502159202
1523	2024648785	9157394477	6782440387	2315704355	2112080122	6748728603
2464	5933599211	7914139804	8006246675	3968818368	8883583545	6250887805
3987	7958247997	7071534282	4788687062	6284522724	0995663668	2999616408
6452	3891847209	4985674087	2794933738	0253341092	9879247213	9250504213
10440	1850095207	2057208369	7583620800	6537863817	0874910882	2250120621
16892	5741942416	7042882457	0378554538	6791204910	0754158096	1500624834
27332	7592037623	9100090826	7962175339	3329068727	1629068978	3750745455
44225	3333980040	6142973283	8340729878	0120273637	2383227074	5251370289
71558	0926017664	5243064110	6302905217	3449342364	4012296052	9002115744
115783	4259997705	1386037394	4643635095	3569616001	6395523127	4253486033
187341	5186015369	6629101505	0946540312	7018958366	0407819180	3255601777
303124	9446013074	8015138899	5590175408	0588574367	6803342307	7509087810
490466	4632028444	4644240404	6536715720	7607532733	7211161488	0764689587
793591	4078041519	2659379304	2126891128	8196107101	4014503795	8273777397
1284057	8710069963	7303619708	8663606849	5803639835	1225665283	9038466984
2077649	2788111482	9962999013	0790497978	3999746936	5240169079	7312244381
3361707	1498181446	7266618721	9454104827	9803386771	6465834363	6350711365

F_{321} to F_{360}

	5439356	4286292929	7229617735	0244602806	3803133708	1706003443	3662955746
	8801063	5784474376	4496236456	9698707634	3606520479	8171837807	0013667111
	14240420	0070767306	1725854191	9943310440	7409654187	9877841250	3676622857
	23041483	5855241682	6222090648	9642018075	1016174667	8049679057	3690289968
	37281903	5926008988	7947944840	9585328515	8425828855	7927520307	7366912825
	60323387	1781250671	4170035489	9227346590	9442003523	5977199365	1057202793
	97605290	7707259660	2117980330	8812675106	7867832379	3904719672	8424115618
	157928677	9488510331	6288015820	8040021697	7309835902	9881919037	9481318411
	255533968	7195769991	8405996151	6852696804	5177668282	3786638710	7905434029
	413462646	6684280323	4694011972	4892718502	2487504185	3668557748	7386752440
	668996615	3880050315	3100008124	1745415306	7665172467	7455196459	5292186469
	1082459262	0564330638	7794020096	6638133809	0152676653	1123754208	2678938909
	1751455877	4444380954	0894028220	8383549115	7817849120	8578950667	7971125378
	2833915139	5008711592	8688048317	5021682924	7970525773	9702704876	0650064287
	4585371016	9453092546	9582076538	3405232040	5788374894	8281655543	8621189665
	7419286156	4461804139	8270124855	8426914965	3758900668	7984360419	9271253952
1	2004657173	3914896686	7852201394	1832147005	9547275563	6266015963	7892443617
1	9423943329	8376700826	6122326250	0259061971	3306176232	4250376383	7163697569
3	1428600503	2291597513	3974527644	2091208977	2853451796	0516392347	5056141186
5	0852543833	0668298340	0096853894	2350270948	6159628028	4766768731	2219838755
8	2281144336	2959895853	4071381538	4441479925	9013079824	5283161078	7275979941
13	3133688169	3628194193	4168235432	6791750874	5172707853	0049929809	9495818696
21	5414832505	6588090046	8239616971	1233230800	4185787677	5333090888	6771798637
34	8548520675	0216284240	2407852403	8024981674	9358495530	5383020698	6267617333
56	3963353180	6804374287	0647469374	9258212475	3544283208	0716111587	3039415970
91	2511873855	7020658527	3055321778	7283194150	2902778738	6099132285	9307033303
147	6475227036	3825032814	3702791153	6541406625	6447061946	6815243873	2346449273
238	8987100892	0845691341	6758112932	3824600775	9349840685	2914376159	1653482576
386	5462327928	4670724156	0460904086	0366007401	5796902631	9729620032	3999931849
625	4449428820	5516415497	7219017018	4190608177	5146743317	2643996191	5653414425
1011	9911756749	0187139653	7679921104	4556615579	0943645949	2373616223	9653346274
1637	4361185569	5703555151	4898938122	8747223756	6090389266	5017612415	5306760699
2649	4272942318	5890694805	2578859227	3303839335	7034035215	7391228639	4960106973
4286	8634127888	1594249956	7477797350	2051063092	3124424482	2408841055	0266867672
6936	2907070206	7484944762	0056656577	5354902428	0158459697	9800069694	5226974645
11223	1541198094	9079194718	7534453927	7405965520	3282884180	2208910749	5493842317
18159	4448268301	6564139480	7591110505	2760867948	3441343878	2008980444	0720816962
29382	5989466396	5643334199	5125564433	0166833468	6724228058	4217891193	6214659279
47542	0437734698	2207473680	2716674938	2927701417	0165571936	6226871637	6935476241
76924	6427201094	7850807879	7842239371	3094534885	6889799995	0444762831	3150135520

F_{361} to F_{400}

124466	6864935793	0058281560	0558914309	6022236302	7055371931	6671634469	0085611761
201391	3292136887	7909089439	8401153680	9116771188	3945171926	7116397300	3235747281
325858	0157072680	7967370999	8960067990	5139007491	1000543858	3788031769	3321359042
527249	3449209568	5876460439	7361221671	4255778679	4945715785	0904429069	6557106323
853107	3606282249	3843831439	6321289661	9394786170	5946259643	4692460838	9878465365
1380356	7055491817	9720291879	3682511333	3650564850	0891975428	5596889908	6435571688
2233464	0661774067	3564123319	0003800995	3045351020	6838235072	0289350747	6314037053
3613820	7717265885	3284415198	3686312328	6695915870	7730210500	5886240656	2749608741
5847284	8379039952	6848538517	3690113323	9741266891	4568445572	6175591403	9063645794
9461105	6096305838	0132953715	7376425652	6437182762	2298656073	2061832060	1813254535
15308390	4475345790	6981492233	1066538976	6178449653	6867101645	8237423464	0876900329
24769496	0571651628	7114445948	8442964629	2615632415	9165757719	0299255524	2690154864
40077886	5046997419	4095938181	9509503605	8794082069	6032859364	8536678988	3567055193
64847382	5618649048	1210384130	7952468235	1409714485	5198617083	8835934512	6257210057
104925269	0665646467	5306322312	7461971841	0203796555	1231476448	7372613500	9824265250
169772651	6284295515	6516706443	5414440076	1613511040	6430093532	6208548013	6081475307
274697920	6949941983	1823028756	2876411917	1817307595	7661569981	3581161514	5905740557
444470572	3234237498	8339735199	8290851993	3430818636	4091663513	9789709528	1987215864
719168493	0184179482	0162763956	1167263910	5248126232	1753233495	3370871042	7892956421
1163639065	3418416980	8502499155	9458115903	8678944868	5844897009	3160580570	9880172285
1882807558	3602596462	8665263112	0625379814	3927071100	7598130504	6531451613	7773128706
3046446623	7021013443	7167762268	0083495718	2606015969	3443027513	9692032184	7653300991
4929254182	0623609906	5833025380	0708875532	6533087070	1041158018	6223483798	5426429697
7975700805	7644623350	3000787648	0792371250	9139103039	4484185532	5915515983	3079730688
1 2904954987	8268233256	8833813028	1501246783	5672190109	5525343551	2138999781	8506160385
2 0880655793	5912856607	1834600676	2293618034	4811293149	0009529083	8054515765	1585891073
3 3785610781	4181089864	0668413704	3794864818	0483483258	5534872635	0193515547	0092051458
5 4666266575	0093946471	2503014380	6088482852	5294776407	5544401718	8248031312	1677942531
8 8451877356	4275036335	3171428084	9883347670	5778259666	1079274353	8441546859	1769993989
14 3118143931	4368982806	5674442465	5971830523	1073036073	6623676072	6689578171	3447936520
23 1570021287	8644019141	8845870550	5855178193	6851295739	7702950426	5131125030	5217930509
37 4688165219	3013001948	4520313016	1827008716	7924331813	4326626499	1820703201	8665867029
60 6258186507	1657021090	3366183566	7682186910	4775627553	2029576925	6951828232	3883797538
98 0946351726	4670023038	7886496582	9509195627	2699959366	6356203424	8772531434	2549664567
158 7204538233	6327044129	1252680149	7191382537	7475586919	8385780350	5724359666	6433462105
256 8150889960	0997067167	9139176732	6700578165	0175546286	4741983775	4496891100	8983126672
415 5355428193	7324111297	0391856882	3891960702	7651133206	3127764126	0221250767	5416588777
672 3506318153	8321178464	9531033615	0592538867	7826679492	7869747901	4718141868	4399715449
1087 8861746347	5645289761	9922890497	4484499570	5477812699	0997512027	4939392635	9816304226
1760 2368064501	3966468226	9453924112	5077038438	3304492191	8867259928	9657534504	4216019675

F_{401} to F_{440}

2848	1229810848	9611757988	9376814609	9561538008	8782304890	9864771956	4596927140	4032323901	
4608	3597875350	3578226215	8830738722	4638576447	2086797082	8732031885	4254461644	8248343576	
7456	4827686199	3189984204	8207553332	4200114456	0869101973	8596803841	8851388785	2280667477	
12064	8425561549	6768210420	7038292054	8838690903	2955899056	7328835727	3105850430	0529011053	
19521	3253247748	9958194625	5245845387	3038805359	3825001030	5925639569	1957239215	2809678530	
31586	1678809298	6726405046	2284137442	1877496262	6780900087	3254475296	5063089645	3338689583	
51107	4932057047	6684599671	7529982829	4916301622	0605901117	9180114865	7020328860	6148368113	
82693	6610866346	3411004717	9814120271	6793797884	7386801205	2434590162	2083418505	9487057696	
133801	1542923394	0095604389	7344103101	1710099506	7992702323	1614705027	9103747366	5635425809	
216494	8153789740	3506609107	7158223372	8503897391	5379503528	4049295190	1187165872	5122483505	
350295	9696713134	3602213497	4502326474	0213996898	3372205851	5664000218	0290913239	0757909314	
566790	7850502874	7108822605	1660549846	8717894289	8751709379	9713295408	1478079111	5880392819	
917086	7547216009	0711036102	6162876320	8931891188	2123915231	5377295626	1768992350	6638302133	
1483877	5397718883	7819858707	7823426167	7649785478	0875624611	5090591034	3247071462	2518694952	
2400964	2944934892	8530894810	3986302488	6581676666	2999539843	0467886660	5016063812	9156997085	
3884841	8342653776	6350753518	1809728656	4231462144	3875164454	5558477694	8263135275	1675692037	
6285806	1287588669	4881648328	5796031145	0813138810	6874704297	6026364355	3279199088	0832689122	
10170647	9630242446	1232401846	7605759801	5044600955	0749868752	1584842050	1542334363	2508381159	
16456454	0917831115	6114050175	3401790946	5857739765	7624573049	7611206405	4821533451	3341070281	
26627102	0548073561	7346452022	1007550748	0902340720	8374441801	9196048455	6363867814	5849451440	
43083556	1465904677	3460502197	4409341694	6760080486	5999014851	6807254861	1185401265	9190521721	
69710658	2013978239	0806954219	5416892442	7662421207	4373456653	6003303316	7549269080	5039973161	
112794214	3479882916	4267456416	9826234137	4422501694	0372471505	2810558177	8734670346	4230494882	
182504872	5493861155	5074410636	5243126580	2084922901	4745928158	8813861494	6283939426	9270468043	
295299086	8973744071	9341867053	5069360717	6507424595	5118399664	1624419672	5018609773	3500962925	
477803959	4467605227	4416277690	0312487297	8592347496	9864327823	0438281167	1302549200	2771430968	
773103046	3441349299	3758144743	5381848015	5099772092	4982727487	2062700839	6321158973	6272393893	
1250907005	7908954526	8174422433	5694335313	3692119589	4847055310	2500982006	7623708173	9043824861	
2024010052	1350303826	1932567177	1076183328	8791891681	9829782797	4563682846	3944867147	5316218754	
3274917057	9259258353	0106989610	6770518642	2484011271	4676838107	7064664853	1568575321	4360043615	
	5298927110	0609562179	2039556787	7846701971	1275902953	4506620905	1628347699	5513442468	9676262369
	8573844167	9868820532	2146546398	4617220613	3759914224	9183459012	8693012552	7082017790	4036305984
1	3872771278	0478382711	4186103186	2463922584	5035817178	3690079918	0321360252	2595460259	3712568353
2	2446615446	0347203243	6332649584	7081143197	8795731403	2873538930	9014372804	9677478049	7748874337
3	6319386724	0825585955	0518752770	9545065782	3831548581	6563618848	9335733057	2272938309	1461442690
5	8766002170	1172789198	6851402355	6626208980	2627279984	9437157779	8350105862	1950416358	9210317027
9	5085388894	1998375153	7370155126	6171274762	6458828566	6000776628	7685838919	4223354668	0671759717
15	3851391064	3171164352	4221557482	2797483742	9086108551	5437934408	6035944781	6173771026	9882076744
24	8936779958	5169539506	1591712608	8968758505	5544937118	1438711037	3721783701	0397125695	0553836461
40	2788171022	8340703858	5813270091	1766242248	4631045669	6876645445	9757728482	6570896722	0435913205

F_{441} to F_{480}

65	1724950981	3510243364	7404982700	0735000754	0175982787	8315356483	3479512183	6968022417	0989749666
105	4513122004	1850947223	3218252791	2501243002	4807028457	5192001929	3237240666	3538919139	1425662871
170	6238072985	5361190588	0623235491	3236243756	4983011245	3507358412	6716752850	0506941556	2415412537
276	0751194989	7212137811	3841488282	5737486758	9790039702	8699360341	9953993516	4045860695	3841075408
446	6989267975	2573328399	4464723773	8973730515	4773050948	2206718754	6670746366	4552802251	6256487945
722	7740462964	9785466210	8306212056	4711217274	4563090651	0906079096	6624739882	8598662947	0097563353
1169	4729730940	2358794610	2770935830	3684947789	9336141599	3112797851	3295486249	3151465198	6354051298
1892	2470193905	2144260821	1077147886	8396165064	3899232250	4018876947	9920226132	1750128145	6451614651
3061	7199924845	4503055431	3848083717	2081112854	3235373849	7131674799	3215712381	4901593344	2805665949
4953	9670118750	6647316252	4925231604	0477277918	7134606100	1150551747	3135938513	6651721489	9257280600
8015	6870043596	1150371683	8773315321	2558390773	0369979949	8282226546	6351650895	1553314834	2062946549
12969	6540162346	7797687936	3698546925	3035668691	7504586049	9432778293	9487589408	8205036324	1320227149
20985	3410205942	8948059620	2471862246	5594059464	7874565999	7715004840	5839240303	9758351158	3383173698
33954	9950368289	6745747556	6170409171	8629728156	5379152049	7147783134	5326829712	7963387482	4703400847
54940	3360574232	5693807176	8642271418	4223787621	3253718049	4862787975	1166070016	7721738640	8086574545
88895	3310942522	2439554733	4812680590	2853515777	8632870099	2010571109	6492899729	5685126123	2789975392
143835	6671516754	8133361910	3454952008	7077303399	1886588148	6873359084	7658969746	3406864764	0876549937
232730	9982459277	0572916643	8267632598	9930819177	0519458247	8883930194	4151869475	9091990887	3666525329
376566	6653976031	8706278554	1722584607	7008122576	2406046396	5757289279	1810839222	2498855651	4543075266
609297	6636435308	9279195197	9990217206	6938941753	2925504644	4641219473	5962708698	1590846538	8209600595
985864	3290411340	7985473752	1712801814	3947064329	5331551041	0398508752	7773547920	4089702190	2752675861
1595161	9926846649	7264668950	1703019021	0886006082	8257055685	5039728226	3736256618	5680548729	0962276456
2581026	3217257990	5250142702	3415820835	4833070412	3588606726	5438236979	1509804538	9770250919	3714952317
4176188	3144104640	2514811652	5118839856	5719076495	1845662412	0477965205	5246061157	5450799648	4677228773
6757214	6361362630	7764954354	8534660692	0552146907	5434269138	5916202184	6755865696	5221050567	8392181090
10933402	9505467271	0279766007	3653500548	6271223402	7279931550	6394167390	2001926854	0671850216	3069409863
17690617	5866829901	8044720362	2188161240	6823370310	2714200689	2310369574	8757792550	5892900784	1461590953
28624020	5372297172	8324486369	5841661789	3094593712	9994132239	8704536965	0759719404	6564751000	4531000816
46314638	1239127074	6369206731	8029823029	9917964023	2708332929	1014906539	9517511955	2457651784	5992591769
74938658	6611424247	4693693101	3871484819	3012557736	2702465168	9719443505	0277231359	9022402785	0523592585
121253296	7850551322	1062899833	1901307849	2930521759	5410798098	0734350044	9794743315	1480054569	6516184354
196191955	4461975569	5756592934	5772792668	5943079495	8113263267	0453793550	0071974675	0502457354	7039776939
317445252	2312526891	6819492767	7674100517	8873601255	3524061365	1188143594	9866717990	1982511924	3555961293
513637207	6774502461	2576085702	3446893186	4816680751	1637324632	1641937144	9938692665	2484969279	0595738232
831082459	9087029352	9395578470	1120993704	3690282006	5161385997	2830080739	9805410655	4467481203	4151699525
1344719667	5861531814	1971664172	4567886890	8506962757	6798710629	4472017884	9744103320	6952450482	4747437757
2175802127	4948561167	1367242642	5688880595	2197244764	1960096626	7302098624	9549513976	1419931685	8899137282
3520521795	0810092981	3338906815	0256767486	0704207521	8758807256	1774116509	9293617296	8372382168	3646575039
5696323922	5758654148	4706149457	5945648081	2901452286	0718903882	9076215134	8843131272	9792313854	2545712321
9216845717	6568747129	8045056272	6202415567	3605659807	9477711139	0850331644	8136748569	8164696022	6192287360

F_{481} to F_{520}

1	4913169640	2327401278	2751205730	2148063648	6507112094	0196615021	9926546779	6979879842	7957009876	8737999681
2	4130015357	8896148408	0796262002	8350479216	0112771901	9674326161	0776878424	5116628412	6121705899	4930287041
3	9043184998	1223549686	3547467733	0498542864	6619883995	9870941183	0703425204	2096508255	4078715776	3668286722
6	3173200356	0119698094	4343729735	8849022080	6732655897	9545267344	1480303628	7213136668	0200421675	8598573763
10	2216385354	1343247780	7891197468	9347564945	3352539893	9416208527	2183728832	9309644923	4279137452	2266860485
16	5389585710	1462945875	2234927204	8196587026	0085195791	8961475871	3664032461	6522781591	4479559128	0865434248
26	7605971064	2806193656	0126124673	7544151971	3437735685	8377684398	5847761294	5832426514	8758696580	3132294733
43	2995556774	4269139531	2361051878	5740738997	3522931477	7339160269	9511793756	2355208106	3238255708	3997728981
70	0601527838	7075333187	2487176552	3284890968	6960667163	5716844668	5359555050	8187634621	1996952288	7130023714
113	3597084613	1344472718	4848228430	9025629966	0483598641	3056004938	4871348807	0542842727	5235207997	1127752695
183	4198612451	8419805905	7335404983	2310520934	7444265804	8772849607	0230903857	8730477348	7232160285	8257776409
296	7795697064	9764278624	2183633414	1336150900	7927864446	1828854545	5102252664	9273320076	2467368282	9385529104
480	1994309516	8184084529	9519038397	3646671835	5372130251	0601704152	5333156522	8003797424	9699528568	7643305513
776	9790006581	7948363154	1702671811	4982822736	3299994697	2430558698	0435409187	7277117501	2166896851	7028834617
1257	1784316098	6132447684	1221710208	8629494571	8672124948	3032262850	5768565710	5280914926	1866425420	4672140130
2034	1574322680	4080810838	2924382020	3612317308	1972119645	5462821548	6203974898	2558032427	4033322272	1700974747
3291	3358638779	0213258522	4146092229	2241811880	0644244593	8495084399	1972540608	7838947353	5899747692	6373114877
5325	4932961459	4294069360	7070474249	5854129188	2616364239	3957905947	8176515507	0396979780	9933069964	8074089624
8616	8291600238	4507327883	1216566478	8095941068	3260608833	2452990347	0149056115	8235927134	5832817657	4447204501
13942	3224561697	8801397243	8287040728	3950070256	5876973072	6410896294	8325571622	8632906915	5765887622	2521294125
22559	1516161936	3308725126	9503607207	2046011324	9137581905	8863886641	8474627738	6868834050	1598705279	6968498626
36501	4740723634	2110122370	7790647935	5996081581	5014554978	5274782936	6800199361	5501740965	7364592901	9489792751
59060	6256885570	5418847497	7294255142	8042092906	4152136884	4138669578	5274827100	2370575015	8963298181	6458291377
95562	0997609204	7528969868	5084903078	4038174487	9166691862	9413452515	2075026461	7872315981	6327891083	5948084128
154622	7254494775	2947817366	2379158221	2080267394	3318828747	3552122093	7349853562	0242890997	5291189265	2406375505
250184	8252103980	0476787234	7464061299	6118441882	2485520610	2965574608	9424880023	8115206979	1619080348	8354459633
404807	5506598755	3424604600	9843219520	8198709276	5804349357	6517696702	6774733585	8358097976	6910269614	0760835138
654992	3758702735	3901391835	7307280820	4317151158	8289869967	9483271311	6199613609	6473304955	8529349962	9115294771
1059799	9265301490	7325996436	7150500341	2515860435	4094219325	6000968014	2974347195	4831402932	5439619576	9876129909
1714792	3024004226	1227388272	4457781161	6833011594	2384089293	5484239325	9173960805	1304707888	3968969539	8991424680
2774592	2289305716	8553384709	1608281502	9348872029	6478308619	1485207340	2148308000	6136110820	9408589116	8867554589
4489384	5313309942	9780772981	6066062664	6181883623	8862397912	6969446666	1322268805	7440818709	3377558656	7858979269
7263976	7602615659	8334157690	7674344167	5530755653	5340706531	8454654006	3470576806	3576929530	2786147773	6726533858
11753361	2915925602	8114930672	3740406832	1712639277	4203104444	5424100672	4792845612	1017748239	6163706430	4585513127
19017338	0518541262	6449088363	1414750999	7243394930	9543810976	3878754678	8263422418	4594677769	8949854204	1312046985
30770699	3434466865	4564019035	5155157831	8956034208	3746915420	9302855351	3056268030	5612426009	5113560634	5897560112
49788037	3953008128	1013107398	6569908831	6199429139	3290726397	3181610030	1319690449	0207103779	4063414838	7209607097
80558736	7387474993	5577126434	1725066663	5155463347	7037641818	2484465381	4375958479	5819529788	9176975473	3107167209
130346774	1340483121	6590233832	8294975495	1354892487	0328368215	5666075411	5695648928	6026633568	3240390312	0316774306
210905510	8727958115	2167360267	0020042158	6510355834	7366010033	8150540793	0071607408	1846163357	2417365785	3423941515

F_{521} to F_{560}

341252285	0068441236	8757594099	8315017653	7865248321	7694378249	1967156997	5838863744	9718960282	8075121882	7164657336
552157795	8796399352	0924954366	8335059812	4375604156	5060388283	3816616204	5767256336	7872796925	5657756097	3740715821
893410080	8864840588	9682548466	6650077466	2240852478	2754766532	5783773202	1606120081	7591757208	3732877980	0905373157
1445567876	7661239941	0607502833	4985137278	6616456634	7815154815	7750930199	7444983826	7310717491	1807999862	8070030493
2338977957	6526080530	0290051300	1635214744	8857309113	0569921348	3534703401	9051103908	4902474699	5540877842	8975403650
3784545834	4187320471	0897554133	6620352023	5473765747	8385076164	1285633601	6496087735	2213192190	7348877705	7045434143
6123523792	0713401001	1187605433	8255566768	4331074860	8954997512	4820337003	5547191643	7115666890	2889755548	6020837793
9908069626	4900721472	2085159567	4875918791	9804840608	7340073676	6105970605	2043279378	9328859081	0238633254	3066271936
1 6031593418	5614122473	3272765001	3131485560	4135915469	6295071189	0926307608	7590471022	6444525971	3128388802	9087109729
2 5939663045	0514843945	5357924568	8007404352	3940756078	3635144865	7032278213	9633750401	5773385052	3367022057	2153381665
4 1971256463	6128966418	8630689570	1138889912	8076671547	9930216054	7958585822	7224221424	2217911023	6495410860	1240491394
6 7910919508	6643810364	3988614138	9146294265	2017427626	3565360920	4990864036	6857971825	7991296075	9862432917	3393873059
10 9882175972	2772776783	2619303709	0285184178	0094099174	3495576975	2949449859	4082193250	0209207099	6357843777	4634364453
17 7793095480	9416587147	6607917847	9431478443	2111526800	7060937895	7940313896	0940165075	8200503175	6220276694	8028237512
28 7675271453	2189363930	9227221556	9716662621	2205625975	0556514871	0889763755	5022358325	8409710275	2578120472	2662601965
46 5468366934	1605951078	5835139404	9148141064	4317152775	7617452766	8830077651	5962523401	6610213450	8798397167	0690839477
75 3143638387	3795315009	5062360961	8864803685	6522778750	8173967637	9719841407	0984881727	5019923726	1376517639	3353441442
121 8612005321	5401266088	0897500366	8012944750	0839931526	5791420404	8549919058	6947405129	1630137177	0174914806	4044280919
197 1755643708	9196581097	5959861328	6877748435	7362710277	3965388042	8269760465	7932286856	6650060903	1551432445	7397722361
319 0367649030	4597847185	6857361695	4890693185	8202641803	9756808447	6819679524	4879691985	8280198080	1726347252	1442003280
516 2123292739	3794428283	2817223024	1768441621	5565352081	3722196490	5089439990	2811978842	4930258983	3277779697	8839725641
835 2490941769	8392275468	9674584719	6659134807	3767993885	3479004938	1909119514	7691670828	3210457063	5004126950	0281728921
1351 4614234509	2186703752	2491807743	8427576428	9333345966	7201201428	6998559505	0503649670	8140716046	8281906647	9121454562
2186 7105176279	0578979221	2166392463	5086711236	3101339852	0680206366	8907679019	8195320499	1351173110	3286033597	9403183483
3538 1719410788	2765682973	4658200207	3514287665	2434685818	7881407795	5906238524	8698970169	9491889157	1567940245	8524638045
5724 8824587067	3344662194	6824592670	8600998901	5536025670	8561614162	4813917544	6894290669	0843062267	4853973843	7927821528
9263 0543997855	6110345168	1482792878	2115286566	7970711489	6443021958	0720156069	5593260839	0334951424	6421914089	6452459573
14987 9368584922	9455007362	8307385549	0716285468	3506737160	5004636120	5534073614	2487551508	1178013692	1275887933	4380281101
24250 9912582778	5565352530	9790178427	2831572035	1477448650	1447658078	6254229683	8080812347	1512965116	7697802023	0832740674
39238 9281167701	5020359893	8097563976	3547857503	4984185810	6452294199	1788303298	0568363855	2690978808	8973689956	5213021775
63489 9193750480	0585712424	7887742403	6379429538	6461634460	7899952277	8042532981	8649176202	4203943925	6671491979	6045762449
102728 8474918181	5606072318	5985306379	9927287042	1445820271	4352246476	9830836279	9217540057	6894922734	5645181936	1258784224
166218 7668668661	6191784743	3873048783	6306716580	7907454732	2252198754	7873369261	7866716260	1098866660	2316673915	7304546673
268947 6143586843	1797857061	9858355163	6234003622	9353275003	6604445231	7704205541	7084256317	7993789394	7961855851	8563330897
435166 3812255504	7989641805	3731403947	2540720203	7260729735	8856643986	5577574803	4950972577	9092656055	0278529767	5867877570
704113 9955842347	9787498867	3589759110	8774723826	6614004739	5461089218	3281780345	2035228895	7086445449	8240385619	4431208467
1139280 3768097852	7777140672	7321163058	1315444030	3874734475	4317733204	8859355148	6986201473	6179101504	8518915387	0299086037
1843394 3723940200	7564639540	0910922169	0090167857	0488739214	9778822423	2141135493	9021430369	3265546954	6759301006	4730294504
2982674 7492038053	5341780212	8232085227	1405611887	4363473690	4096555628	1000490642	6007631842	9444648459	5278216393	5029380541
4826069 1215978254	2906419752	9143007396	1495779744	4852212905	3875378051	3141626136	5029062212	2710195414	2037517399	9759675045

F_{561} to F_{571}

7808743	8708016307	8248199965	7375092623	2901391631	9215686595	7971933679	4142116779	1036694055	2154843873	7315733793	4789055586		
12634812	9923994562	1154619718	6518100019	4397171376	4067899501	1847311730	7283742915	6065756267	4865039287	9353251193	4548730631		
20443556	8632010869	9402819684	3893192642	7298563008	3283586096	9819245410	1425859694	7102450322	7019883161	6668984986	9337786217		
33078369	8556005432	0557439403	0411292662	1695734384	7351485598	1666557140	8709602610	3168206590	1884922449	6022236180	3886516848		
53521926	7188016301	9960259087	4304485304	8994297393	0635071695	1485802551	0135462305	0270656912	8904805611	2691221167	3224303065		
86600296	5744021734	0517698490	4715777967	0690031777	7986557293	3152359691	8845064915	3438863503	0789728060	8713457347	7110819913		
140122223	2932038036	0477957577	9020263271	9684329170	8621628988	4638162242	8980527220	3709520415	9694533672	1404678515	0335122978		
226722519	8676059770	0995656068	3736041239	0374360948	6608186281	7790521934	7825592135	7148383919	0484261733	0118135862	7445942891		
366844743	1608097806	1473613646	2756304511	0058690119	5229815270	2428684177	6806119356	0857904335	0178795405	1522814377	7781065869		
593567263	0284157576	2469269714	6492345750	0433051068	1838001552	0219206112	4631711491	8006288254	0663057138	1640950240	5227008760		
960412006	1892255382	3942883360	9248650261	0491741187	7067816822	2647890290	1437830847	8864192589	0841852543	3163764618	3008074629		

Puzzles and Problems

1. *A Solid Dissection Problem*

By means of three straight cuts dissect a 24x9x8 rectangular block into the fewest possible pieces that will form a cube.

(Harry Lindgren; Canberra, Australia)

* * * *

2. *The Yacht Races*

Lord Ambling, Mrs. Beetle, Captain Clam, and Admiral Dreery arranged a series of yacht races. In the first of four races each was to sail his or her own yacht; in the remaining three races each owner was to sail each of the other yachts in turn.

In the second race, Mrs. Beetle sailed *Firefly* and the admiral sailed *Gazelle.* In the third race, Captain Clam sailed *Hydra* and the admiral *Irene.* In the last race, *Hydra* was sailed by Mrs. Beetle and *Gazelle* by Captain Clam.

Who owned which yacht?

(H. E. Tester; Middlesex, England)

* * * *

3. *Singletree Farm*

This farm is said to have got its name from a walled park in it having one lone tree in the middle.

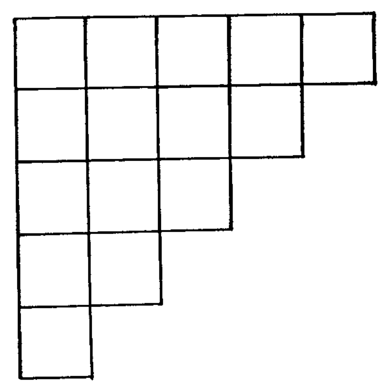
The park is exactly the same area as a square with 143 yard sides and, like a square, one point within it is the same distance from each of its four corners. Unlike a square, however, only two equal sides meet at right angles: the other two sides are not equal.

Since the lone tree stands at this rather special point, it may be that it was planted there to mark the spot.

All four sides are whole numbers of yards in length, so perhaps you can discover what these lengths are.

(Sinclair Grant; Perth, Scotland)

* * * *



4. *The Postmaster's Problem*

The diagram shows all that is left of a sheet of postage stamps. If a customer ordered three stamps, how many different ways could the postmaster select three connected stamps? The stamps must be connected on at least one side.

How many ways could he select four connected stamps? Five connected stamps?

(C. R. Dickinson; Camas, Washington)

5. *Here you Meet Resistance*

In an electric resistor assembly, invented by Thomas C. Wright of the International Resistance Co., four resistors are encased side by side with lead-wires free as in Figure 1. They can be joined in 51 different ways, some of which are shown in Figure 2. If the component resistances are 1, 2, 4, and 8 ohms, the resistances of the combinations have a "spectrum" as in Figure 3, partly crowded and partly bare.

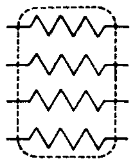


FIG. 1

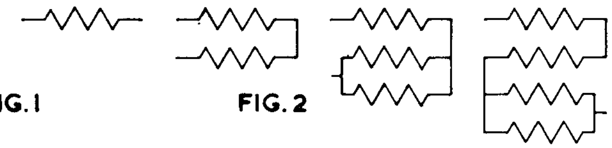
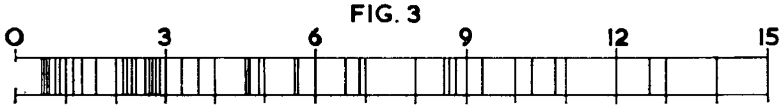


FIG. 2

FIG. 3



The problem is this. Find values of the component resistances that make the spectrum more regular. To keep the problem within bounds, let us restrict them to integral values of at most 10 ohms. But if anyone finds a set of non-integral values that gives a remarkably regular spectrum, well and good !

Most problems have one solution that is right, all others being wrong, but this is not one of them. The Editor will publish the solutions submitted that seem best to him. His task will be made easier if your solution includes a spectrum.

(*Harry Lindgren;* Canberra, Australia)

* * * *

6. *Area Duplication Puzzle*

Mr. Smith, our accountant friend, was quite proud of his mathematically talented son. Each would toss puzzles at each other to test the other's alertness.

"I have here a piece of cardboard," asked Mr. Smith of his son one day, "measuring 20x20 inches. Can you make up five separate cardboard squares having the same total area?"

In a few moments, his son said, "Yes, I have a solution in which three of the squares are the same size."

"Very good," said Mr. Smith, "Does each of those measure 5x5x5 inches?"

"No, they're more than four times greater in area."

Surprised, Mr. Smith checked his son's solution and found that it was correct.

Since Mr. Smith had made no error, either, maybe you can find the solutions.

(*D. C. Cross;* Birmingham, England)

* * * *

7. *A Problem in Logic*

In a row of five houses live five couples: Mr. and Mrs. Green, Brown, Smith, Jones, and Cook. There are five tradesmen, none of whom are married, who call at these houses, namely, grocer, coalman, baker, butcher, and

milkman. The tradesmen's names are Green, Brown, Smith, Jones, and Cook, but not respectively.

The butcher's married sister lives in No. 1.
Mr. Jones lives next door but one to the coalman's namesake.
The milkman's namesake has no relatives.
The butcher's namesake lives at No. 2.
Mr. Jones goes to work with the butcher's brother-in-law.
Mr. Brown helps the coalman's namesake in the garden.
Mr. Smith lives next door but one to the milkman's namesake.
Mrs. Green and Mrs. Jones are sisters.
The baker's namesake has only one brother-in-law, who lives in No. 3.
Mr. Cook lives next door to the coalman's namesake.

What is the name of each tradesman? In what numbered house do each of the couples reside?

(*Howard C. Saar;* Petoskey, Michigan)

* * * *

8. *Weatherman's Report*

On the day before yesterday, the weatherman said, "Today's weather is different from yesterday's. If the weather is the same tomorrow as it was yesterday, the day after tomorrow will have the same weather as the day before yesterday. But if the weather is the same tomorrow as it is today, the day after tomorrow will have the same weather as yesterday."

It is raining today, and it rained on the day before yesterday. What was the weather like yesterday? (*Note:* The prediction was correct!)

(*Howard C. Saar;* Petoskey, Michigan)

Book Reviews

The USSR Olympiad Problem Book by D. O. Shklarsky, N. N. Chentzov, I. M. Yaglom; translated by John Maykovich, revised and edited by Irving Sussman; W. H. Freeman and Co., 1962, 462 pages. $9.00.

Irving Sussman is indeed to be congratulated on his noteworthy contribution to both serious and recreational mathematics in his handling of this selection of problems from mathematical competitions held in the USSR.

Here we have a collection of 320 quite unconventional problems in the fields of elementary Number Theory, algebra, and arithmetic, with fully detailed solutions for all. Not one of these problems could be classed as dull, or as the type of problem to be found in normal textbooks. And the solutions, some occupying several pages, are all most unusually clear in the expositions.

The authors claim that the book is designed for students at high-school level, in the USSR, and most of its contents should certainly be within the capability of our grade 12 students who may have some flair for mathematics.

Thirteen well-known and popular "old-timers" comprise the first chapter. Amongst these we find the problem of the counterfeit coin, with unusual variations on the same theme: the solutions of these are very detailed, with interesting and understandable discussion of the more general aspects of such problems.

Then we have 80 more pages of problems, grouped by chapters under headings such as: "The Divisibility of Integers", "Equations Having Integer Solutions", "Some Distinctive Inequalities", "Difference Sequences and Sums". Some of these chapters open with introductory comments which may to some extent prepare the enthusiast for what follows.

The rest of this quite large book is devoted to solutions, and with a final summarized list of "answers".

The wide variety of problems, and the very lucid solutions, should provide almost endless scope for student and for teacher in opening up avenues for stimulating research and discussion. For the recreational math enthusiast there is a wealth of material, enough almost to last a lifetime!

This book should be found in every high school and university library, as it will surely be found in years to come on the shelves of true recreational mathematics addicts—well worn and much thumbed.

J. A. Hunter; Toronto, Ontario

* * * *

Elementary Vector Geometry by Seymour Schuster, John Wiley & Sons, 1962. $4.95.

As an outgrowth of several lectures given at an NSF summer institute, Seymour Schuster, of the Carleton College mathematics staff, has developed a cohesive, well-organized treatise in *Elementary Vector Geometry.* Beginning with fundamental definitions and operations with vectors, the author proceeds to develop analytical geometry and certain phases of trigonometry from the vector viewpoint. The reviewer offers particular commendation to the author for his clearly written chapter on inner products.

Although the material is not totally new to the seasoned mathematician, it does develop under one cover the significant applications of elementary vector analysis. Further, this development is written at a level which is easily comprehended by a good senior high school student of mathematics or by the undergraduate. For those high schools seeking a "good" semester course in mathematics at the junior-senior level, this text should be given serious consideration.

The format is excellent and the many figures and diagrams are well marked, notationwise. There are sufficient examples and exercises to render the work quite teachable in a classroom situation and useless redundancy of problem types has been minimized. Although aimed primarily at an academic audience, the avocational mathematician would certainly enjoy this delightful contribution to "modern" mathematics.

Howard C. Saar; Petoskey, Michigan

Numbers, Numbers, Numbers

INTEGRAL CUBE AS THE SUM OF THREE CUBES

$$W^3 = X^3 + Y^3 + Z^3$$

In the June 1962 RMM (page 46) we showed how the cube of 870 can be expressed in ten different ways as the sum of three cubes, this result being due to *Dr. Leon Bankoff.* We challenged readers to better this with some other integer, *W,* less than 1000.

David A. Klarner, of *Humboldt State College,* has met this challenge with amazing success. He shows that the cube of 492 can be so expressed in ten different ways, and the cube of 792 in *eleven* different ways.

Here we give the tabulation for $W = 492$ and for $W = 792$.

$W = 492$			$W = 792$		
X	Y	Z	X	Y	Z
24	204	480	30	456	738
48	85	491	88	528	704
72	384	396	108	184	788
113	264	463	188	298	774
144	360	414	189	387	756
176	204	472	225	279	774
207	297	438	288	414	738
226	332	414	292	540	680
246	328	410	374	429	715
281	322	399	396	528	660
			480	542	610

Readers may use the fact that $X + Y + Z - W$ is divisible by 6 as a check on any results they may discover.

* * * *

CURIOSITIES

Two-Way Consecutives

J. A. Lindon, of *Weybridge, England* reports that there are numbers formed by writing down two consecutive integers in ascending order, which can also be expressed as the product of two consecutive factors. They were found by using a simple homemade mechanical device explained in Uspensky & Heaslet's *Elementary Number Theory* (McGraw-Hill). Some of the results are listed.

$12 = 3 \times 4$ $56 = 7 \times 8$ $6162 = 78 \times 79$
$65479\ 65480 = 80919 \times 80920$ $84289\ 84290 = 91809 \times 91810$
$106609\ 106610 = 326510 \times 326511$ $453589\ 453590 = 673490 \times 673491$
$1869505\ 1869506 = 4323777 \times 4323778$
$1667833021\ 1667833022 = 4083911141 \times 4083911142$
$9225507211\ 9225507212 = 9604950396 \times 9604950397$

Again, readers may care to search for some of the others not reported here.

Bisecting the Square

Mr. Lindon also reports a number of unusual and interesting squares. He finds many squares which are composed of consecutive pairs of integers or which are composed of identical pairs of integers. Here are just a few of the many examples Mr. Lindon has found. Readers may feel the urge to find other examples.

Squares of the form N|N + 1:

$$428^2 = 183\ 184$$
$$846^2 = 715\ 716$$
$$7810^2 = 6099\ 6100$$
$$36365^2 = 13224\ 13225$$
$$63636^2 = 40495\ 40496$$
$$326734^2 = 106755\ 106756$$
$$4705882 3^2 = 22145328\ 22145329$$
$$331983807^2 = 110213248\ 110213249$$
$$422892898^2 = 178838403\ 178838404$$
$$615384615^2 = 378698224\ 378698225$$
$$961722488^2 = 924910143\ 924910144$$
$$3960396041^2 = 1568473680\ 1568473681$$
$$9118892970^2 = 8315420899\ 8315420900$$

Squares of the form N|N − 1:

$$91^2 = 82\ 81$$
$$9079^2 = 8242\ 8241$$
$$9901^2 = 9802\ 9801$$
$$88225295^2 = 77837026\ 77837025$$
$$99990001^2 = 99980002\ 99980001$$
$$8900869208^2 = 7922547265\ 7922547264$$

Squares of the form N|N

$$36363636364^2 = 13223140496\ 13223140496$$
$$63636363637^2 = 40495867769\ 40495867769$$
$$90909090910^2 = 82644628100\ 82644628100$$
$$428571428571428571429^2 = 183673469387755102041\ 183673469387755102041$$
$$857142857142857142858^2 = 734693877551020408164\ 734693877551020408164$$

* * * *

Number Odditity

J. A. H. Hunter reports the following oddity, the only 3-digit pairs exhibiting this odd property:

$$243 \times 432 = 324^2$$
$$486 \times 864 = 648^2$$

The Series $2n^2$-1

by Sinclair Grant

It is well known that simple, integral right-angled triangles can be derived by partitioning the square of any odd integer into two consecutive integers, thus:

n	(n^2)	n	$1/2(n^2 - 1)$	$1/2(n^2 + 1)$
1	(1)	1	0	1
3	(9)	3	4	5
5	(25)	5	12	13
7	(49)	7	24	25
9	(81)	9	40	41

If we now form a series by adding together the *legs* of each triangle (i.e., base and perpendicular) we get 1, 7, 17, 31, 49, etc. This is the series $2n^2 - 1$.

Let us experiment with it, by adding together the squares of successive terms:

$$1^2 + 7^2 = 50 = 2 \times 5^2$$
$$7^2 + 17^2 = 338 = 2 \times 13^2$$
$$17^2 + 31^2 = 1250 = 2 \times 25^2$$

In each case, the double squares shown on the right correspond to the hypotenuses of our original series of right-angled triangles.

It is also obvious that this "hypotenuse" series gives a set of integers satisfying the equation $x^2 + y^2 = 2z^2$. It therefore produces a series of squares whose diagonals equal the diagonals of integral rectangles, e.g.:

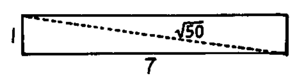


Figure 1

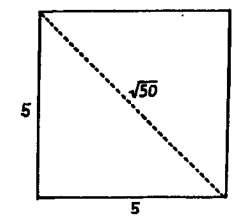


Figure 2

It thus leads to a series of cyclic quadrilaterals, e.g.:

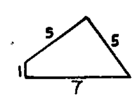


Figure 3

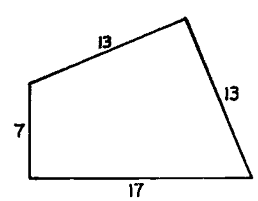


Figure 4

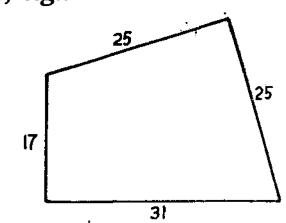


Figure 5

the areas of which equal *(average of unequal sides)*^{2}. The areas of these cyclic figures thus produce the further series: 16, 144, 576, 1600, etc., which can be written otherwise as $16(1^2, 3^2, 6^2, 10^2$, etc.) Note here that 1, 3, 6, 10, etc., is the series of *triangular numbers.*

Let us experiment further, by adding 2 to each term of the series $2n^2 - 1$:

1, 7, 17, 31, 49, ...
3, 9, 19, 33, 51, ...

Taking the *difference* between the squares of corresponding terms, we get: $9^2 - 7^2 = 32 = 2 \times 4^2$, $33^2 - 31^2 = 128 = 2 \times 8^2$, etc., so giving a series that satisfies $x^2 - y^2 = 2w^2$.

Geometrically, therefore, we now have a series giving the Main Diagonal, Vertical Height, and Base Length for a series of square prisms in *integers.* Figure 6, for example.

Now, if four such equal prisms are set up on their bases so that a common edge is the central vertical axis, we obtain the Slant Edge, Vertical Height, and Base Length of an *integral* regular pyramid as shown in Figure 7.

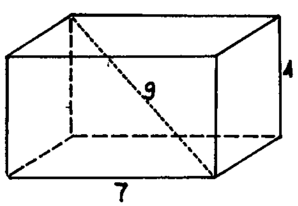


Figure 6

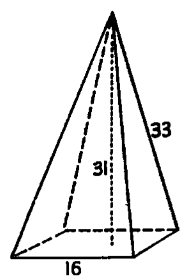


Figure 7

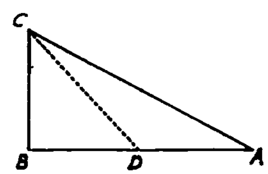


Figure 8

The series 1, 7, 17, 31, etc., also enables us to provide an *integral* solution to the problem of finding a point D in the side of a right-angled triangle ABC so that $DA = DC$.

We assign to BD the value ab, where ab is the product of any two consecutive terms of the series, e.g. $ab = 1 \times 7 = 7$, $ab = 7 \times 17 = 119$, etc. Then $DC = 1/2\ (a^2 + b^2)$, $BC = 1/2\ (a^2 - b^2)$.

The properties we have noted in the foregoing are not peculiar to the series $2n^2 - 1$, however. If a series of right-angled is built up with sides $4n$ and $4n^2 \pm 1$, for successive values of n, we get:

$4n$	$4n^2 - 1$	$4n^2 + 1$
4	3	5
8	15	17
12	35	37
16	63	65

If, as before, we add the *legs,* we derive the series 7, 23, 47, 79, etc., which also satisfies the equation $x^2 + y^2 = 2z^2$, where z is again a hypotenuse. The cycle quadrilaterals are now such as:

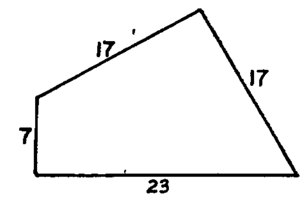


Figure 9

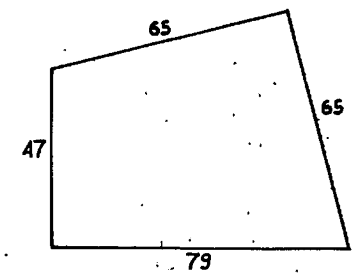


Figure 10

and here the *perimeters* gives the series 64, 144, 256, etc., which is $16(2^2, 3^2, 4^2$, etc.).

The series 7, 23, 47, 79 etc. also solves the previous triangle problem is we multiply together any two consecutive terms, e.g.

$ab = 7 \times 23 = 161$, $\quad 1/2\ (a^2 + b^2) = 289$, $\quad 1/2\ (a^2 - b) = 240$.

The "why" of all these diverting results will be apparent if the various operations that have been outlined numerically are simulated in general terms algebraically, starting from the original expressions which have been given for the sides of the basic triangles.

SOLUTION TO PERMUTACROSTIC ON PAGE 10

(1) *Rational*
(2) *Mantissa*
(3) *Median*
(4) *Isoceles*
(5) *Straight edge*
(6) *Trapezoid*
(7) *Rhombus*
(8) *Equation*
(9) *Modulus*
(10) *Eliminant*
(11) *Numerator*
(12) *Differentiation*
(13) *Ordinate*
(14) *Unknown*
(15) *Subtraction*
(16) *Fundamentals*
(17) *Units*
(18) *Napierian*

Recent Research in Mersenne Numbers, $M_p = 2^p - 1$

by Sidney Kravitz and Murray Berg

The purpose of this brief note is to bring readers of RMM up to date on recent work done on the Mersenne numbers.

In Reference 1, Alexander Hurwitz reported that he had tested prime exponents up to 5000, using Lucas' test, and had discovered that $2^{4253} - 1$ and $2^{4423} - 1$ are prime numbers. In a letter from the UCLA Computing Facility, dated March 12, 1962, Mr. Hurwitz further states:

"We have already tested the Mersenne numbers for exponent p less than 6000. No new prime were discovered. We will probably not continue searching for primes with exponent p greater than 6000. For exponent p around 6000 the test takes approximately 2-1/2 hours on the IBM 7090.

"The factors which you mention in your letter were among those communicated to us by John Brillhart. All the factors F of the Mersenne numbers for p prime, p less than 20,000, and F less than 2^{30} are known."

Thanks to a generous donation of time by the Standard Oil Company of California the authors are currently testing prime exponents between 6000 and 7000 using the IBM 7090 at the company's Electronic Computing Center in San Francisco.* At the moment prime exponents between 6000 and 6337 inclusive have been tested without finding any new Mersenne Primes. The prime 6337 required 2 hours 48 minutes. If a new Mersenne Prime is discovered in subsequent testing RMM readers will be the first to receive the news. The authors have verified that Riesel's M_{3217} and Hurwitz's M_{4253} and M_{4423} are prime.

Progress has also been made in finding divisors of Mersenne numbers. In Reference 2, H. Riesel publishes all divisors less than 10^8 for p less than 10^4. It is expected that John Brillhart will also publish an extensive list of divisors soon.

REFERENCES FROM *MATHEMATICS OF COMPUTATION*

1. A. Hurwitz, "New Mersenne Primes" April 1962, page 249
2. H. Riesel, "All Factors q less than 10^8 in All Mersenne Numbers, p Prime less than 10^4" October 1962, page 478.

See also "The 20 Mersenne Primes" *RMM No. 8, April 1962, pages 25 to 28*

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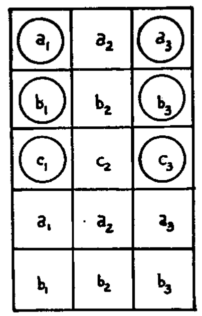
* The IBM 709 Facility at Picatinny Arsenal, Dover, New Jersey, has also contributed time to this project.

A Fair Exchange

by Peter Hagis, Jr.
Temple University

The well-known diagrammatic procedure for evaluating a third order determinant can be phrased in the language of chess as follows. Consider a chessboard consisting of fifteen squares arranged in five rows and three columns. Let the squares in the first three rows be assigned numerical values which are equal to the elements which occupy the corresponding positions in the determinant to be evaluated. In the remaining two rows rewrite the elements of the first two rows. Now place three white bishops on the board, one on each of the first three squares of the first column. Similarly, place three black bishops in the third column, one on each of the first three rows. We define the "weight" of each bishop as the product of three elements. The first of these is the element in the square occupied by the bishop. The remaining two are the elements in those squares which lie diagonally *below* the bishop and are under attack by the bishop. Thus, in the accompanying diagram, in which the positions of the pieces are indicated by circles, the weight of the white bishop in row one, column one is $a_1b_2c_3$. The weight of the black bishop in row two, column three is $b_3c_2a_1$. We easily verify that the total weight of the white bishops minus the total weight of the black bishops is equal to the value of the determinant.



Now most students of the game of chess are in agreement that the bishop and the knights are of equal value. That is, if a white (black) bishop is exchanged for a black (white) knight then the relative strength of the opponents remains unchanged. A bishop for a knight is an even trade. Since mathematics and chess are two closely allied disciplines it is perhaps of interest to investigate the mathematical effect of such an exchange on the above procedure for evaluating a determinant.

Therefore, let us replace all the white bishops by black knights and all the black bishops by white knights in our diagram. Thus, the circles in columns one and three now represent black and white knights respectively. As before we define the weight of each knight as the product whose factors are the element in the square occupied by the knight and the elements in the two squares lying *below* the knight and under attack by it. Thus, the weight of the black knight in row two, column one is $b_1a_2c_3$, while the weight of the white knight in row three, column three is $c_3b_2a_1$. We easily verify that the total weight of the white knights minus the total weight of the black knights is $a_3b_1c_2 + b_3c_1a_2 + c_3a_1b_2 - a_1b_3c_2 - b_1c_3a_2 - c_1a_3b_2$. This, of course, is the value of the determinant as any devotee of chess could have predicted. We conclude that a knight for a hishop is a fair exchange in mathematics as well as in chess.

ANSWERS FOR THE AUGUST 1962 ISSUE OF RECREATIONAL MATHEMATICS MAGAZINE

ALPHAMETICS (Page 11 — August 1962 RMM)

(1) SEND MORE MONEY = 9567 1085 10652
(2) EIGHT FIVE FOUR = 12780 6321 6549
(3) TWO TWO THREE = 138 138 19044
(4) TWO SEVEN TWO BOB JOE = 237 56169 237 474 876
(5) EVE DID .TALKTALKTALK = 212 606 .349834983498. . .
= 242 303 .798679867986. . .
(6) POST TOPS STOP = 3285 5238 8523
(7) NOW TWO IN TWIN ORBIT = 312 921 73 9273 10579
= 214 941 52 9452 10659

Cover Alphametic: PLEASE PARDON DELAYS = 451681 463927 915608

* * * *

PUZZLES AND PROBLEMS (Pages 34-36 — August 1962 RMM)

1. *More Number Curiosities:* What are the lowest and highest square numbers containing the nine digits (excluding 0) exactly twice, three times, four times, etc.?

No answers to this one, yet.

2. *Another Balance Scale Problem — Eight Coins:* Since a relatively short time between this issue and the last would prevent many readers from working out this problem before seeing the answer, we will postpone this answer until the February 1963 issue of RMM. Mr. Fujimura has given us a complete analysis of determining, in the least number of weighings, the relative order of weights of N coins, all differing in weight, using only a balance scale without weights.

3. *Publisher's Dilemma:* There were 86 \$0.95 subscriptions, 29 1-year subscriptions, 27 2-year subscriptions and 9 3-year subscriptions.

4. *Trouble at the Trestle:* Mr. Smith can run 15 miles per hour.

5. *Problem of the Generations:* The first grandfather was the father of seven children.

6. *Toy Bricks:* The problem is one of finding solutions to the equation $a^3 + b^3 = c^3 + d^3$. The only solutions involving sums less than 10,000 are $(a, b, c, d,) = (12, 1, 10, 9)$ or $(16, 2, 15, 9)$ where $a^3 + b^3 = 1729$ or 4104, respectively. As the boy said he had over 2000 bricks, the first solution is eliminated and he must have had 4104 bricks.

7. *The Farmer's Fields:* We must find three Pythagorean triangles of equal area, but of different dimensions. The smallest possible area is 840 with the sides of the triangles (42, 40, 58); (70, 24, 74); (112, 15, 113).

8. *Simplified Multiplication:* Another thirteen digit number which can be multiplied by eight by merely moving the digit on the right to the front: 1139240506329.

You can multiply 1034482758620689655172413793 by 3 merely moving the 3 at the end up to the front, forming 3103448275862068965517241379.

There are many other numbers with similar properties and we will leave it to the readers to discover them.

RECREATIONAL MATHEMATICS magazine
JUNIOR DEPARTMENT

All correspondence and material relating to the Junior Department should be sent to:

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The editor wishes to take this opportunity of expressing his thanks to the several readers who have written to him regarding the JUNIOR DEPARTMENT. I would further like to encourage all our readers to make use of this section of RMM by contributing articles, problems, solutions, and any other material which they feel would be suitable for inclusion. The editor has never quite ceased to be amazed at the respectable mathematics which some of our younger readers are capable of producing. For example, consider our mailbox this month which brings us the following article from twelve year old Avner Ash of Santa Monica, California.

* * * *

The Magic of One Ninety-Seventh

by Avner Ash

Among the many decimals found in existence, there are a few that possess certain special properties. Invariably, their denominators are prime and their numerators are unity. Their properties are very amazing and will be discussed in the text that follows.

Before going on, it should be stated that this article's title is somewhat misleading. Its primary use is to catch the reader's eye. However, as it is a title, it must (and does) contain a shred of truth. Namely, the period of 1/97 will be discussed. However, also involved will be 1/7, 1/17, 1/19, 1/29, our friend 1/97, probably 1/13, and perhaps a few others. Of course, nothing about them is magical, only mathematical.

First, consider some general properties. (1) All numerators are unity and all denominators are prime. In other words, the quantities under consideration are reciprocals of primes. However, it will not work (the so-called magic) for any prime reciprocal. The magic ones are hand-picked, and between 1 and 1/100 there are only five known to the author to be magic, although I have my personal suspicions about a few others. (2) Take the period of a magic reciprocal and drop the decimal point. Now there exists a number with digits numbering one less than the prime of which it is a reciprocal. Call this number m. Multiply m by any integer from 1 to the length of m, call this k. The product is m in a cyclic arrangement of itself. Note that this is the same as saying that $1/p$ (p is the prime), $2/p, 3/p, \ldots (p-1)/p$ are all cyclic arrays of each other. If m were multiplied by p, there would be $(p-1)$ nines. (3) Call each digit, starting from the left, $d_1, d_2, \ldots d_n, (n = p - 1)$. If m is split in two, the first digit of the first half and the first digit of the second half equals nine. The same is true for the second digit of both halves, the third and so on. If m were divided into equal groups (of two each, of three each if $p-1$ is divisible by three, of four groups if $p-1$ is divisible by four, etc.), and then add them together, the result is a number divisible by nine, usually in an interesting manner. (4) Item (3) also holds when you use the remainders in the division of p: $r_1, r_2, r_3, \ldots r_n$ $(n = p - 1)$. However, add carefully. If the remainders are 1, 6, 10, and 18, and if they are taken in pairs as follows: 1 & 18 plus 6 & 10, it would not be 118 added to 610, but $(1\times10) + 18$ plus $(6 \times 10) + 10$, or 28 plus 70 equals 98. In this case, the sum is always evenly divisible by p.

Now we proceed to some examples. The simplest of these magic reciprocals is 1/7. Its period is 142857. Note that it has six digits in its period, (7—1). Multiply it by two, the answer is 285714, a cyclic arrangement of the numbers 1 4 2 8 5 7, in that order. In fact, the cyclic property of these decimals can be shown by means of the chart at the right. To multiply start at the digit next to which is the multiplier in parenthesis, and continue clockwise around the arragnment. For instance, to multiply by four, start at the 5 and read off 571428. If you multiply by 7, the result is 999999.

1(1)
7(5) 4(3)
5(4) 2(2)
8(6)

For the next test we add the first digit of each half, i.e. $1 + 8 = 9$. Also, we get $4 + 5 = 9$ and $2 + 7 = 9$. Taking m in groups of three (e.g. 142 and 857) and in different cyclic orders (e.g. 428 and 571 or 285 and 714) we find the sums of these pairs equal 999. Next, taking them in groups of two and in different cyclic orders (e.g. 14, 28, 57 or 42, 85, 71) we find their sums are 99 or multiples of 99. And, of course, the sum of all six digits is 27 (3×9). During the process of dividing 1 by 7, the remainders are 1, 3, 2, 6, 4, and 5. Try the same procedure with them as we have just done with the digits in the original period.

Try doing the same thing with the period of 1/17 which is 0588235294117647. Everything as stated in sections (1) through (4) holds. To get started notice that 05882352+94117647=99999999. In this multiplication table for seventeen, note that each product is a cyclic order of the original and of each other.

$1\times0588235294117647=0588235294117647$
$2\times0588235294177647=1176470588235294$
$3\times0588235294117647=1764705882352941$
$4\times0588235294117647=2352941176470588$
$5\times0588235294117647=2941176470588235$
$6\times0588235294117647=3529411764705882$
$7\times0588235294117647=4117647058823529$
$8\times0588235294117647=4705882352941176$
etc.

And, of course, $17 \times 0588235294117647 = 9999999999999999$. Try all the other magic things in (1) through (4). If you're energetic, divide and find the remainders, and make magic with them, too.

The period of 1/19 is 052631578947368421, its remainders are 0, 5, 12, 6, 3, 11, 15, 17, 18, 9, 14, 7, 13, 16, 8, 4, 2, 1, and that is magic. 1/13 is divided into two halves with both somewhat magical in nature. The period of 1/13 is only half of what it might be, six digits instead of the expected twelve. It is 076923. Most of the magic things hold true. Add, in threes, 076+923=999, for example; in twos, 07, 69, and 23 equal 99 and 76, 92, and 30 equal 198; the sum of the six digits equals 27. Its remainders upon division are 10, 9, 12, 3, 4, and 1. These can be manipulated in the same manner as the period is manipulated. However, heed the warning, take care in adding. For instance, if the remainders are taken by threes, we do not get 10912 and 341, but 1000, 90, 12, 300, 40, and 1 equal to 1443 (111×13). Different cyclic orders give similar results. And the sum of the remainders is 39 or 3×13.

So we see that (1) through (4) with the exception of (2) all hold for 1/13.

Now we come to the portion of the problem concerning the halves of the period of 1/13. When we multiply by 1, 3, 4, 9, 10, and 12, m is obtained in cyclic orders of itself. However, when multiplied by 2, 5, 6, 7, 8, and 11, a new period, 153846 is obtained which repeats itself in a cyclic manner, also. Observe the following table:

$1\times076923=076923$	$7\times076923=538461$
$2\times076923=153846$	$8\times076923=615384$
$3\times076923=230769$	$9\times076923=692307$
$4\times076923=307692$	$10\times076923=769230$
$5\times076923=384615$	$11\times076923=846153$
$6\times076923=461538$	$12\times076923=923076$

This condition arises naturally because there are only six ways 1/13 can be cyclic, but there exists twelve multipliers.

Last, but certainly not least, 1/97 is king. 1/97=0103092783505154639175257731958762886597938143298969072164948453608247422680412371134020618567. You're on your own. Good luck!

Problem Corner

Problems for Solution

1. Three countrymen met at a cattle market. "Look here," said Hodge to Jakes, "I'll give you six of my pigs for one of your horses, and then you'll have twice as many animals as I've got." "If that's your way of doing business," said Durrant to Hodge, "I'll give you fourteen of my sheep for a horse, and then you'll have three times as many animals as I." "Well, I'll go better than that," said Jakes to Durrant, "I'll give you four cows for a horse, and then you'll have six times as many animals as I've got here."

How many animals did Jakes, Hodge, and Durrant take to the cattle market?

2. It is required to place arithmetical signs between the nine figures so that they shall equal 100. Of course you must not alter the present numerical arrangement of the figures. Use the fewest possible signs.

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 = 100$$

3. If the product of 673,106 and 4,783,205,468 is 3,219,60..,299,743,608 can you supply the missing digit without actually multiplying the numbers?

4. "I have come to consult you about William Weston's will," said the surrogate to the mathematician. "William Weston was fatally injured in an accident while on his way to the hospital where his wife was confined, expecting a child. He lived long enough to make a will which provided that, if his child is a boy, the estate is to be divided in the proportion of 2/3 to the boy and 1/3 to his widow. But, if the child is a girl, she is to receive only 1/4 and the widow 3/4 of the estate. Now, Mrs. Weston has given birth to twins, a boy and a girl. What would be the correct division of the estate to carry out Weston's intentions?"

5. In a box there are six apples. Divide these equally among six boys in such a way as to leave one apple in the box, but do not cut any apple.

6. Seven pennies are arranged in two rows, with three pennies in one row and four pennies in the other row. Can you move just one penny and make two rows with four pennies in each row?

Game Corner

Scramble

The object of this game is to match a drawing, picture, or article with a familiar mathematical term. To assist the players the mathematical terms to be used are supplied in a scrambled spelling and in scrambled order. The items listed below are suggestions.

Picture or Exhibit	*Scrambled Spelling*	*Mathematical Term*
sunburned farmer	gnanett	tangent
advertising sign	nise	sines
electric lines	wrope	power
identical twins	raimlis segifur	similar figures
Stalin	soluc	locus
house boat	car	arc
oleomargarine	bistutes	substitute
empty parrot perch	yonplog	polygon
prison	simpr	prism

Ibn Haitham

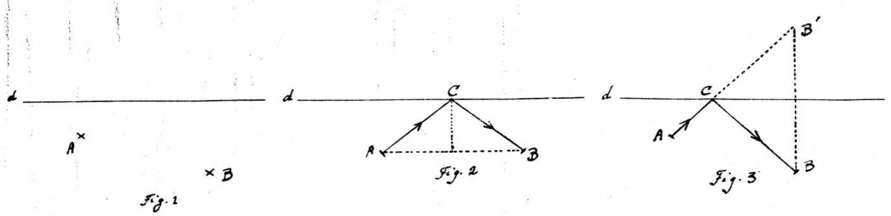
by Ali R. Amir-Moez

An Arab is in the field and must water his camel at the river before he goes home. What path should he take in order to travel the shortest distance? This problem intrigued young Hassan very much. He thought:

"Let the Arab be at A, his house at B, and the river be the line d (Figure 1). Note that the Arab's house is not at the same distance from the river as he himself is. This makes the problem somewhat harder. If A and B were at the same distance from the river, the problem would have been very easy. In this case we draw the perpendicular bisector of AB (Figure 2). This line cuts d at c. Then $AC + CB$ will be the path. Oh well! I'll find the solution. I'm sure it must not be very hard."

Then he went playing. But this problem was on his mind for a couple of days, and he paid little attention to anything else.

"I found it," he shouted, "Light travels the shortest path; if we think of d as a mirror and A as a center of light, then we have to choose the direction which connects A to the reflection of B with respect to d (Figure 3). So we draw a perpendicular through B to d until it cuts d at P. Then we extend this line up to B' such that $PB = PB'$. This way we get B' which is the mirror reflection of B with respect to d. Now the line AB' intersects d at C. The path $AC + CB$ is the shortest."



We are sure that the reader would like to prove this proposition. Thus we will not give the proof.

Aboo Ali Hassan ibn Mohammad Ibn Haitham was born about 965 in Basra. The khalif of Egypt invited him to come and live in Egypt for Ibn Haitham had a plan for building a dam to raise the level of the water of the Nile. But when he arrived at the river he found that his plan was impractical. He pretended to be crazy to escape the khalif's punishment and went to Damascus.

Ibn Haitham's name is sometimes written Al-Hazan. Ibn Haitham wrote over one hundred books and papers, but we shall not list his works here since this brief discussion does not have room for it. His most interesting book is in perspective geometry.

In geometry there are two problems which are known as Al-Hazan's problems. One of them has been solved several ways but the other one is still unsolved.

1. The circle *(O)* with center at *O* and the point *P* inside *(O)* are given (Figure 4). Find the direction of the ray through *P* such that after two reflections it passes again through *P*.

2. Given points *A* and *B* inside of the circle *(O)* with center *O*, find the ray through *A* such that its reflection on *(O)* passes through *B* (Figure 5).

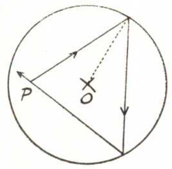


Fig. 4

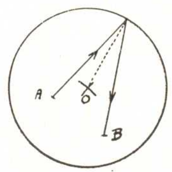


Fig. 5

In problem 2 Al-Hazan uses a spherical mirror, but we see that it is similar to the problem he solved with the flat mirror and two points.

* * * *

"I GET 312 SQUARE FEET, TOO. THAT MAKES THREE OUT OF FIVE, SO LET'S PUT IT DOWN."

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