

☆ RMM - February 1961 ☆

The demand for a reprint (with errors corrected) of the first RECREATIONAL MATHEMATICS MAGAZINE has been so great that the task will be undertaken and copies of the reprint will be ready for mailing on or about March 15, 1962. All readers who have sent in their paid reservations are to be thanked for their immediate response which made the project possible. Those who would like to receive a copy of the reprint of the February 1961 issue of RMM should send in their 65 cents (cash, check or money order) to RMM, Box 1876, Idaho Falls, Idaho. Only a limited number of copies can be printed - enough to see that all orders are filled, of course - so to make sure you don't miss out again please send in payment immediately. **THERE WILL BE NO OTHER REPRINT OF ANY SINGLE ISSUE OF RMM IN THE FUTURE.**

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Effective immediately and until March 1, 1962, however, all new or renewal subscriptions can be obtained for only \$3.00 a year ("general public" subscribers should note this - now only \$3.00 a year until March 1, 1962.) A renewal subscription for \$3.00 a year can be obtained regardless of when your present subscription is due to terminate! And you can renew for 1 year (\$3.00), or 2 years (\$6.00), or as many years as you wish (at \$3.00 per year).

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RECREATIONAL MATHEMATICS magazine

ISSUE NO. 7
FEBRUARY 1962

65¢



PUZZLE SOLVERS: The solvers of the various puzzles in the different departments are listed below with the numbers of the puzzles solved. See the preceeding pages for the answers to all the problems proposed in the December 1961 RMM.

WORD GAMES

"7" LETTER SCRAMBLE: S. Baker, Montreal, Quebec (A,B - Proposer); W. A. Robb, Ottawa, Ontario (A;2a;3abc;4ab;5abcd;7abcd;8a;9a); David Shulman, N. Y., N. Y. (A;4a;6a); Donald V. Trueblood, Bellevue, Wash. (A;5c;6a).
CHANGE A LETTER: S. Baker, Montreal, Quebec (Proposer); Dr. Murray Berg Oakland, Calif. (1a,2a,3a,4a); W. A. Robb, Ottawa, Ontario (1b,2a,3d); Donald V. Trueblood, Bellevue, Wash. (1b,2b,2c,3b,3c,4b,4c).

PUZZLES AND PROBLEMS

Anneliese Zimmermann, Montreal, Quebec (2,3,4,5,7,9); Frederick Roy Wiener, Erlton, N. J. (1,2,3,4,9); Jim Wilson, Rock Hill, Mo. (7); Neil Wanamaker, Concord, Calif. (2,3,4,7,9); Amos Voll, Azusa, Calif. (2,3,7,9); Alfred Vasko, Swanton, Ohio (2,3,4,7,9); W. J. Sweitzer, San Diego, Calif. (2,3,4,5,7,9); John H. Sweitzer, Princeton, N. J. (2,3,4,5,7,9); J. M. Spaeth, East Alton, Ill. (2,3,4,7,9); Jim Smith, Marshalltown, Iowa (2,3,9); David L. Silverman, Beverly Hills, Calif. (1,2,3,5,6,9); Robert Ruderman, New Hyde Park, N. Y. (7,9); Tom Rieder, Toronto, Ontario (1 - Proposer); Robert Frall, Harvard, Nebr. (2,3,5,9); Dianne Pickering, Natick, Mass. (2,3,4,7,9); Paul Nemecek, Riverside, Ill. (2,3,7,9); Harry L. Nelson, Livermore, Calif. (1,2,3,4,5,6,7,9); Bertha McDaniel, Stayton, Oregon (2,3,5,7,9); Dorman Luke, West Palm Beach, Fla. (2 - Proposer); Remy Landau, Montreal, Quebec (7,9); Louis Kasper, Philadelphia, Pa. (7 - Proposer); Robert S. Johnson, Town of Mt. Royal, Quebec, (1,2,3,4,5,6,7,9); J. A. H. Hunter, Toronto, Ontario (4,5,6 - Proposer,7,9); David B. Hollander, Martinsville, Va. (2,3,4,7,9); Jack H. Halliburton, Los Angeles, Calif. (2,3,4,5,7,8 - Proposer,9); David C. Haines, Wooster, Ohio (2,3,7,9); Harvey Hahn, Valparaiso, Ind. (2,3,4,9); Lester Gabriel, Osceola, Nebr. (2,3,7,9); Kobon Fujimura, Osaka, Japan (6 - Proposer); Roger Fairbairn, San Francisco, Calif. (2,3,5,7,9); Ronald L. Enyeart, Los Angeles, Calif. (2,3,4,5,7,9); John Eckelman, Westport, N. Y. (2,3,5,7,9); Juli Dieckman, St. Louis, Mo. (7); John H. Davids, Ardmore, Pa. (2,3,5,7,9); Howard Cohodas, Cleveland Heights, Ohio (2,3,5,7,9); Sam Cannella, Atlanta, Ga. (2,3,9); Brother D. Joseph, Canton, Ohio (1,5,9); Corine Bickley, Warson Woods, Mo. (2,3); William Baumgartner, Detroit, Mich. (3 - Proposer); Bernard J. Battersby, Ft. Sill, Okla. (5 - Proposer); Merrill Barnebey, Grand Forks, N. D. (2,3,5,7,9); Elizabeth H. Baehr, Loveland, Ohio (2,3,4,7,9); John G. Armstrong, Rexdale, Ontario (2,3).

COIN-GAME COFFEE-WINNERS

STACK 'EM UP! Roger Fairbairn, San Francisco, Calif.; Jack H. Halliburton, Los Angeles, Calif.; J. A. H. Hunter, Toronto, Ontario; Robert Ruderman, New Hyde Park, N. Y.; and the proposer Maxey Brooke, Sweeny, Texas.

SHIFTY: Howard Cohodas, Cleveland Heights, Ohio; Jack H. Halliburton, Los Angeles, Calif.; Harry L. Nelson, Livermore, Calif.; and the proposer Maxey Brooke, Sweeny, Texas.

DOTS AND LINES

Marvin C. Graham, Thule, Greenland (1); Jack H. Halliburton, Los Angeles, Calif. (8,9,11); David B. Hollander, Martinsville, Va. (8,10); and the proposer Maxey Brooke, Sweeny, Texas.

ALPHAMETICS

Elizabeth H. Baehr, Loveland, Ohio (1,2a,3,4a); Merrill Barnebey, Grand Forks, N. D. (1,2a,3,4a); Dr. Murray Berg, Oakland, Calif. (1,2a,3,4a); Corine Bickley, Warson Woods, Mo. (1,3); Fred B. Blosser, Goshen, Ind. (2a,4a); Blair Bowling, Cowansville, Quebec (1,2a,3,4a); A. G. Bradbury, North Bay, Ontario (3 - Proposer); Brother D. Joseph, Canton, Ohio (1,3,4a); Mrs. June Burgess, Atlanta, Ga. (1,2a,3,4a); Gary L. Carson, Palmyra, Nebr. (1,2a,3,4a); Howard Cohodas, Cleveland Heights, Ohio (1,2a,3,4a); John H. Davids, Ardmore, Pa. (2a,3,4a); Herman Ebberts, Picton, Ontario (4a - Proposer); John Eckelman, Westport, N. Y. (1,2a,3,4a); Ronald L. Enyeart, Los Angeles, Calif. (1,2a,3,4a); Harold Federow, Springfield, Mo. (4); Harry M. Gehman, Buffalo, N. Y. (1,2a,3,4a); Harvey Hahn, Valparaiso, Ind. (1,2b,3,4b); David C. Haines, Wooster, Ohio (1,2a,3,4a); Jack H. Halliburton, Los Angeles, Calif. (1,2a,3,4a); Joe Haseman, Davidson, N. C. (1,2a,3,4a); J. A. H. Hunter, Toronto, Ontario (1 & 2 - Proposer, 3,4a); Bertha McDaniel, Stayton, Oregon (1,2a,3,4a); Harry L. Nelson, Livermore, Calif. (1,2c,3b,4a); Dianne Pickering, Natick, Mass. (1,3,4a); Robert Frall, Harvard, Nebr. (1,3,4a); W. A. Robb, Ottawa, Ontario (1,2a,3,4a); Lloyd V. Rogers, Menlo Park, Calif. (1,2a,3,4a); Robert Ruderman, New Hyde Park, N. Y. (1,3,4a); W. J. Sweitzer, San Diego, Calif. (1,2a,3,4a); Daniel Suty, East Detroit, Mich. (1,3,4a); Donald V. Trueblood, Bellevue, Wash. (1,2a,3,4a); Alfred Vasko, Swanton, Ohio (1,2a,3,4a); Anneliese Zimmermann, Montreal, Quebec (1,2a,3,4a).

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From the Editor

The start of the second year of RMM begins with this issue. A slightly different page set-up has been utilized, but the 56 pages here have as much material as any past 60-page issue. The list of puzzle solvers has been made a part of Cover II to add over a full page of text internally. It had been the intention to put this list on Cover III, but a little printers' devil switched plates on us. To ease in the final make-up of RMM - and make deliveries sooner - all answers to the December 1961 RMM have been grouped together in one section on the last pages.

This particular issue introduces a relatively new idea in magic squares: *anti-magic* squares on page 16. We hope readers will do some research on the problems outlined by Mr. Lindon.

* * * * *

Attention is directed to the newly established *RMM Book Department* (see the inside back cover).

* * * * *

The response to the *Junior Department* (see December 1961 RMM, page 2) has been very encouraging indeed. Some publishable material has already been received and, of course, more is desired. It is hoped that the first section of the Junior Department can appear in the June or August 1962 issue of RMM.

* * * * *

In the April 1962 issue of RMM we shall feature another article by J. A. H. Hunter, "*The Problemist at Work*", in which he tells us just how he devises puzzles having such neat answers. Mr. Golomb, in response to the demands of RMM readers, will present a Part 4 of his *General Theory of Polyminoes*. New solutions to previously unsolved problems will be shown, along with other observations. Sidney Kravitz will have an article on the Mersenne Primes and the Editor will publish the full values of the 18 known Mersenne Primes along with a few other large primes of interest to RMM readers. Alan L. Brown will present his work on Multiperfect Numbers.

* * * * *

The cover, "*Chess Fantasy*", by John McClennan (see his *Recreations for Space Travel* on page 7 of this issue) is a representation, on a Moebius chessboard, of a problem by Morphy as given in Lasker's *Manual of Chess* on page 294. Only the white pawn at the upper right has been moved out of position for the sake of visibility.

1 February 1962

J.S.M.

That "Remainder" Business

by J. A. H. Hunter

The mathematician will call this Congruence Theory, a description that may sound alarming to many people. So let's look into some of the elementary aspects of this important branch of Number Theory in terms of the familiar remainder that may result from the process of simple division.

To understand and apply the principles in the solving of simple problems there is little need to delve deeply into abstruse theoretical considerations. And, in the very brief survey that space permits here, I intend to avoid almost all theory. Also, it must be understood that we shall be concerned only with integers, i.e. whole numbers.

It is essential, of course, to use and understand the special notation. If we divide 23 by 7, the remainder is 2. This would be shown as

$$23 \equiv 2 \pmod{7}$$

which is read as: "23 is congruent to 2, to modulus 7". In other words: "23 leaves a remainder of 2 when divided by 7."

Stretching our concept of "remainder", it is obviously true that an infinite number of integers are congruent to 2, to modulus 7. For example:

$$30 \equiv 2 \pmod{7}, 9 \equiv 2 \pmod{7}, 2 \equiv 2 \pmod{7}, -5 \equiv 2 \pmod{7}$$

In fact, all of those integers are solutions of the equation $x = 7k + 2$, where k can be any integer, +ve or -ve, or zero. Try a few examples, to familiarize yourself with these ideas.

Now we have seen that, to modulus 7, 23 and 2 are equivalent. If we multiply one side of our congruence equation by 2 and the other side by 23, the new congruence equation will be valid. For example:

$$46 \equiv 46 \pmod{7}, \text{ and } 529 \equiv 4 \pmod{7}$$

The first of these is too obvious for comment, but notice the second for future reference.

Clearly, $12 \equiv 5 \pmod{7}$, which can also be shown as $12 \equiv -2 \pmod{7}$. And an infinite number of integers are congruent to -2, to modulus 7. For example:

$$19 \equiv -2 \pmod{7}, \quad 5 \equiv -2 \pmod{7}, \quad -2 \equiv -2 \pmod{7}, \text{ etc.}$$

and all of these integers are solutions of the equation:

$$x = 7k - 2 \text{ where } k \text{ can be any integer, +ve or -ve, or zero.}$$

And, since 19 and -2 are equivalent, to modulus 7, we can multiply one side of $19 \equiv -2 \pmod{7}$ by 19 and the other side by -2, so obtaining a new congruence equation:

$$361 \equiv 4 \pmod{7}$$

Note that $529 = 23^2$ and $361 = 19^2$. Now say we have to find all integer solutions of the equation $x^2 - 4 = 7y$, where x and y have to be integers.

Then this can be shown as $x^2 \equiv 4 \pmod{7}$, which is satisfied by $x = 7k \pm 2$, or using congruence notation $x \equiv \pm 2 \pmod{7}$. If positive values for x are required, we have an infinite number of such values: 2, 5, 9, 12, 16, 19, 23, etc. And, in particular, 19 and 23 have been found previously by quite a different method.

In solving some problem we might derive the rather more complex equation $x^2 - 6x + 6 = 11y$, say, and we might be required to find the general solution for integral values of x and y .

This equation can be written as $(x-3)^2 = 11y + 3$, so $(x-3)^2 \equiv 3 \pmod{11}$. But $25 \equiv 3 \pmod{11}$, hence we have $(x-3)^2 \equiv 25 \pmod{11}$, giving $(x-3) \equiv \pm 5 \pmod{11}$. The required general solution follows, as $x \equiv 8$ or $9 \pmod{11}$, which may be shown as $x = 11k - 3$, or $x = 11k - 2$. Hence, particular solutions are $x = 8, 9, 19, 20$ etc., with corresponding values for y .

At this point I suggest you write down a few typical equations of this type, based on prime-number moduli, and obtain their general solutions. You will almost certainly find that some of them have no whole-number solutions at all. Full discussion of this point must be outside the scope of this survey. However, say we have $X^2 \equiv a \pmod{m}$, which is equivalent to $X^2 = mk + a$: then, if there is an integral value of k that will make the right-hand side a square, the smallest such value must lie within the range of integral values $k = 0$ to $2k \leq m$. So, to find that square value if indeed it does exist, it is only necessary to test for values of k within that range.

So far we have dealt with prime moduli. Now we can consider how to deal with a case where the modulus is a power of 2, such as 8 for example. Say we have $x^2 \equiv 1 \pmod{8}$. Then we must have $x^2 \equiv 9$, and 17, and 25, etc. $\pmod{8}$, from taking successive values for k up to $k = 3$, in the implied $x^2 = 8k + 1$. So here we have two entirely different solutions, $x \equiv \pm 1 \pmod{8}$, and $x \equiv \pm 5 \pmod{8}$. The first few positive values for x are therefore 1, 3, 5, 7, etc., and we see that the two different solutions do in fact cover all odd numbers. Hence, $x^2 \equiv 1 \pmod{8}$ modifies down to $x \equiv 1 \pmod{2}$.

If, however, we have $x^2 \equiv 4 \pmod{8}$, this leads to $x^2 \equiv 12$, and 20, and 28, and 36, etc. $\pmod{8}$. In turn these values give as the general solution, $x \equiv \pm 2$ and the exactly equivalent $x \equiv \pm 6 \pmod{8}$, so that we need only consider the solution $x \equiv \pm 2 \pmod{8}$ with positive values 2, 6, 10, 14, etc. Here we see that the solution of $x^2 \equiv 4 \pmod{8}$ modifies down to $x \equiv 2 \pmod{4}$.

Without delving into the theoretical considerations, any case where the modulus is a power of 2 can be dealt with along the same lines.

Cases of composite moduli must be considered too. In such a case the modulus must be broken down into its different prime factors, noting if a prime factor appears as its square or higher power. For example, the modulus 72 would be noted as $2^3 \times 3^2$.

Now say $x^2 \equiv 1 \pmod{72}$. Obviously this implies $x^2 \equiv 1 \pmod{8}$ and also $x^2 \equiv 1 \pmod{9}$. Hence, to deal with a composite modulus we must first set up the requisite number of resulting congruence equations, depending on the number of different prime factor elements in the modulus. It will then be necessary to solve each congruence equation, using the methods that have been outlined, and finally those

separate solutions will have to be co-ordinated to produce a general solution that will cover all of them.

For example, say $x^2 \equiv 11 \pmod{35}$, the congruence equation implied in $x^2 - 11 = 35y$. Here the modulus is $35 = 5 \times 7$. Therefore, $x^2 \equiv 11 \pmod{5}$, and $x^2 \equiv 11 \pmod{7}$, which become $x^2 \equiv 1 \pmod{5}$ and $x^2 \equiv 4 \pmod{7}$. The solutions of these are: $x \equiv \pm 1 \pmod{5}$, and $x \equiv \pm 2 \pmod{7}$.

The surest method for co-ordinating these two separate solutions is as follows, and I give it in detail because its application is involved so often in solving indeterminate equations of this original type.

Say $x = 5u + a$, and $x = 7v + b$, where u and v are whole numbers, and where $a = \pm 1$, $b = \pm 2$, the + and - signs not necessarily corresponding.

Then

$$5u - 7v = b - a.$$

Dividing through by 5, $(2v - a + b)/5$ must be a whole-number. Then $(6v - 3a + 3b)/5$ must be a whole-number, so $(v - 3a + 3b)/5$ must be a whole number, say t (i.e. +ve, -ve or zero). Hence, $v = 5t + 3a - 3b$, and we had $x = 7v + b$. So, $x = 35t + 21a - 20b$, whence $x \equiv [21a - 20b] \pmod{35}$. Now we assign their values to a and b , as $a = \pm 1$, $b = \pm 2$, the + and - signs being mutually independent, to get:

$$x \equiv [\pm 21 + \pm 40] \pmod{35}$$

which simplifies to $x \equiv \pm 9$ or $\pm 16 \pmod{35}$. So, the general solution of $x^2 - 11 = 35y$ is $x = 35k \pm 9$ or $35k \pm 16$, with corresponding values for y . The first few positive values for x are: 9, 16, 19, 26, etc.

There could well be more than two prime factor elements in a composite modulus. For example, the equation $x^2 - 4 = 105y$ leads to co-ordination of the solutions of three separate congruence equations. The same method can be used, however. First co-ordinate the solutions of one pair of equations, and then co-ordinate the result with the solution of the third equation.

This short survey has covered only a tiny part of the wide field in even the elementary applications of "that remainder business" in the solving of indeterminate equations, and in it many niceties have been glossed over of necessity. But it may have served its purpose if only as an introduction to the practical applications of a most interesting branch of Number Theory.

Two problems are presented here to show how "that remainder business" can be used. One problem is worked out — but the second is left for RMM readers to work out.

* *

"I wonder if we'll be as spry as they are when we're that old," mused Mary. "Just one year between their ages, the same as us."

Doug took back the snapshot of his parents. "They're so happy too," he said. "Maybe that's why."

His wife smiled, "But you're a long lived family," she told him. "Look at your Uncle Fred. He's older still, though I can't remember exactly."

"A good deal older than them," chuckled Doug. "If you write down their ages, one beside the other, you get Uncle Fred's age multiplied by itself."

That didn't help Mary! But how old was Doug's uncle?

Solution

Say parents' ages were x and $x \pm 1$ years, Uncle Fred's age y years, and from "a good deal older" $y < 100$. Then, $y^2 = 100x + x \pm 1 = 101x \pm 1$, so $y^2 \equiv \pm 1 \pmod{101}$. Say, $y^2 \equiv +1 \pmod{101}$, then $y \equiv \pm 1 \pmod{101}$, whence $y = 100$ or 102 , both unacceptable. Say, $y^2 \equiv -1 \pmod{101} \equiv 100 \pmod{101}$, then $y \equiv \pm 10 \pmod{101}$, whence $y = 91$, leading to parents' ages 82 and 81.

* * *

"Loan me two bucks, Jack," said Jill.

"Haven't got that much," her brother replied. He checked his cash and thought a moment. "I'll lend you what I've got if you guess what it is. If you added half the square of half of it, there'd be an exact number of dollars."

Maybe Jill peeked, but she did get the right answer. Can you do so?

"Your age, please." asked the census taker.

"Well, now, let me figure it out," said the housewife, "I was 18 when I was married and my husband was 30. He is 60 now, or twice as old as he was when we were married, so I must be twice as old now as when we were married. That makes me 36."

* * * * *

$$11 + 2 - 1 = 12$$

ELEVEN plus TWO equals ELEVENTWO, which, minus ONE equals LEVETW. Rearrange these letters and get TWELVE.

Recreations for Space Travel

by John McClellan

It is high time someone put some thought in on the subject of recreational activity for space travellers. Astronauts are blazing unseen trails through the upper air, and after astronauts, of course, *we* come. We will travel at fantastic speeds, yes: but it will still take a very long time to reach the nearest planet,¹ and the question is, what are we going to do with ourselves as we zoom along through space?

Pilot and crew will be busy enough with the details of navigation, and we do not have to worry about them from a recreational standpoint. This will be true, too, of the spaceship stewardess, to a lesser degree. (We expect she will be as pretty as her present-day counterpart. More precisely, we like to think of her as an 'invariant' in a world where many values will have changed radically.) Passengers, however, will tire of looking out at the vast reaches of outer space, and this paper attempts to present a few ideas for their amusement.

Looking over our stock of time-passing devices gathered during long trips of the past, we find: talking, reading, and game-playing. That is, over and above the purely animal necessities of eating, drinking, and sleeping. Creative activity may be included under reading, both being rather solitary occupations.

Space-travel conversation will probably not depart much from the familiar subjects - ailments, sports, stock-market, children, etc. On stories and jokes we need not elaborate. Many passengers will, as usual, spend much time in reading, and the curtailment of current books will give us the opportunity to catch up on the Classics (as we always meant to do, but never quite got around to). It is suggested that each 'ship' install a book-shelf in the form of a Moebius strip - the famous 'Five-Foot-Shelf' would turn into the 'Infinite Shelf' of the future.² A reader who forgot at which volume he began would continue reading around and around, and the weightlessness which he experiences will enable him to follow the curve of the strip with ease.

In the creative field, there will be ample time for artist-travellers to work on personal projects, although sculptors may have a hard time of it on account of the nature of their materials. Theoreticians will theorize, and there can be no way of testing the validity of their ideas - the ivory tower will remain inviolate, at least for the duration of the trip.

But let us get to some recreations, and not ones of short duration, either, or we defeat the purpose of passing a maximum of time with a maximum of pleasure. Tic-Tac-Toe, even in 3 dimensions, can hardly fill the bill, and this goes, too, for Old Maid, Tiddly-Winks, Pig, Jack-Straws, Mumbly-Peg, and a host of other 'quickies'. Darts, and similar games, while good 'socializers', may have to be passed over: the diminishing of the gravitational force may make the dart's trajectory erratic and dangerous - but the question needs more study before a definite Yes or No can be given.

However, a problem such as 'The Tower of Hanoi', which we are told³ would take industrious Buddhist monks several billions of years to

complete (provided they made only correct moves!) will certainly be usable as a game. The equipment for it may be of light material, and would not present a weight problem for our space-craft.

Some games of solitaire may appeal to the more reticent passengers; and particularly popular might be the small, linked shapes of metal which can be taken apart if one knows how: and the various sets of wooden blocks which, when put together properly, become animal or abstract forms, and are all locked together by a final piece.

Word-games should be considered. Scrabble, along with Anagrams, Cross-Words (diagrammed and diagramless), all have many devotees. An adequate supply of these puzzles should be kept handy in the ship's locker.

Some board-games, such as Dominoes, are completed fairly quickly, although the substitution of Trominoes, Tetrominoes, Pentominoes, etc. will lead to lengthy variations. The writings of Mr. Golomb on this subject are of great interest - even if you are *not* going on a space-journey - and may be found in several other issues of RMM. Chess, Checkers, and Go may be of long duration, and are suitable for our purpose. It is suggested that a chessboard be made of metal, in the form of a Moebius strip, and the pieces magnetized. Figure 1 shows a possible design. If two such strip-boards are joined, as in Figure 2, four persons could play.

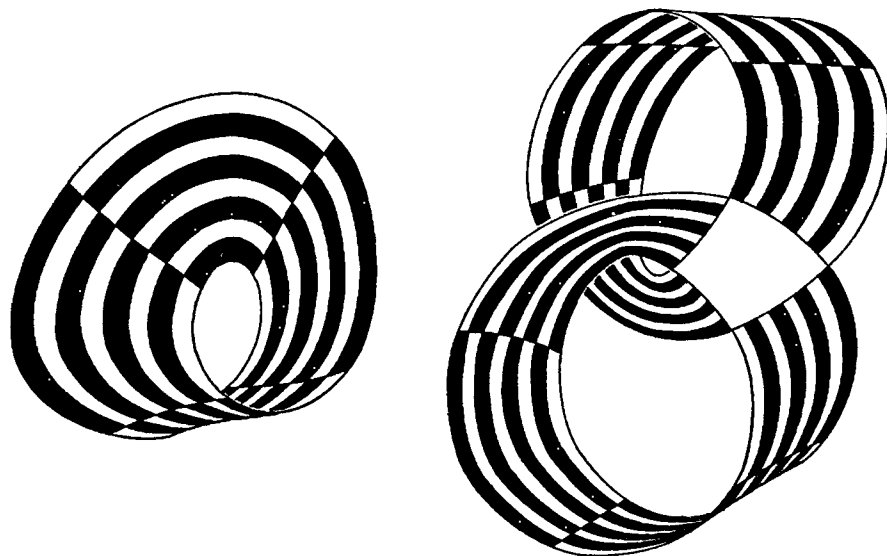


Figure 1

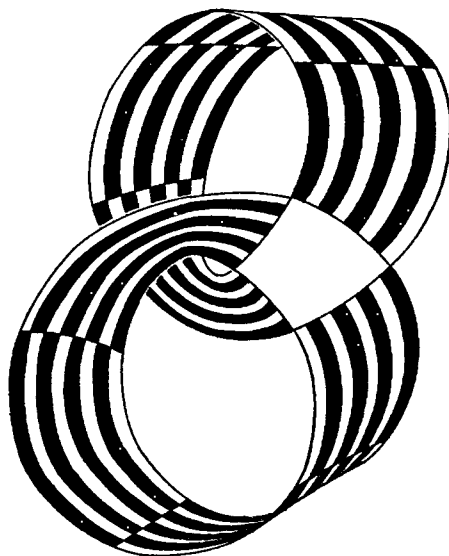


Figure 2

Rules for these variations have not been established to my knowledge. The single strip presents no particular problem except that of seeing the moves readily, which difficulty will be partly obviated by the player's weightless ability to move around, and in and out of, the strip. There will be no reason to change the characters of the pieces, although a certain elongation may be desirable to match the elongation of the squares. At the commencement of the game, Black and White will be set up back to back: they will also be facing each

other, as in the conventional game, although it does not look it. In the game for four players, Black and White are aligned on opposite sides of the large, white 'no-man's-land' at the intersection of the two strips.

Those who find the Moebius variation unpalatable for one reason or another may consider the 8-board arrangement of Figure 3.

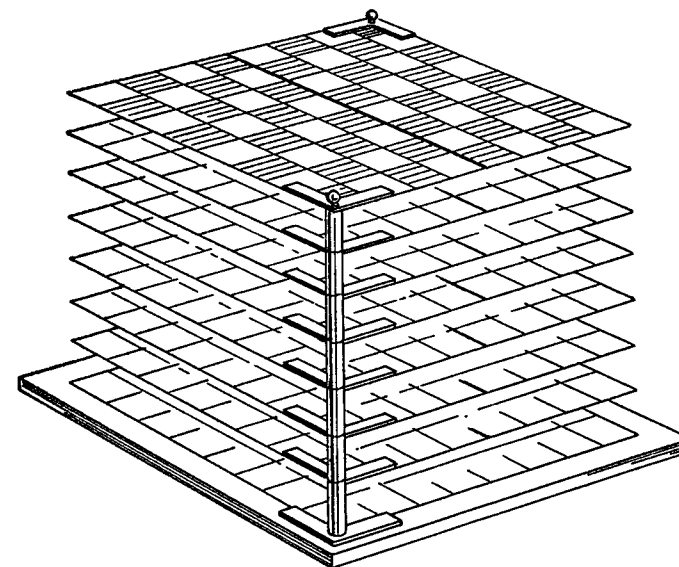


Figure 3

Each board, of clear plastic, is divided in half, and all the halves pivot at corresponding, opposite corners - as shown in the drawing. The game begins in the conventional manner, on the lowest board, and all the half-boards above are swung to one side to allow a clear view. Play continues until a mate is effected, at which point the defeated King acquires 3-dimensionality and may escape by *one* square to the next board above. However, as the King acquires this soaring ability, so do the other pieces remaining in the game.

If the mate occurs in the eighth row, the King will have six escape-routes: if on the seventh, or less, he will have nine routes open to him. It is calculated that a 3-D Knight, in the center of the board, will have twenty-four possible jumps, as opposed to his eight on a 2-D board. Readers may like to calculate the possible moves of the other pieces in 3-D, and to formulate some precise rules of play.

A time-clock for Space-Chess may involve questions of relativity, which is beyond the scope of this paper.

Group-activities should be staged from time to time, similar to the 'get-together' parties put on by cruise-directors on boats and social-directors of summer hotels. These activities would be handled by the stewardess who must see to it that her passengers get to know one another while they hurtle through the void. A space-pool may be organized daily for those who wish to bet on the day's run. Competitions may be arranged, such as the filling of Klein bottles, with a small prize going

to the winner. Charades might be popular; and guessing games on the order of TV's "What's My Line?" would be enjoyed.

Essay contests on famous problems would constitute a 'change-of-pace', and the literary-minded will like the opportunity of writing at length on such celebrated affairs as, 'The Marie Celeste' mystery, Fermat's Last Theorem, Was Lizzie Borden Guilty?, The Shakespeare-Bacon Controversy, and many other thought-provoking subjects.

Games of chance must be provided. In addition to the traditional dice-game played with cubes, we should consider the possibilities of the octahedral form, to be called, perhaps, 'Octadice'.⁴ (Figure 4)

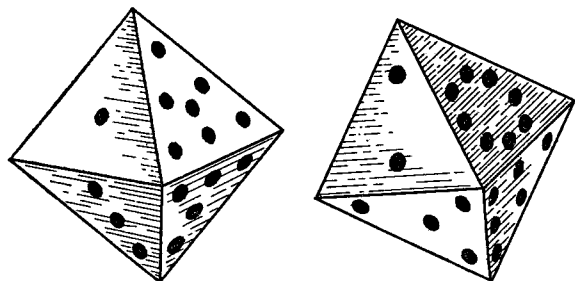


Figure 4

It is amusing to speculate on the phrases that may be added to the familiar jargon of the game through the addition of the two extra faces.⁵

As faces increase in the 'regular' family of solids (or, as they approach the spherical form) their 'rollability' increases, and we may also consider the use of 'Dodecadice' and 'Icosadice'. The spots of these should be replaced by numerals, for easier identification.

If it is required that all faces of our dice have equal chances of turning up, the dihedral angles between 'significant' faces must be equal, as is true in the case of the Platonic solids. The pyramidal and dipyramidal forms of Fig. 5 also fill this requirement.



Figure 5

If we are not concerned that each face have an equal chance of appearing, we may use, as dice, one of the Archimedean group of semi-regular solids. Each member of this group contains at least two different types of polygonal faces, and of dihedral angles. As dice tend to roll through the greater dihedral angle, if more than one type is present, we can easily determine the odds for the appearance of certain faces. The several interesting variations of this field are left to the curious reader to explore.

While on the subject of polyhedra, we may note another group - the 'Space-Fillers' - and consider their game-possibilities. These solids, of which there are five,⁶ have this characteristic in common: if we take

an infinite number of any one of them, and pack them together, we may completely fill space, leaving no cracks or crannies. The cube is an example of a space-filler - if we put an infinite number of them together tightly, they will completely fill as much space as we have time or inclination to fill. Hexagonal prisms do the same thing, and the three other, rather more complicated, forms.

Now, if a great number of each of these five solids are made, a fine game presents itself! A player will draw, say, ten pieces. He then begins to 'close-pack' the pieces of which he has drawn the most: if he has drawn six cubes, for example, he will put them together, and discard the other four pieces. The next player will do the same, and so on around the table. When the turn comes again to the first player he will draw four new pieces from the 'boneyard' in return for his four discards, adding new cubes (if he gets them) to his original pile. He will again discard unusable pieces. Play will continue around the table until . . . Well, some kind of a limit must be set, either of time or size of pile. Perhaps transparent plastic containers should be supplied to keep the piles within reason. The name 'Close-Pack-It' is suggested as appropriate to this pastime.

The foregoing notes may seem somewhat incomplete as to procedure, equipment, and other details - we should point out that the author's purpose is chiefly to indicate certain, developable lines of inquiry. We shall be happy if this paper stimulates an interest in recreations for a new age, and elicits further suggestions from space-minded readers.

NOTES

1. Interplanetary distances and the times necessary to traverse them at given speeds are readily available. Here we will be content to just speak of 'a long time', or 'a long ways', even though a certain loss of rigor results therefrom.
2. The complete works of Charles Dickens will serve equally well.
3. See Martin Gardner, in *The Scientific American Book of Mathematical Puzzles and Diversions*, Simon & Schuster, 1959, p. 55.
4. Note that the digits on opposite faces of these dice may be so arranged that their sums equal 9.
5. 'Little Joe', 'Snake-Eyes', '8-er from Decatur', etc., will remain valid - but what phrases will the new double-eight and double-seven produce?
6. They are the Cube, Hexagonal Prism, Truncated Octahedron, and the Rhombic and Elongated Dodecahedron.
7. He must, of course, draw 'blind'.

* * * * *

Golfer: One who yells "Fore!", takes six, and put down five.

Cross-Number Puzzle

by Walter F. Penney

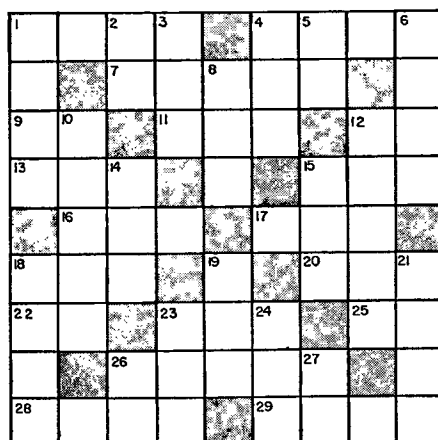
The entries in this Cross-Number puzzle are letters - but instead of calling this a crossword puzzle we still use the term *cross-number* since the letters are numbers (Roman numerals). A bit of logic will enable one to start and, finally, finish.

HORIZONTAL

1. 12 Horizontal + 20
4. 23 Horizontal + 10
7. Square of 27 Vertical
9. 5 times 29 Horizontal
11. 16 Horizontal ÷ 3
12. 3 times 4 Horizontal
13. 2 times 17 Horizontal
15. 13 Horizontal + 14 Vertical
16. 26 Horizontal + 3
17. 10 times 11 Horizontal
18. 6 Vertical - 26 Vertical
20. 15 Vertical ÷ 4
22. 19 Vertical - 20 Horizontal
23. 4 times 2 Vertical
25. Square root of 10 Vertical
26. 3 times 25 Horizontal
28. 2 times 21 Vertical
29. 18 Horizontal + 25 Horizontal

VERTICAL

1. 5 times 23 Horizontal
2. 6 Vertical + 27 Vertical
3. 23 Vertical - 7 Horizontal
4. 13 Horizontal - 18 Vertical
5. 12 Horizontal ÷ 10
6. 14 Vertical - 20 Horizontal
8. 10 Vertical ÷ 3
10. 22 Horizontal + 3 Vertical
12. Square of 3 Vertical
14. 5 Vertical + 10
15. 10 times 8 Vertical
18. 19 Vertical - 8 Vertical
19. 29 Horizontal + 24 Vertical
21. 18 Vertical - 18 Horizontal
23. 1 Horizontal ÷ 5
24. 17 Horizontal - 5 Vertical
26. 15 Vertical ÷ 8
27. 26 Horizontal ÷ 2



Alphametics

A little note to be added to the brief historical sketch in the December 1961 RMM: the *Strand Magazine* for July 1924 published the following alphametic:

$$(TWO)(TWO) = THREE$$

J. A. H. Hunter, Maxey Brooke and the Editor (and, undoubtedly, many other RMM readers) are still trying to locate the earliest published *alphametic* (a cryptarithm with meaningful words). Also, when did the classic $SEND + MORE = MONEY$ first appear? Any help by RMM readers would be appreciated by all.

Unless otherwise noted: (1) all alphametics are to be solved in our base-10, or decimal system and (2) it is understood that no solution is accepted if the initial letter of a word is zero (e.g. any solution of the YOH O HEAVE HO alphametic below where either Y, H or U is zero is not acceptable).

By sheer coincidence, only, does it happen that the next four *alphametics* come from Ontario, Canada.

A N) E A S Y (O N E

This is the first issue of the year, so let's TRY AN EASY ONE. (A. G. Bradbury; North Bay, Ontario)

$$\begin{array}{r} \text{x T} \\ \text{x x x} \\ \text{x x R} \\ \hline \text{x x} \\ \text{x Y} \end{array}$$

B I G) N U G S (O R

"Is that new?" Tony asked, as he strolled through the zoo.

And the keeper replied: "It's not new, it's a gnu."

(J. A. H. Hunter; Toronto, Ontario)

$$\begin{array}{r} \text{B I G} \\ \text{G N U S} \\ \text{G N U S} \end{array}$$

After two such easy alphametics, see how well you do with the following two.

This is a wild one, all right! How about taming the TIGERS?

(Alan Gold; Downsview, Ontario)

$$\begin{array}{r} \text{S E E} \\ \text{T H E} \\ \hline \text{x x x x} \\ \text{x x S x} \\ \text{x x x H} \\ \hline \text{T I G E R S} \end{array}$$

Y O H O) H E A V E (H O

Mr. Bradbury gives us another poser. Just don't get seasick on this one!

(A. G. Bradbury; N. Bay, Ontario)

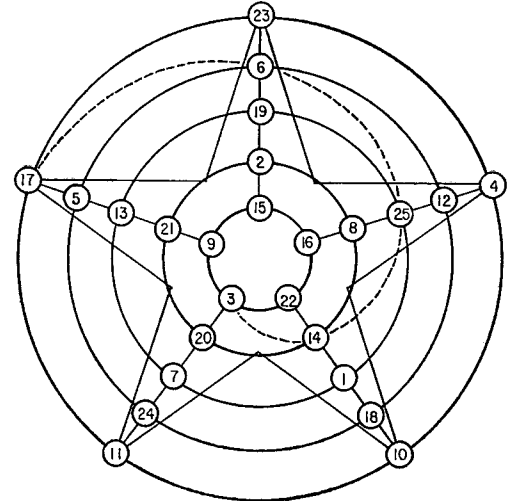
$$\begin{array}{r} \text{x x x x} \\ \text{x x x x} \\ \text{x x x x} \\ \hline \text{U P} \end{array}$$

Answers to the December 1961 RMM *Alphametics* will be found on pages 50 and 51.

More Strictly for Squares!

The October 1961 RMM (pages 24 to 29) had a collection of unusual Magic Squares. A request was made for contributions from RMM readers for other unusual types. The collection below and Mr. Lindon's article *Anti-Magic Squares* on pages 16 to 19 include some of the more interesting of those received.

Lloyd V. Decker, of Goddard, Kansas, gives us the magic star shown below which has a magic constant of 65 for every five equally spaced numbers on the lines, circles or spirals both clockwise and counter-clockwise (one such spiral is indicated).



17	5	13	21	9
23	6	19	2	15
4	12	25	8	16
10	18	1	14	22
11	24	7	20	3

The configuration is derived from the 5th order pandiagonal (or *nasik*) magic square shown to right above. The pandiagonal magic square has a magic constant of 65 for every row, column and main diagonal - and also for every broken diagonal, such as 17, 24, 1, 8, 15 and 4, 18, 7, 21, 15.

1	42	29	7	36	35
48	9	20	44	13	16
5	38	33	3	40	31
43	14	15	49	8	21
6	37	34	2	41	30
47	10	19	45	12	17

The pandiagonal magic square to the left has some remarkable constellations which may be located by using the patterns on page 15. W. W. Horner (see his *Addition-Multiplication Magic Squares* on pages 30 to 32 of the October 1961 RMM) sent in this one. If any pattern is placed anywhere upon the magic square, the sum of the numbers under the x's is equal to 25 times the number of x's in the pattern. Of course the grid lines of the pattern must lie on those of the magic square. If a pattern extends beyond a side of the magic square, the missing numbers may be found by sliding the extended part (without rotation) to the opposite side of the square. There are many other patterns not shown here. Perhaps some reader interested in such useless research may amuse himself by finding others.

1

2

3

4

5

6

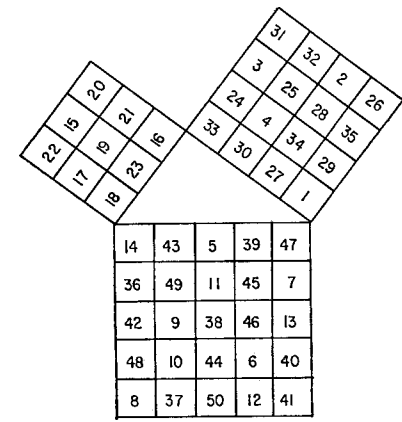
7

8

9

10

Jack Halliburton, of Los Angeles, California, shows below a Pythagorean magic square similar to Ben Laposky's (October 1961 RMM, page 25) but using only the integers 1 to 50. The individual 3x3, 4x4 and 5x5 squares are magic. The sums of the magic constant of the two smaller squares equal the magic constant of the larger square.



* * * * *

Or as the geometry Professor, who dearly enjoyed his work, said:
 "I love my wife, but Oh Euclid!"

Anti-Magic Squares

by J. A. Lindon, Surrey, England

Editor's Note: *The interest in Magic Squares dates back thousands of years, but neither the Editor nor the author of the following article know of the existence of any literature on anti-magic squares - number arrays where the sums of the rows, columns and main diagonals are different. The construction of such anti-magic squares would not be difficult, except that the author in the following article restricts the different sums to certain limits and the numbers used to construct the squares are likewise restricted. Both the Editor and the author would welcome comments and further observations - and especially earlier references to the subject.*

A magic square of order n consists of a square array of the first n integers, such that their totals along any row, column or principal diagonal are the same. In an anti-magic square these totals are all different and form a scrambled sequence of consecutive integers.

Thus the square shown, which gives the totals 11, 12, 13, (15), 15, 16, 17, 18 is almost, but not quite, anti-magic: there is no 14 and the 15 is repeated.

1	8	7	16
9	5	4	13
3	2	6	11
13	15	17	12

Anti-Magic Squares of orders 1, 2 and (I think) 3 are impossible, but all higher orders occur. Analysis shows that for each order there are two, and only two, feasible sequences. A simple way of obtaining these is as follows: For

any given order n write down $2n+1$ natural numbers (integers) where the middle term is $\frac{1}{2}n(n^2+1)$. (This, incidentally, is the magic constant of an ordinary magic square of order n .) These $2n+1$ integers we shall call our generating sequence. Then the two feasible sequences for our anti-magic square are those obtained by extending the generating sequence in either direction by one term. The sums of the diagonals will be equal to this additional term plus the middle term of our generating sequence. For example, suppose we are trying 4th order anti-magic squares. The middle term of our generating sequence will be $(\frac{1}{2})(4)(4^2+1) = 34$ and the sequence will be 30, 31, 32, 33, 34, 35, 36, 37, 38. So in a 4th order anti-magic square the ten line-totals must form a scramble of one of the sequences:

29 to 38 inclusive, diagonals totalling $29+34 = 63$ or
30 to 39 inclusive, diagonals totalling $39+34 = 73$.

We therefore choose our sequence, abstract two terms having the right total for diagonals, and separate the remaining $2n$ terms into two equal groups (having the same total) for rows and columns. There will be numerous permutations, so mathematical analysis is difficult. Fortunately, trial and error can be used.

Here is the general method (illustrated for 4th order squares) which has given me the examples in this article. To double our chance of

finding a result we will choose 34 as one of our two diagonals, since this number can be associated with either 29 or 39. We will separate the remaining eight numbers of our generating sequence into two groups

30, 33, 36, 37 and 31, 32, 35, 38

though several other combinations are possible. Prepare a 4-by-4 blank diagram on a card and number the rows and columns at the side as shown in the first figure below:

30
33
36
37

14	5	1	10
2	12	4	15
8	3	16	9
11	13	7	6

Rows only
shuffled
in.

10	5	14	1
12	4	2	15
3	16	8	9
6	7	11	13

Rows and
columns
shuffled
in.

31 32 35 38

Shuffle in the numbers 1 to 16 so that the rows have the required totals. This is very quick and easy. Then reshuffle horizontally until the columns are also right. This usually presents no trouble. We now test the diagonals. The two principal ones have totals 35 and 25, neither of which is of use to us. However, we can perhaps use the broken diagonals, such as 5, 2, 9, 6 or 14, 15, 3, 7 and this idea yields the two sets of numbers 22, 39, 40 and (in the opposite direction) 40, 37, 34. The two underlined numbers have the values we want, but unfortunately they will not do, as they have a number in common, the 14, and so cannot be rearranged as principal diagonals.

But we are far from beaten yet. Any set of four numbers, one from each row and column, taken from our square, can be rearranged as a principal diagonal, and we find several hopeful values among the 24 possibilities: in all, four 34's and a 39. We take each in turn and test it. Thus the 39 gives, on rearrangement, the square to the left in which the other principal diagonal has a total of 37. There are also the two broken diagonals 2, 1, 6, 16 and (less obviously) 8, 10, 13, 4 with totals of 25 and 35 respectively - no good, as it happens. But either could have been rearranged as a principal diagonal crossing our 39. But we are still not beaten.

14	1	10	5
2	15	12	4
8	9	3	16
11	13	6	7

30
33
36
37

35 38 31 32 (39)

It is usually possible to make various alterations in a square which leave a given diagonal unchanged. We can, for example, give the bottom-left-to-top-right diagonal its wanted value by interchanging the two uppermost rows in their entirety. This, by substituting 2, 1 for 14, 15, drastically alters our 39-diagonal. Since $2+14$ equals $15+1$, we can make the horizontal interchanges 2—15, 14—1, and so correct our 39-diagonal without altering the values of any row or column, or that of the other diagonal. So we have found our first anti-magic square, the one shown to the right, which these changes give.

15	2	12	4
1	14	10	5
8	9	3	16
11	13	6	7

(34) 35 38 31 32 (39)

The table below shows the twenty feasible (but not necessarily all possible*) combinations of numbers forming a 4th order anti-magic square sequence. The four rows or columns may of course be rearranged in any order, and rows or columns interchanged. One example from each of the six differently diagonalised groups follows the table.

Sequence	Diagonals	Rows	Columns	Code
29 to 38	29/34	30 31 37 38	32 33 35 36	A
		30 32 36 38	31 33 35 37	B
		30 33 35 38	31 32 36 37	C
		30 33 36 37	31 32 35 38	D
	30/33	29 32 37 38	31 34 35 36	E
		29 34 35 38	31 32 36 37	F
		29 34 36 37	31 32 35 38	G
	31/32	29 33 36 38	30 34 35 37	H
		29 34 35 38	30 33 36 37	I
		29 34 36 37	30 33 35 38	J
30 to 39	34/39	30 31 37 38	32 33 35 36	K
		30 32 36 38	31 33 35 37	L
		30 33 35 38	31 32 36 37	M
		30 33 36 37	31 32 35 38	N
	35/38	30 31 36 39	32 33 34 37	O
		30 33 34 39	31 32 36 37	P
		30 33 36 37	31 32 34 39	Q
	36/37	30 32 35 39	31 33 34 38	R
		30 33 34 39	31 32 35 38	S
		30 33 35 38	31 32 34 39	T

11 14 1 4 5 2 16 10 15 3 8 9 6 12 7 13 37 31 32 36 33 35 38 29 34	8 2 16 3 4 15 6 13 12 14 1 10 7 5 11 9 31 36 34 35 37 32 30 33	1 13 3 12 15 9 4 10 7 2 16 8 14 6 11 5 37 30 34 35 38 36 32 31
C	E	H
4 5 7 14 6 13 3 15 11 12 9 1 10 2 16 8 31 32 35 38 34 36 39 34	7 5 2 16 1 15 6 11 13 3 12 8 10 9 14 4 31 32 34 39 37 38 35 38	1 12 4 16 13 9 6 2 10 8 11 5 7 3 14 15 31 32 35 38 36 39 37 36
N	Q	S

* In fact all are possible except (B) and (L) in the table, which fail for reasons of parity.

Readers may like to try and construct examples illustrating the other code letters. And now for some higher order squares. As n increases each square takes progressively longer to complete, but your first attempt will usually yield a result. With 4th order squares many failures occur.

21 18 6 17 4 7 3 13 16 24 5 20 23 11 1 15 8 19 2 25 14 12 9 22 10 66 63 60 69 67 65 64	10 25 32 13 16 9 22 7 3 24 21 30 20 27 18 26 11 6 1 31 23 33 17 8 19 5 36 12 15 29 34 14 2 4 35 28 105 107 108 113 116 117 118 110	14 3 34 21 47 29 22 43 16 13 25 6 26 44 30 48 24 8 12 9 45 10 5 11 38 49 46 19 4 41 37 36 33 27 1 39 17 40 20 7 35 23 31 42 18 32 28 2 15 170 173 176 178 179 181 168 183 175
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5th Order (59 to 70) 6th Order (105 to 118) 7th Order (168 to 183)

49 16 50 10 19 28 24 56 42 43 11 15 44 38 55 5 25 21 48 46 9 37 6 63 29 47 8 40 51 30 52 1 45 22 54 23 20 34 2 62 14 59 18 33 41 26 61 13 36 12 58 32 27 64 3 35 17 39 7 57 53 4 60 31 252 253 255 258 262 265 267 268 269 260	52 19 81 22 29 15 42 31 76 61 10 67 23 54 79 25 33 16 57 9 71 24 38 1 51 47 75 26 78 7 69 66 77 13 27 12 39 21 74 20 37 17 49 55 64 8 65 4 62 50 34 73 41 40 56 68 2 63 14 72 35 44 6 53 30 60 32 36 3 46 43 58 11 70 5 59 48 80 28 45 18 367 368 373 375 376 377 360 361 364 379 369
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8th Order (252 to 269)

(9th Order (360 to 379))

This article must be regarded as no more than a preliminary skirmish with the subject. Suggestions for further research:

- (1) General properties of anti-magic squares.
- (2) Systematic methods of formation (routes, lattices, subsidiary squares, etc.).
- (3) Methods of converting magic squares into anti-magic squares, and *vice versa*.
- (4) Methods of converting small squares into large ones.

Editor's Comment: *The ramifications of mathematics are indeed ubiquitous! Mathematical computers tell us how much we're worth (paychecks), what our chances are of completing college, they mail most of our magazines to us by mathematical code, planes and cars are designed mathematically for our comfort (or is it to ensure purchase and use?), and so on for just about every activity in which we may become involved. But even such a commonplace event as getting caught in the rain can be treated mathematically. Specifically, how to keep dry in the rain - or at least how to keep drier than those persons who are so thoughtless as to forget their slide rule, log and trig tables, anemometer, and graph paper!*

A Walk in The Rain

by Alan Sutcliffe — Yorkshire, England

When it's raining, I often see people hurrying, running even, apparently in the belief that by doing so they will not become so wet. But are they right? Anyone who has been on a motorcycle in the rain will know that it is a very efficient way of getting *very* wet. By analyzing the situation I have been able to show that it is not always the fastest who keep the driest, but that there is often an optimum speed for keeping comparatively dry.

If, in a north wind, rain is falling with a speed y and at an angle α to the vertical, and if a man has to make a trip through the rain in a direction making an angle β with north, then the situation is as shown in Figure 1.

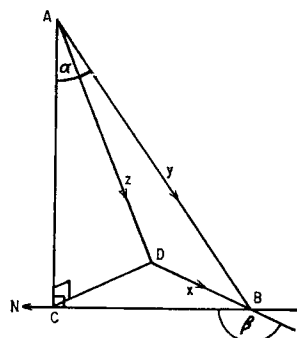


Figure 1

The question is, assuming that conditions remain constant, at what speed must the man walk, or run, in order to keep as dry as possible? The amount of rain, R , he encounters will be proportional to:

$$(\text{Time}) (\text{Area exposed to rain}) (\text{Flow of rain relative to himself}) \quad (1)$$

In the first place we shall assume that the man is a sphere of unit radius, so that he will have a constant area π exposed normally to the rain. Without losing anything in the way of generality, we may take the density of the rain (i.e. the volume of water in a given volume of air) and the distance to be travelled as units. To allow for these in practice, we simply multiply our answer by the actual density and

distance involved in the particular case.* If x is the man's speed, and z is the speed of the rain relative to him, then y may be found by reference to Figure 1, where the lengths are taken as being proportional to the corresponding speeds. Using the cosine rule we have:

$$CD^2 = x^2 + y^2 \sin^2 \alpha + 2xy \sin \alpha \cos \beta$$

Applying Pythagoras' Theorem gives:

$$z^2 = x^2 + y^2 \sin^2 \alpha + 2xy \sin \alpha \cos \beta + y^2 \cos^2 \alpha = x^2 + 2xy \sin \alpha \cos \beta + y^2$$

Since, at unit density, flow is equal to speed, it follows that

$$R = (1/x) (\pi) (z) \\ = (\pi/x) (\sqrt{x^2 + 2xy \sin \alpha \cos \beta + y^2})$$

Differentiating gives:**

$$(dR/dx) = \frac{-\pi y (x \sin \alpha \cos \beta + y)}{x^2 \sqrt{x^2 + 2xy \sin \alpha \cos \beta + y^2}} \\ = 0 \text{ when } x = (-y) / (\sin \alpha \cos \beta) \quad (2)$$

This can easily be shown to correspond to the minimum value of R . When $\beta > 90^\circ$, that is when the wind is to some extent behind the man, $\cos \beta$ is negative, so that a positive value of x can be found giving the value

$$R_{\min} = \pi \sqrt{1 - \sin^2 \alpha \cos^2 \beta}$$

Looking at Figure 2, where E is the foot of the perpendicular from D to BC , and where x is taken so that $\angle AEB = \alpha$, we have

$$y / \sin \alpha = EB = -x \sin \beta$$

Thus the minimum value of R is given when $\angle BAE = 90^\circ$.

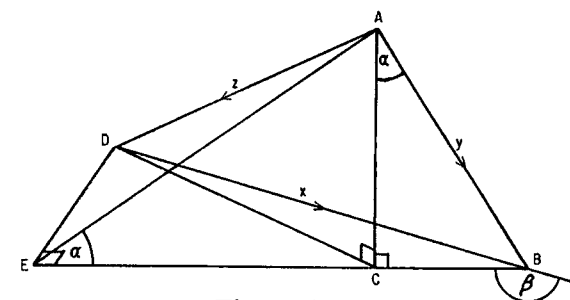


Figure 2

Now in $\triangle ABD$ we have $y^2 = x^2 \sin^2 \alpha \cos^2 \beta$ and also $z^2 = x^2 (1 - \sin^2 \alpha \cos^2 \beta)$ since $(\pi z/x) = R = \pi (1 - \sin^2 \alpha \cos^2 \beta)$. Therefore $x^2 = y^2 + z^2$, so that $\angle BAD = 90^\circ$.

We can now give the following rule for keeping as dry as possible, when the wind is behind:

The man must increase his speed until the direction of the rain relative to himself is at right angles to its direction relative to the ground.

*Hence the need for slide rule, log and trig tables, anemometer, graph paper. (Editor's Note)

**All this mathematics is making our poor man mighty wet. (Editor's Note)

When $\beta \leq 90^\circ$, that is when the wind is more or less facing him, there is no positive value of x satisfying equation (2), since $\cos\beta \geq 0$. The best the man can do in these circumstances is to travel as fast as possible.*** This gives, as his speed x increases to infinity, $R = \pi$. (This limit holds for all values of β , but only when $\beta \leq 90^\circ$ does it give a minimum value of R .)

Let us take a few examples and see what variation there is in R at different values of x . We shall take $\alpha = 30^\circ$, so that $\sin\alpha = \frac{1}{2}$, and $\beta = 0^\circ, 60^\circ, 120^\circ$ and 180° , giving $\cos\beta = 1, \frac{1}{2}, -\frac{1}{2}$ and -1 , respectively. Then we have

$$R = (\pi/x)(\sqrt{x^2 + xy\cos\beta + y^2})$$

and putting in different values of x for each value of β allows us to draw the graph of R against x shown in Figure 3.

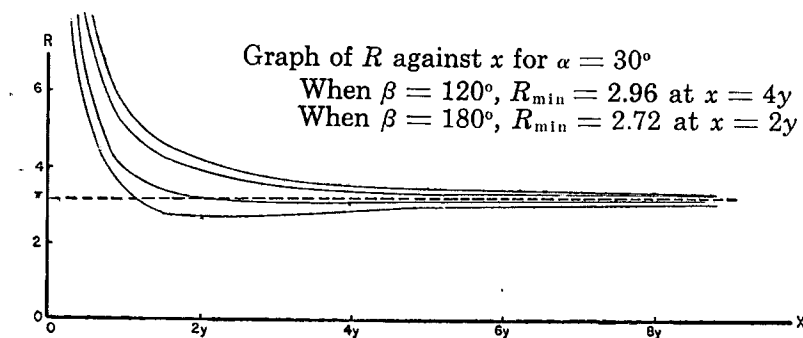


Figure 3

This shows more clearly than the equations just what are the benefits and penalties of walking at different speeds. As a general rule it is clear that there is little or no benefit in walking faster than about twice the speed with which the rain is travelling. I've not made any accurate calculation of this speed, y , in practice, but presumably it is of the order of 5 to 10 miles per hour, in which case hurrying, running even, is justified.

Considering now the case of a man more realistically shaped than our sphere, say a vertical circular cylinder, we must take into account the changes in the area exposed normally to the rain for different values of the angle, and as the speed x varies. Following the analysis given above, the interested reader should be able to deal with these cases himself.****

And remember, before you go hurrying through the rain, make sure first to see which way the wind is blowing!

And if there is no wind at all, stand still and you'll keep drier!*****
 (Wouldn't it be simpler just to stay home and watch TV? *Editor's Thought*)

***Really, he should simply turn around and go back home. (*Editor's Note*)

****While getting soaking wet, of course! (*Editor's Note*)

*****In which case don't be so thoughtless as to forget your umbrella. (*Editor's Note*)

Editor's Note: Many RMM readers have asked for some kinds of mathematical tricks or shortcuts in arithmetic operations — the emphasis on mental manipulation. Mr. Branscome's little article is presented here for those who want to learn how to square numbers rapidly.

Mental Squaring

by C. E. Branscome

Mental squaring is a challenge to any teacher or student even of high school level. Some majors in the field some times reach an impasse in mastering the simple mental expansion of squaring a number. The associative and memory capacity must be cultivated.

The ability to do mental squaring saves much time and a student can eliminate the use of the table of squares up to one hundred and even above that bracket of numbers. Mental squaring has great practical value in many branches of mathematics.

Teachers and students have found the techniques intriguing. In high school and college where this method has been presented, the students often pass the technique and principles to other persons interested in mathematics. The author has found that students in classes in the following terms have learned the technique from former students.

The algebraic form of the product of the sum and difference taught in algebra was used for the formula plus the square of the second term. Through mental analysis the result is determined in a moment or in a very few seconds.

$$x^2 = (x-y)(x+y) + y^2$$

The value of y is selected so that the first or second term ends with a zero, thus the multiplying can be done mentally and the square of y can be added mentally in that same step of multiplication. Numbers ending with a 5 require less concentration. The first number is multiplied by the next larger number and then 25 is added. Thus 75^2 becomes 70 times 80 plus 25.

$$(75)^2 = (70)(80) + 5^2 = 5625$$

$$(22)^2 = (20)(24) + 2^2 = 480 + 4 = 484$$

Mental squaring is of great value in checking Pythagorean triplets:

$$(1013)^2 = (1012)^2 + (45)^2$$

$$(1026)(1000) + 169 = (1024)(1000) + 144 + 2025$$

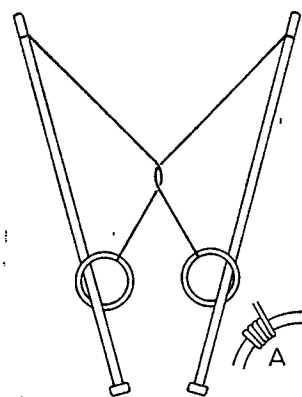
$$1026169 = 1024144 + 2025$$

Puzzles and Problems

February 1962

RECREATIONAL MATHEMATICS MAGAZINE

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1. Needle-and-Ring

For this puzzle you require two knitting-needles, two curtain-rings, and two short pieces of string. File a groove round the top end of each needle so that a string can be firmly tied to it. Loosely tie a ring to the other end of the string, which is made just too short to allow the ring to be slipped over the bottom end of the needle. (The only reason for loosely tying the rings is to avoid coiling as at A - there is no trick in it.)

If you have made the two assemblies separately, try to loop them as shown, and if they are looped, try to separate them.

(Harry Lindgren; Canberra, Australia)

2. Lamebrain Practices Arithmetic

In order to get some practice in arithmetic Mr. Lamebrain took some fraction, subtracted it from 1, and found the reciprocal of the result. He then took the number thus obtained, subtracted it from 1 and again found the reciprocal. The process which he had just completed twice so fascinated him that he kept on repeating it over and over. Twelve hours, twenty-three minutes and six seconds later, he had just completed the process for the one thousandth time, and surprisingly enough he found the net result to be 1000. Can you tell what number Lamebrain started with?

(Leo Moser; Edmonton, Alberta)

3. Neverest

Krush, Macdud and Perez are climbing Neverest, all three gaining height steadily (they never rest) at speeds which are in each case an exact small number of hundreds of feet per hour. Prez started at 6 a.m. on Monday, Krush 24 hours later, and Macdud 24 hours later still. At midnight on Wednesday they are still climbing, with Prez 600 feet above Krush and 6000 feet above Macdud. When the successful climber reaches the Summit, his rivals are again 600 and 6000 feet below him. How high is Neverest, who first reaches the Summit, and at what time?

(J. A. Lindon; Surrey, England)

4. Cube Formation, Again!

A rectangular piece of paper, 1 unit by 3 units, can be cut into two identical pieces which, by folding only, will form a cube, with no overlapping. The problem is to find at least one of the two known solutions.

(Stephen Barr; Woodstock, N. Y.)

5. Efficient Balancing

With five weights totaling 121 pounds, and an accurate two-pan balance, one can correctly make integral weighings from 1 to 121 pounds. What are the five weights?

(John H. Sweitzer; Princeton, New Jersey)

6. Two-Animal Puzzle

There are two animals, A and B, on a flat continuous unobstructed plain. They are about 100 feet apart, but if A wants to catch B (who will try to escape) he invariably can. Likewise, if B wants to catch A (who will likewise try to escape) he, too, invariably can.

How come? And what kind of animals are they? There is a valid and real answer to this puzzle.

(Stephen Barr; Woodstock, N. Y.)

7. Death in the Decanter

Messrs. Beer, Brandy, Chianti, Cider, Port, Sherry, and Whiskey of the Bottle Club hold their annual booze-up, the drinks being beer, brandy, etc. In conformity with the rules, each samples exactly three different drinks, but nothing beginning with his own initial. After the party three genuine corpses are found among the seven dead-drunks under the table, and investigation shows that one of the drinks had been maliciously tampered with. The following facts come to light: (a) the three victims among them sampled all seven drinks; (b) the brandy-drinker was a married man who refused sherry; his name-drink was taken by Mr. Brandy only; (c) Mr. Whiskey did not drink port; (d) of the seven only Messrs. Port, Sherry and Whiskey have never married; (e) the port-drinkers outnumbered the whiskey-drinkers by one; (f) the namesake of the only drink sampled by all three bachelors drank Chianti; Mr. Chianti himself was the only other to try this drink; (g) the murderer did not touch the name-drink of any of his victims.

Who poisoned whom, and which was the fatal tippie?

(J. A. Lindon; Surrey, England)

8. A Minor Computer Catastrophe

Mr. Smith, our accountant acquaintance, had a job that had to be done as fast as possible. Knowing that it would take him 10 hours to do it on his new computer and 12 hours to complete it on his old computer, he started the job on the new computer.

After he had been working a while, a breakdown occurred requiring repairs that would take 30 minutes.

Mr. Smith did some fast mental calculation and figured out that he could repair the circuits and still finish 10 minutes earlier than he could if he transferred the job to the old computer.

How long had he been working on the job before the computer broke down?

(Bertram Leadbeater; Beverly, Massachusetts)

9. *Really, Now!*

Bob is twice as old as Sally will be when Bob is one-half as old as Sally will be when Bob is four times as old as he is now. Sally will be three times as old as Bob was when Sally was one-third as old as Bob will be when Sally is half as many years older than the number of years she is younger than Bob.

Sally is in her teens, and of course we have disregarded odd months in their ages.

I hate to ask, but how old are Sally and Bob?

(Alan Gold; Downsview, Ontario)

10. *Brain Twister*

It was the twins' birthday and Mary had been given a kitchen scale of the type with two large balancing pans. Tony's present was a very well-made set of 12 wooden blocks each of a different color. They could be built up into brightly colored pyramids and the largest block was 12 inches on a side, the next 11 inches, then 10 inches, and so on by inches down to the smallest one inch block. They were colored silver, gold, crimson, yellow, violet, white, black, emerald, indigo, orange, purple and red.

The twins were happily playing on the lawn within sight of my window. After tiring of building towers and pyramids they began to balance the blocks on Mary's scales. My study was too far from them to see the size of the blocks but I could easily distinguish the colors. During the next hour I knew that with each squeal of delight the twins had managed a balance with some of the blocks. I was surprised at the number of different balances they were able to find. I quickly started jotting down the colors of various sets of balanced blocks. However, I only managed to jot down three different balances from the dozen or so they found before they were called in for lunch. Here are the three sets I managed to record:

- (1) Yellow and Black balanced Gold and Emerald.
- (2) Purple balanced Red, Silver and Violet.
- (3) Black, Orange, Purple and Violet balanced Emerald, Gold, Indigo, Silver, White and Yellow (this was their best effort).

Unfortunately a heavy thunderstorm produced such a downpour during lunch that when the twins went out to play after the weather had cleared they found that the rain had completely washed the paint off all the blocks. They were desolate and begged me to repaint their blocks as they were colored originally.

After some calculations I found that I had luckily recorded sufficient data for this.

What were the original colors of each of the wooden blocks according to size?

(R. H. Hide; Devonshire, England)

11. *Brain Strainer*

In our strange world of imagination, there are found, among other things, three categories of beings: (1) Purple people-eaters - who eat people, naturally; (2) People - who eat purple-people eaters; and (3) Purple-people eaters who eat purple people-eaters. (These diets are not necessarily exclusive, but they are inclusive.) The great Brain-Trust of this unique cosmos has gathered the following statistics:

Purple people-eaters reproduce at the rate of 10 per 1000 per year.

People reproduce at the rate of 25 per 1000 per year.

Purple-people eaters reproduce at the rate of 100 per 1000 per year.

Also, these three categories die from natural causes as follows:

Purple people-eaters at the rate of 1 per 1000 per year.

People at the rate of 3 per 1000 per year.

Purple-people eaters at the rate of 5 per 1000 per year.

At a given point in history, it is found that if x = the number of purple people-eaters, y = the number of people and z = the number of purple-people eaters, then $x:y:z = 1:5:10$. (For purposes of simplification (!) let's assume that x , y and z are constant over a period of a year.)

The great Brain-Trust finds these ratios to his liking and would like to keep the numbers of these three populations stationary for all time. At what rates (expressed in number per thousand per year) may purple people-eaters eat people, people eat purple-people eaters and purple-people eaters eat purple people-eaters?

(Brother Alfred; St. Mary's College, California)

12. *What's that again?*

Her husband had a 1960 car. A dealer offered him \$2000 for it toward a new \$3000 1961 model, leaving a balance of \$1000 to be paid on the new car. This also happens to be the amount hubby still owes the bank on his present car. Her plan was:

"It's simple", she explained, "We tell the man we accept the deal. He gives us \$2000 and we give him the car. We go to the bank and pay off the \$1000 balance. Then we take the other \$1000 cash and walk out with our new car. It hasn't cost us a cent. We don't even owe the bank a thing!"

(Contributor of this 'gem' prefers to remain anonymous)
(I don't blame him! — Editor's Note)

77	216
12	1293
121	131
8	68
139	4099

Tell a friend you are going to give him ten numbers quickly. After each number he is to say the next highest - quickly. Bet him he won't get every number right. Use the numbers shown. The trick is that most people will say 5000 after the the last number.

How to Get into an Argument With a Moebius Stripper

by Stephen Barr

You: How do you do, Mr. M-S; I've always wanted to meet someone who doesn't exist.

M-S: What do you mean - I don't exist?

You: Well, there's no such thing as a Moebius strip, so . . .

M-S: But there *is*! See: I take this strip, and -

You: What "strip"? That piece of paper has 6 faces: top, bottom, the two sides and the two ends - like a box. As I understand it, you claim it has only two faces - and four "edges," two of which you are about to join.

M-S: (With a superior tone) We ignore its thickness.

You: That's all very well to say, Mr. M-S, but its section, when drawn as it *really* is, looks like this:



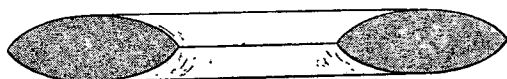
As you see, it has 4 sides, and 4 edges. When you twist it and join the ends, you make A coincide with C, and B with D. This results in 2 sides: a wide one and one very narrow one, and 2 edges, (AC) and (BD).

M-S: I never heard such nonsense! I -

You: As a matter of fact, this is the only way a piece of paper *can* have two edges and two sides. Of course, if it's a disc it has 3 sides and two edges. On the other hand, if you don't care about it's being a strip, you could have a torus with this cross-section:



It has one side and one edge, fulfilling your Moebius definition without any twisting. On the other hand - that makes three hands - if you *must* twist, you could use this torus:



Cut it radially, give it the half twist and join. Then you get one edge and one side. It had better be rubber, though.

M-S: But that's a torus: the moebius deformation doesn't apply to it. Only to paper . . .

You: Okay, but paper comes in different thicknesses, and if the paper were very thick and the strip narrow enough, so that $AB = AD$, then a quarter twist (90°) would connect A to B, B to C, C to

D and D to A. Then you would have one side and one edge, only I don't think even you would call it a "strip."

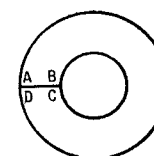
M-S: Of course not! What we do is imagine the thickness is zero.

You: All right, but if you do that, then A would coincide with D, and B with C, without any twisting and joining, and A would *equal* D, and B ditto C. Also all points, a, a', a'' , etc., on AB would coincide with corresponding points, d, d', d'' , etc., and therefore the surface AB would consist of a set of points all of which were also on the surface CD, and visa versa, and therefore AB would be the same as CD. One side. I don't think that giving it the old Moebius Twist would make anything very different; you'd get what you started with - a one-sided piece of paper.

M-S: (Craftily) Yes, but with only one edge!

You: You don't need the Moebius twist for that - all you had to do was take your scissors and round off the corners. Which reminds me: supposing for a minute I agree to ignore those very narrow sides - which you call "edges," - and go along with your myth about a two-sided, two-edged strip; how about making Moebius strip out of a paper annulus?

M-S: Simplicity itself, my dear Sir. You see this?



Well, cut along the radius, twist and join A to C, and B to D. *Viola!*

You: Very pretty. Now, about that hole in the middle - can't you get rid of it?

M-S: Get *rid* of it? Why?

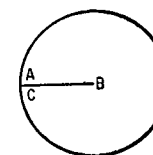
You: Because then you'd be starting out with a disc; a one-edged piece of paper, which might have an advantage. Maybe you could get rid of *it*, and you'd have a no-edged piece!

M-S: (With a faint sneer) Now *you're* talking nonsense. Anyway you need that hole to -

You: (Interrupting) How big does it have to be?

M-S: Well . . . er, let me see, now . . . (Mutters to himself.)

You: It seems to me you don't need any hole at all. Here - (taking his paper and scissors,) - we have a disc, and we cut a radial slit:



Now all we have to do is twist, and join the radial edges, only with directions reversed; AB against BC.

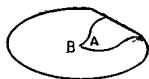
M-S: But then they'd all meet in one point!

You: So they would. First bring A up and over to B:

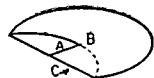


Now bend C, only the other way, also forming a circle, so that it coincides with the first circle, in reversed sense:

(turned back to front)



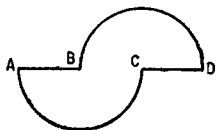
Then you join along the cut edges. The easiest way is to flatten it out:



As you see, A goes up and over to B, whereas C goes down, under and back to B. Now we round it out again, and we see it's developed a sort of hole.

M-S: If you join the ends of *any* strip you get a hole where there wasn't one before. (Examines your handiwork.) Yes . . . not bad - it has one side and one edge. Kind of tricky to make, though.

You: Not really. Look; we make a new radial cut opposite the original one, and flatten it out, getting this:



It's a shape we could have started with: two semi-circles connected radius to radius, and you merely bend it over and place the unjoined radii next to one another, with A at D, and B at C, and then join them. Gives the same thing.

M-S: Hmm . . . Wonder what shape it gives when we cut it down the center . . . (Takes back scissors, and cuts.) Why, it's the most ridiculous thing I ever saw! (He's right: it is.)

You: Try doing it with a Klein bottle some time. It's real nervous, man. But speaking of cutting; you can cut a regular Moebius strip in two by cutting along one straight line, no?

M-S: Naturally: parallel to the edge, and near to it than to the middle.

You: What middle?

M-S: Why, half-way between . . . er . . .

You: Between the edge? (M-S frowns horribly.) All right; I know: like the center of a circle is, you might say, half way between the edge. It's like, how far can a rabbit run into a forest?

M-S: What's that got to do with -

You: (Innocently) Nothing. What I'm driving at is to make two pieces of *equal area* -

M-S: (Interrupting) Easy: you make the cut one-quarter the width away from the edge. Goes around twice.

You: (Continuing) - starting the cut at a point on the edge. In other words, what's the angle of the cut?

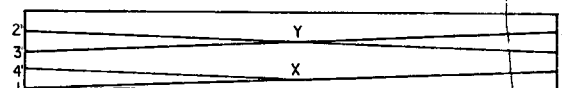
M-S: (There is a long silence).

(The floor is now littered with bits of paper.)

You: Give up? (M-S glowers.) It's not really as hard as it sounds, but there's a trick to it when you come to the cutting. The problem as stated was to draw one continuous straight line, starting at the edge, and then cut along it - getting two pieces of equal area. You have to do it in that way, otherwise you have trouble. The answer is this: lay the original unjoined strip flat, mark 4 equal divisions on the ends, then rule the line from 1 to 2. That's the required angle.



Now, if the strip were twisted and the ends joined, the line continued on would reappear at the *upper* left (because of the twist,) at the point marked 2' and continue *down* to 3.



Then reappear at 3', continue up to 4. (crossing itself, at y), and then reappear at 4', and on down to x. If you doubt the diagram, go ahead and rule the line *after* it's joined, and you'll see the line goes the way I said.

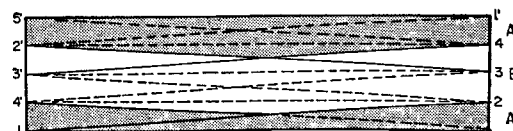
M-S: (After much maneuvering and swearing, the line is drawn.)

M-S: (Continued) There! Now we'll cut along it, and -

You: Hold on a minute: when you cut to a certain point, y, you'll find the strip springs open, but still in one piece. The rest of the line will be crossing it near the middle, at an angle. You'll have the impression you're making *two* cuts, but actually it's a continuation of the first. If, when you cut just past y, you stick it together again temporarily at that point with a small bit of tape, you'll find that eventually you come back to y again, and the rest of the line is right there to continue cutting on.

M-S: And just how do we know the two pieces are indeed equal in area?

You: We make a fatter diagram and put in the remaining diagonals in dotted lines. The shaded parts you can see are joined in the finished Moebius strip: 1-4' to 1'-4, and 2-5 to 2'-5'. That's Piece A:



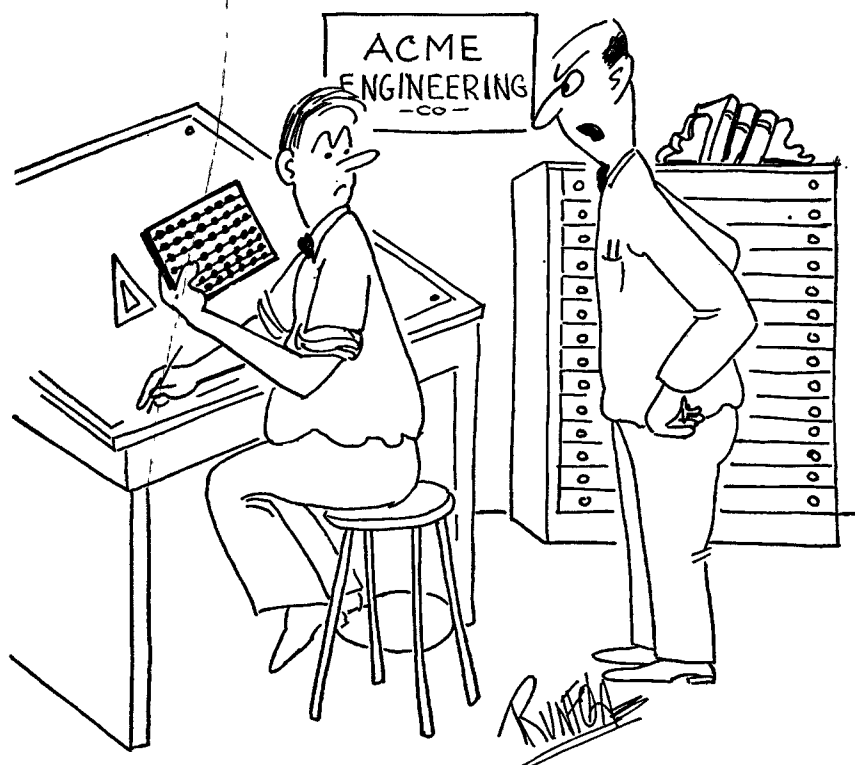
The unshaded parts join 3'-4' to 3-4, and 2-3 to 2'-3'; that's Piece B. Add the triangles and you see A has 8; B has 8.

M-S: (After a pause) So how far *can* a rabbit run into a forest?

(You tip-toe out, leaving him this consolation prize:



* * * * *



RMM Publications

Recreational mathematics is fast becoming a national pastime and RMM intends to fill the need for material to pass the time! A series of publications of the same size as RMM but containing 80-100 pages will be produced during the next few years. The general plan is to publish individual booklets about single topics in recreational mathematics and puzzles or games. One title is already in completed manuscript form and will be published soon. Another is being written and others are in the planning stages. Here's a list:

GEOMETRIC DISSECTIONS by Harry Lindgren. The authority on a favorite recreational mathematics pastime. Transformation of one polygon to another, amazing arrangements, etc. . . . "Harry Lindgren is the world's leading expert on such problems . . ." - Martin Gardner.

FUN FOR THE MONEY by Maxey Brooke. A collection of games and puzzles which can be played with coins. This booklet will be illustrated and printed along the same lines as RMM, and will be available about July 1962.

TOPOLOGY CAN BE FUN by Maxey Brooke and Joseph S. Madachy. All about topology, Moebius strips, paper chains, and associated phenomena. A thorough bibliography will be included so readers can explore the field much more thoroughly.

Other tentative titles include: **THE GOLDEN TREASURE OF THE GOLDEN RATIO**, **NUMBERS SYSTEMS** (and how to handle them), **THE BEST OF RMM** (a collection of the most popular items in RMM along with a full measure of added material supplied by enthusiastic readers), **ALMOST ALL ABOUT PRIME AND PERFECT NUMBERS**, **RECREATIONAL MATHEMATICAL TABLES** (the kind of mathematics tables useful to the incurable recreational mathematics fiend), and others.

The general price range for these booklets will be \$1.00 to \$1.25, though some may cost more. Would you like to examine these booklets as they appear? (Approximately two a year). Fill in your name and address below and you will be notified as soon as booklets are published. Or send in a post card if you don't like to tear up your copy of RMM.

Please let me know about the new RMM Publications as soon as they appear.

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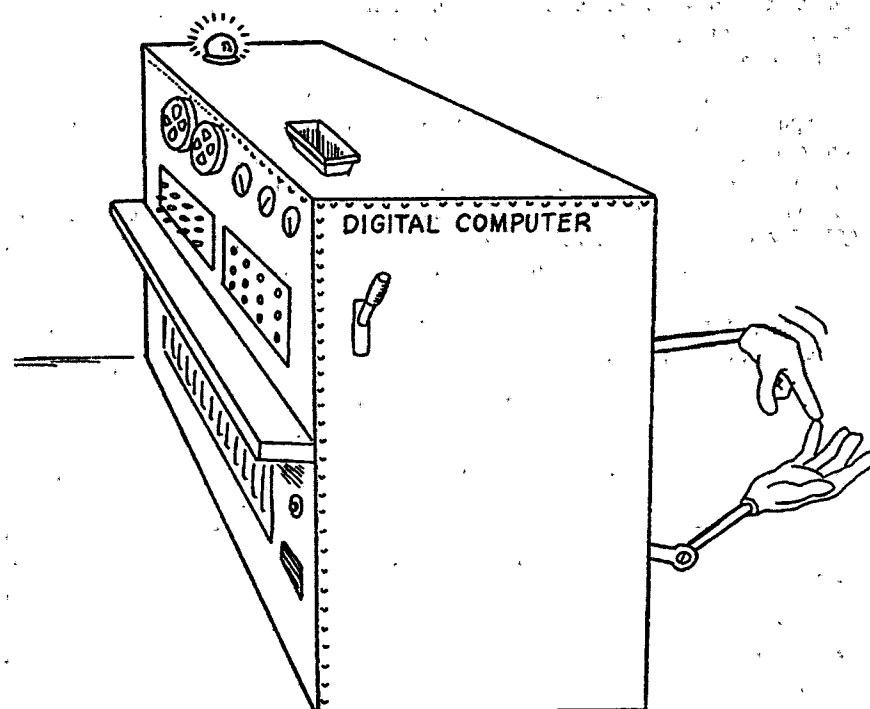
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Paige

Numbers, Numbers, Numbers

Editor's Note: Charles W. Trigg, of Los Angeles City College, is an inexhaustible source of number phenomena. This department of RMM starts with his work on solutions of $ABC + DEF = GHK$ with distinct digits. Mr. Trigg then continues with some amazing number manipulations with the year 1962.

SOLUTIONS OF $ABC + DEF = GHK$ WITH DISTINCT DIGITS

Charles W. Trigg

In the equation $ABC + DEF = GHK$, $A + B + C + D + E + F \equiv G + H + K \pmod{9}$. If each of the letters represents a distinct positive digit, then, since the sum of the nine digits is 45, $G + H + K \equiv 0 \pmod{9}$.* From the seven sets of three distinct digits with a sum of 18, the forty-one basic solutions of the equation are found easily. Each of these gives three additional solutions by interchanging the ten's digits and by interchanging the unit's digits of ABC and DEF . In the 164 solutions which follow, it will be observed that there are only thirty-one distinct values of GHK .

$$\begin{aligned} 173 + 286 &= 176 + 283 = 183 + 276 = 186 + 273 = 459 \\ 173 + 295 &= 175 + 293 = 193 + 275 = 195 + 273 = 468 \\ 127 + 359 &= 129 + 357 = 157 + 329 = 159 + 327 = 486 \\ 127 + 368 &= 128 + 367 = 167 + 328 = 168 + 327 = 495 \\ 162 + 387 &= 167 + 382 = 182 + 367 = 187 + 362 = 549 \\ 128 + 439 &= 129 + 438 = 138 + 429 = 139 + 428 = 567 \\ 218 + 349 &= 219 + 348 = 248 + 319 = 249 + 318 = 567 \\ 182 + 394 &= 184 + 392 = 192 + 384 = 194 + 382 = 576 \\ 216 + 378 &= 218 + 376 = 276 + 318 = 278 + 316 = 594 \\ 152 + 487 &= 157 + 482 = 182 + 457 = 187 + 452 = 639 \\ 251 + 397 &= 257 + 391 = 291 + 357 = 297 + 351 = 648 \\ 218 + 439 &= 219 + 438 = 238 + 419 = 239 + 418 = 657 \\ 182 + 493 &= 183 + 492 = 192 + 483 = 193 + 482 = 675 \\ 281 + 394 &= 284 + 391 = 291 + 384 = 294 + 381 = 675 \\ 215 + 478 &= 218 + 475 = 275 + 418 = 278 + 415 = 693 \\ 143 + 586 &= 146 + 583 = 183 + 546 = 186 + 543 = 729 \\ 142 + 596 &= 146 + 592 = 192 + 546 = 196 + 542 = 738 \\ 214 + 569 &= 219 + 564 = 264 + 519 = 269 + 514 = 783 \\ 124 + 659 &= 129 + 654 = 154 + 629 = 159 + 624 = 783 \\ 134 + 658 &= 138 + 654 = 154 + 638 = 158 + 634 = 792 \\ 352 + 467 &= 357 + 462 = 362 + 457 = 367 + 452 = 819 \\ 243 + 576 &= 246 + 573 = 273 + 546 = 276 + 543 = 819 \\ 241 + 596 &= 246 + 591 = 291 + 546 = 296 + 541 = 837 \\ 142 + 695 &= 145 + 692 = 192 + 645 = 195 + 642 = 837 \\ 317 + 529 &= 319 + 527 = 327 + 519 = 329 + 517 = 846 \end{aligned}$$

* J. A. H. Hunter explains this symbolism on pages 3-6 of this issue.

$$\begin{aligned}
125+739 &= 129+735 = 135+729 = 139+725 = 864 \\
271+593 &= 273+591 = 291+573 = 293+571 = 864 \\
214+659 &= 219+654 = 254+619 = 259+614 = 873 \\
324+567 &= 327+564 = 364+527 = 367+524 = 891 \\
342+576 &= 346+572 = 372+546 = 376+542 = 918 \\
243+675 &= 245+673 = 273+645 = 275+643 = 918 \\
341+586 &= 346+581 = 381+546 = 386+541 = 927 \\
152+784 &= 154+782 = 182+754 = 184+752 = 936 \\
317+628 &= 318+627 = 327+618 = 328+617 = 945 \\
162+783 &= 163+782 = 182+763 = 183+762 = 945 \\
216+738 &= 218+736 = 236+718 = 238+716 = 954 \\
271+683 &= 273+681 = 281+673 = 283+671 = 954 \\
215+748 &= 218+745 = 245+718 = 248+715 = 963 \\
314+658 &= 318+654 = 354+618 = 358+614 = 972 \\
324+657 &= 327+654 = 354+627 = 357+624 = 981 \\
235+746 &= 236+745 = 245+736 = 246+735 = 981
\end{aligned}$$

By interchanging ABC and DEF each of these sums may be arranged in a square matrix in two ways such that the third row is the sum of the first two rows. In only four of these 328 arrays is there a continuous path of single moves, parallel to the sides or diagonally, which covers the digits in increasing order. These are:

$$\begin{array}{cccc}
2 & 1 & 9 & 3 & 1 & 9 & 1 & 3 & 9 & 1 & 2 & 9 \\
3 & 4 & 8 & 2 & 4 & 8 & 4 & 2 & 8 & 4 & 3 & 8 \\
5 & 6 & 7 & 5 & 6 & 7 & 5 & 6 & 7 & 5 & 6 & 7
\end{array}$$

In the first two cases, the paths close by moving from 9 to 1. In the last matrix, the path is the unique rookwise connected chain mentioned by Martin Gardner on page 170 of the June 1961 SCIENTIFIC AMERICAN.

If the digits in order be considered to form a closed chain upon going from 9 to 1, then five more arrays exhibit an opened chain, the breaks occurring after 4, 6, 6, 7, and 7, respectively. These are:

$$\begin{array}{ccccc}
2 & 4 & 9 & 2 & 1 & 9 & 2 & 3 & 9 & 2 & 1 & 8 & 3 & 1 & 8 \\
3 & 1 & 8 & 4 & 3 & 8 & 4 & 1 & 8 & 3 & 4 & 9 & 2 & 4 & 9 \\
5 & 6 & 7 & 6 & 5 & 7 & 6 & 5 & 7 & 5 & 6 & 7 & 5 & 6 & 7
\end{array}$$

Of the eighty-four possible sets of three distinct digits only fifty-eight appear as parts of the sums and an additional four appear in the totals. None occur less than twice, and 4, 5, 6 appears fifteen times whereas 1, 2, 8 and 1, 2, 9 appear 16 times each.

Of the five hundred and four three-digit numbers with distinct digits only one hundred and ninety-three appear as parts of the sums, and an additional twenty-four appear in the sums. Many appear only once, but 273, 327, and 546 appear six times and 218 appears seven times.

PLAYING WITH 1962 AND ITS DIGITS

by Charles W. Trigg

Representation of Integers

Many integers may be expressed by using the digits of 1962, once each, together with standard algebraic symbols. Some integers may be expressed in a variety of ways. In the representations which follow, in some cases a simpler form has been discarded in order to keep the digits in the order, 1, 9, 6, 2. [Note: $!n$ = sub-factorial n , so $!1 = 0$, $!2 = 1$, $!3 = 2$, $!4 = 9$, $!5 = 44$, and $!6 = 265$. $n!$ = factorial $n = (1)(2)(3)\dots(n)$]

$$\begin{aligned}
0 &= -1+9-6-2 & 35 &= -1+9(6-2) \\
1 &= (1)(9)-6-2 & 36 &= (1)(9)(6-2) \\
2 &= 1+9-6-2 & 37 &= 1+9(6-2) \\
3 &= -(1)(9)+(6)(2) & 38 &= [1+(\sqrt{9})(6)]2 \\
4 &= -1+9-6+2 & 39 &= 1+(\sqrt{9}!)(6)+2 \\
5 &= (1)(9)-6+2 & 40 &= (1+9)(6-2) \\
6 &= 1+9-6+2 & 41 &= -(1)(\sqrt{9})+!(6-!2) \\
7 &= 1+9-6/2 & 42 &= 1-\sqrt{9}+!(6-!2) \\
8 &= 1+\sqrt{9}+6-2 & 43 &= -19+62 \\
9 &= (1)(\sqrt{9})(6/2) & 44 &= -1+9+6^2 \\
10 &= 1+\sqrt{9}(6/2) & 45 &= (1)(9)+6^2 \\
11 &= 19-6-2 & 46 &= (-1+9)6-2 \\
12 &= (1)(9)+6/2 & 47 &= -1+96/2 \\
13 &= (1)(9)+6-2 & 48 &= 1(96)/2 \\
14 &= 1+9+6-2 & 49 &= 1+(96)/2 \\
15 &= (1)(\sqrt{9})+(6)(2) & 50 &= (19+6)(2) \\
16 &= -1+9+6+2 & 51 &= -1+(9)(6)-2 \\
17 &= (1)(9)+6+2 & 52 &= (1)(9)(6)-2 \\
18 &= 1+9+6+2 & 53 &= 1+(9)(6)-2 \\
19 &= -1+(\sqrt{9})(6)+2 & 54 &= (1)(9)(6)(!2) \\
20 &= -1+9+(6)(2) & 55 &= -1+(9)(6)+2 \\
21 &= (1)(9)+(6)(2) & 56 &= (1)(9)(6)+2 \\
22 &= 1+9+(6)(2) & 57 &= 1+(9)(6)+2 \\
23 &= 19+6-2 & 58 &= (1+9)(6)-2 \\
24 &= (-1+9)(6/2) & 59 &= -(1)(\sqrt{9})+62 \\
25 &= 19+(6/2)! & 60 &= (1+9)(6/2)! \\
26 &= (19-6)(2) & 61 &= (1+9)(6)+!2 \\
27 &= (1)(9)(6/2) & 62 &= (1+9)(6)+2 \\
28 &= (\sqrt{196})(2) & 63 &= (1)(9)(6+!2) \\
29 &= -1+(9+6)(2) & 64 &= (-1+9)(6+2) \\
30 &= (1+9)(6/2) & 65 &= (1)(\sqrt{9})+62 \\
31 &= 19+(6)(2) & 66 &= 1+\sqrt{9}+62 \\
32 &= (1+9+6)(2) & 67 &= -1+(\sqrt{9})!+62 \\
33 &= (1)(9)+(6-2)! & 68 &= 1(\sqrt{9})!+62 \\
34 &= [-1+(\sqrt{9})(6)]2 & 69 &= 1+(\sqrt{9})!+62
\end{aligned}$$

$$\begin{array}{ll}
70 = -1 + 9 + 62 & 85 = -1 + 92 - 6 \\
71 = -1 + 9(6+2) & 86 = (1 + \sqrt{9})! + 62 \\
72 = 1 + 9 + 62 & 87 = (1)(9^2) + 6 \\
73 = 1 + 9(6+2) & 88 = (-1 + \sqrt{9})[!(6-!2)] \\
74 = [1 + (\sqrt{9!})(6)]2 & 89 = 91 - \sqrt{6-2} \\
75 = (9)(6) + 21 & 90 = (1+9)[!(6-2)] \\
76 = 19(6-2) & 91 = 91 + !{![!(6/2)]} \\
77 = (1+6)(9+2) & 92 = 92 + (6)(!1) \\
78 = (1/9)(6!) - 2 & 93 = -1 + 96 - 2 \\
79 = (1/9)(6!) - !2 & 94 = (1)(96) - 2 \\
80 = (1+9)(6+2) & 95 = 1 + 96 - 2 \\
81 = 19 + 62 & 96 = (-1+9)(6)(2) \\
82 = (1/9)(6!) + 2 & 97 = -1 + 96 + 2 \\
83 = 91 - 6 - 2 & 98 = (196)/2 \\
84 = 96 - 12 & 99 = 1 + 96 + 2
\end{array}$$

Permutations

There are twelve two-digit integers which can be formed from the digits of 1962. One of these, 16, is a square. Eight are otherwise composite. Three are prime and their sum $19 + 29 + 61 = 109$, a prime.

There are $P(4,3)$ or twenty-four three-digit integers which can be formed from the digits of 1962. One of these, 216, is a cube. Three, 169, 196, and 961, are squares. Seventeen are otherwise composite. Three, 269, 619, and 691, are primes. Their sum, 1579, is also prime.

There are $P(4,4)$ or twenty-four four-digit numbers which can be formed from the digits of 1962. All are divisible by 9. One, 1296, is a fourth power; one, 9261, is a cube; and three, 1296, 2916, and 9216, are squares. $1962 + 9621 + 6219 + 2196 = 19998$, and the sum of the twenty-four integers formed by the permutations of 1, 9, 6, 2 is 119988.

$2691 - 1962 = 729$, $927 - 729 = 198$, $891 - 198 = 693$, $693 - 396 = 297$, $792 - 297 = 495$, and $594 - 495 = 99$, a palindromic number after 6 operations.

$1962 + 2691 = 4653$, $4653 + 3564 = 8217$, $8217 + 7128 = 15345$, and $15345 + 54351 = 69696$, a palindromic number after 4 operations.

Representation by arithmetic progressions

1962 is the sum of four sets of consecutive integers, five sets of consecutive even integers, three arithmetic progressions with $d = 3$, five arithmetic progressions with $d = 4$, etc. Thus:

$$\begin{aligned}
1962 &= 653 + 654 + 655 = 214 + 215 + 216 + \dots + 222 \\
&= 158 + 159 + 160 + \dots + 169 = 37 + 38 + 39 + \dots + 72 \\
&= 980 + 982 = 652 + 654 + 656 = 322 + 324 + \dots + 332 \\
&= 210 + 212 + 214 + \dots + 226 = 92 + 94 + 96 + \dots + 126 \\
&= 651 + 654 + 657 = 486 + 489 + 492 + 495 \\
&= 147 + 150 + 153 + \dots + 180 \\
&= 979 + 983 = 650 + 654 + 658 = 317 + 321 + \dots + 337 \\
&= 202 + 206 + 210 + \dots + 234 = 75 + 79 + 83 + \dots + 143 \\
&= 649 + 654 + 659 = 483 + 488 + 493 + 498 \\
&= 136 + 141 + 145 + \dots + 191 \\
&= 198 + 203 + 208 + \dots + 238
\end{aligned}$$

Representation by Squares

Since $1962 = (2)(3^2)(109)$ it is not factorable into a pair of even factors nor into a pair of odd factors, so it cannot be expressed as the difference of two integer squares. However,

$$\begin{aligned}
1962 &= (63\frac{1}{2})^2 - (45\frac{1}{2})^2 = (113\frac{1}{2})^2 - (104\frac{1}{2})^2 \\
&= (166\frac{1}{2})^2 - (160\frac{1}{2})^2 = (328\frac{1}{2})^2 - (325\frac{1}{2})^2 \\
&= (491\frac{1}{2})^2 - (489\frac{1}{2})^2 = (981\frac{1}{2})^2 - (980\frac{1}{2})^2
\end{aligned}$$

1962 can be expressed as the sum of n distinct integer squares ($n = 2, 3, \dots, 15$ in a great number of ways.

Typical examples are:

$$\begin{aligned}
1962 &= 21^2 + 39^2 = 1^2 + 5^2 + 44^2 = 1^2 + 10^2 + 30^2 + 31^2 \\
&= 1^2 + 6^2 + 8^2 + 30^2 + 31^2 = 1^2 + 4^2 + 6^2 + 8^2 + 9^2 + 42^2
\end{aligned}$$

In the next examples for more than 6 terms, the notation $(a, b, \dots, n)^2 \equiv a^2 + b^2 + \dots + n^2$ is used:

$$\begin{aligned}
1962 &= (1, 2, 4, 8, 10, 16, 39)^2 = (1, 2, 3, 4, 6, 10, 14, 40)^2 \\
&= (1, 3, 4, 5, 6, 7, 8, 9, 41)^2 = (1, 3, 6, 7, 9, 11, 12, 13, 14, 34)^2 \\
&= (2, 4, 6, 8, 9, 12, 14, 16, 18, 20, 21)^2 \\
&= (2, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19)^2 \\
&= (1, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 21)^2 \\
&= (1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19)^2 \\
&= (2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18)^2
\end{aligned}$$

Representation by Primes

$$\begin{aligned}
1962 &= (41)(43) + 199 = (43)(47) - 59 = (7)(11)(13) + 31^2 \\
&= (11)(13)(17) - (7)(67) = (3)(5)(7)(13) + (3)(199) \\
&= (3)(5)(7)(19) - (3)(11)
\end{aligned}$$

1962 can be expressed as the sum of n distinct primes ($n = 2, 3, \dots, 31$) in a variety of ways. In the following examples, most of the series have some subsidiary characteristic. Twin primes, consecutive primes, palindromic primes and other types may be identified with the aid of the table of primes in the February 1961 issue of RMM. Representative series of primes for the various values of n are listed in the concise notation:

$$(a, b, c, \dots, y) \equiv a + b + c + \dots + y$$

$$\begin{aligned}
1962 &= 11, 1951 = 2, 659, 1301 = 241, 421, 617, 683 \\
&= 2, 181, 479, 503, 797 = 37, 73, 79, 97, 739, 937 \\
&= 2, 3, 7, 277, 307, 317, 1049 \\
&= 139, 179, 199, 229, 239, 269, 349, 359 \\
&= 2, 7, 131, 181, 191, 313, 353, 383, 401 \\
&= 37, 71, 73, 149, 151, 281, 283, 293, 311, 313 \\
&= 2, 47, 71, 73, 137, 139, 233, 281, 283, 347, 349 \\
&= 41, 61, 71, 101, 131, 151, 181, 191, 211, 241, 251, 331 \\
&= 2, 13, 37, 43, 73, 101, 131, 151, 181, 191, 313, 353, 373 \\
&= 3, 23, 43, 53, 83, 103, 113, 163, 173, 193, 223, 233, 263, 293 \\
&= 2, 13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 131, 179, 311, 701
\end{aligned}$$

$= 11, 13, 17, 19, 23, 101, 103, 107, 109, 191, 193, 197, 199, 223, 227, 229$
 $= 2, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 167$
 $= 17, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157$
 $= 2, 3, 5, 7, 11, 13, 19, 23, 127, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197$
 $= 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 163$
 $= 2, 3, 5, 7, 11, 13, 17, 19, 23, 31, 79, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197$
 $= 3, 11, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149$
 $= 2, 5, 7, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149$
 $= 3, 5, 7, 11, 13, 19, 29, 37, 43, 53, 61, 71, 79, 89, 97, 101, 113, 131, 139, 151, 163, 173, 181, 193$
 $= 2, 3, 31, 37, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139$
 $= 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 601$
 $= 2, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 113, 127, 131, 137, 139$
 $= 11, 13, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 137, 139$
 $= 2, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137$
 $= 3, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137$
 $= 2, 5, 7, 11, 13, 17, 19, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137$

Curious Sequences of Primes by Andrzej Makowski; Warsaw, Poland

1. The numbers $a_k = \frac{1}{3}(10^k - 7)$ are prime for $k = 2, 3, 4, 5, 6, 7, 8$. These numbers are

31 331 3331 33331 333331 3333331 33333331

For $k = 16m + 9$, where $m = 0, 1, 2, \dots$, the formula yields numbers divisible by 17.

2. Write the digits of n successive powers of 3 ($n = 0, 1, 2, 3$) beginning with 3^0 and after them write 01 or .07. The resulting numbers are prime:

101 1301 13901 1392701 and 107 1307 13907 1392707

The Next 605 Prime Numbers - 25111 to 31319

25111	25657	26209	26737	27337	27917	28513	29021	29581	30197	30809
25117	25667	26227	26759	27361	27919	28517	29023	29587	30203	30817
25121	25673	26237	26777	27367	27941	28537	29027	29599	30211	30829
25127	25679	26249	26783	27397	27943	28541	29033	29611	30223	30839
25147	25693	26251	26801	27407	27947	28547	29059	29629	30241	30841
25153	25703	26261	26813	27409	27953	28549	29063	29633	30253	30851
25163	25717	26263	26821	27427	27961	28559	29077	29641	30259	30853
25169	25733	26267	26833	27431	27967	28571	29101	29663	30269	30859
25171	25741	26293	26839	27437	27983	28573	29123	29669	30271	30869
25183	25747	26297	26849	27449	27997	28579	29129	29671	30293	30871
25189	25759	26309	26861	27457	28001	28591	29131	29683	30307	30881
25219	25763	26317	26863	27479	28019	28597	29137	29717	30313	30893
25229	25771	26321	26879	27481	28027	28603	29147	29723	30319	30911
25237	25793	26339	26881	27487	28031	28607	29153	29741	30323	30931
25243	25799	26347	26891	27509	28051	28619	29167	29753	30341	30937
25247	25801	26357	26893	27527	28057	28621	29173	29759	30347	30941
25253	25819	26371	26903	27529	28069	28627	29179	29761	30367	30949
25261	25841	26387	26921	27539	28081	28631	29191	29789	30389	30971
25301	25847	26393	26927	27541	28087	28643	29201	29803	30391	30977
25303	25849	26399	26947	27551	28097	28649	29207	29819	30403	30983
25307	25867	26407	26951	27581	28099	28657	29209	29833	30427	31013
25309	25873	26417	26953	27583	28109	28661	29221	29837	30431	31019
25321	25889	26423	26959	27611	28111	28663	29231	29851	30449	31033
25339	25903	26431	26981	27617	28123	28669	29243	29863	30467	31039
25343	25913	26437	26987	27631	28151	28687	29251	29867	30469	31051
25349	25919	26449	26993	27647	28163	28697	29269	29873	30491	31063
25357	25931	26459	27011	27653	28181	28703	29287	29879	30493	31069
25367	25933	26479	27017	27673	28183	28711	29297	29881	30497	31079
25373	25939	26489	27031	27689	28201	28723	29303	29917	30509	31081
25391	25943	26497	27043	27691	28211	28729	29311	29921	30517	31091
25409	25951	26501	27059	27697	28219	28751	29327	29927	30529	31121
25411	25969	26513	27061	27701	28229	28753	29333	29947	30539	31123
25423	25981	26539	27067	27733	28277	28759	29339	29959	30553	31139
25439	25997	26557	27073	27737	28279	28771	29347	29983	30557	31147
25447	25999	26561	27077	27739	28283	28789	29363	29989	30559	31151
25453	26003	26573	27091	27743	28289	28793	29383	30011	30577	31153
25457	26017	26591	27103	27749	28297	28807	29387	30013	30593	31159
25463	26021	26597	27107	27751	28307	28813	29389	30029	30631	31177
25469	26029	26627	27109	27763	28309	28817	29399	30047	30637	31181
25471	26041	26633	27127	27767	28319	28837	29401	30059	30643	31183
25523	26053	26641	27143	27773	28349	28843	29411	30071	30649	31189
25537	26083	26647	27179	27779	28351	28859	29423	30089	30661	31193
25541	26099	26669	27191	27791	28387	28867	29429	30091	30671	31219
25561	26107	26681	27197	27793	28393	28871	29437	30097	30677	31223
25577	26111	26683	27211	27799	28403	28879	29443	30103	30689	31231
25579	26113	26687	27239	27803	28409	28901	29453	30109	30697	31237
25583	26119	26693	27241	27809	28411	28909	29473	30113	30703	31247
25589	26141	26699	27253	27817	28429	28921	29483	30119	30707	31249
25601	26153	26701	27259	27823	28433	28927	29501	30133	30713	31253
25603	26161	26711	27271	27827	28439	28933	29527	30137	30727	31259
25609	26171	26713	27277	27847	28447	28949	29531	30139	30757	31267
25621	26177	26717	27281	27851	28463	28961	29537	30161	30763	31271
25633	26183	26723	27283	27883	28477	28979	29567	30169	30773	31277
25639	26189	26729	27299	27893	28493	29009	29569	30181	30781	31307
25643	26203	26731	27329	27901	28499	29017	29573	30187	30803	31319

Fibonacci - Mathematical Innovator

by Maxey Brooke

The year is 1180. It has been 800 years since the last European mathematician, Diophantus died, and it will be 400 years until the next is born. Yet in the midst of this intellectual wilderness is born a man who is destined to make most profound changes in our mathematical way of life, Leonardo, called Fibonacci.

If the dark ages can be said to have had a low point, it was the twelfth century. At least it was in the thirteenth that we can first detect any indication of an upswing.

A consequence, rather than a cause of this upswing, was the founding of the first universities; Paris, Oxford, Cambridge, Padua, and Naples. These outgrowths of the church and cathedral schools did little but perpetuate the philosophy of scholasticism. A student studied only the Ancient Authorities; grammar from Donatus, logic from Aristotle, rhetoric from Cicero. His mathematics, if any, consisted of arithmetic from Boethius, music according to Pythagoras, geometry according to Euclid, and astronomy according to Ptolemy. But since the student's goal was metaphysics, dialectics, and theology, little stress was placed upon mathematics. It was possible to obtain a degree with only a smattering of arithmetic and a few Euclidean propositions.

If education was only beginning to sprout, trade and commerce were bursting into full bloom. Medieval man was becoming aware of the products of the East; fabrics, spices, and perfumes. And the merchants were happy to accommodate him, at a price. The Mediterranean seaports; Venice, Genoa, and Pisa bristled with warehouses where goods could be stored and duty paid. The factor of such an establishment was a man of considerable importance.

To one of these, Guglielmo Bonaccio, was born a son. Leonardo was born at Pisa. However, the family must have soon moved to Bugia on the coast of North Africa, a prime source of wax candles (Fr. *bougies*). At any rate, it was here that Leonardo received his schooling from a Moorish school-master and was introduced to the Hindu numerals and the Hindu-Moslem methods of computation.

As a young man, he traveled both for business and pleasure in Europe and the Near East; Egypt, Syria, Greece, Sicily, and southern France. He met with scholars and merchants, observing and analyzing the arithmetical systems used in commerce. All the systems he considered poor compared with that which he had learned in school. The clumsy Roman numerals did not lend themselves to computation, even when assisted by the abacus.

He, therefore, in 1202, wrote a work *Liber Abaci* in which he gave a satisfactory treatment of arithmetic and elementary algebra. This famous book at last converted Europe to the Hindu arithmetic. It is divided into fifteen chapters, as follows:

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1. Reading and writing numbers in the Hindu-Arabic system.
2. Multiplication of integers.
3. Addition of integers.
4. Subtraction of integers.
5. Division of integers.
6. Multiplication of integers by fractions.
7. Further work with fractions.
8. Prices of goods.
9. Barter.
10. Partnership.
11. Alligation.
12. Solutions of problems.
13. Rule of false position.
14. Square and cube roots.
15. Geometry and algebra.

His numerals are not a great deal different from the ones we use today

1 2 3 4 5 6 7 8 9 0

and although there was opposition from the then equivalent chambers of commerce, they did away with the abacus and the counting board.

His computation methods for addition, subtraction, and multiplication are much the same as we use today. He uses "extract" where we use "subtract" and "take" where we use "extract" (for roots.). He uses the galley method for division, which in some ways is more economical than our own. One of his problems, $18,456 \div 17$ is shown below.

```

      6
    149
  18456
    17
   1085
  11/17 1085
  
```

He recognized the method of casting out nines to check his figures and also gives instructions to use the check of sevens and elevens.

Fibonacci's algebra followed that of al-Khowarizimi whose *al-jabr w'al muqubalah* gave name to the science. That is, he used no symbolism in the modern sense except on occasion to substitute a letter for a number. This may be the earliest trace of generality of algebra as opposed to verbally expressed rules for numerical computation.

Another algebraic innovation is his use of the term "minus" in connection with the Rule of False Position ("plus" was not used until the latter part of the fifteenth century).

To a modern day student, it seems improbable that he should ever be troubled by the solution of an equation like $ax - b = 0$, but think of solving this equation without symbolism. Hence the Rule of False Position.

Let g_1 and g_2 be two guesses as to the value of x and let f_1 and f_2 be the failures, that is, the values of $ag_1 + b$ and $ag_2 + b$ which would

be equal to zero if the guesses were right. Then

$$\begin{aligned} ag_1 + b &= f_1 \\ ag_2 + b &= f_2 \\ a(g_1 - g_2) &= f_1 - f_2 \\ ag_1g_2 + bg_2 &= f_1g_2 \\ ag_1g_2 + bg_1 &= f_2g_1 \\ b(g_2 - g_1) &= f_1g_2 - f_2g_1 \end{aligned}$$

dividing:

$$-\frac{b}{a} = \frac{f_1g_2 - f_2g_1}{f_1 - f_2} = x$$

Suppose, for example

$$5x - 10 = 0$$

Make two guesses as to the value of x , say $g_1 = 3$ and $g_2 = 1$

$$\begin{aligned} \text{Then;} \quad (5)(3) - 10 &= 5 = f_1 \\ (5)(1) - 10 &= -5 = f_2 \end{aligned}$$

$$x = \frac{f_1g_2 - f_2g_1}{f_1 - f_2} = \frac{(5)(1) - (-5)(3)}{5 - (-5)} = \frac{20}{10} = 2$$

Awkward as it seems, the use of two false values enables us to find the true result.

When Fibonacci wrote his *Liber Abaci* in 1202, he followed the Arabic custom of paying no attention to negative numbers but in his *Flos* (c. 1225) he interpreted a negative root in a financial problem to mean a loss instead of a gain. This breakthrough marks the beginning of two distinct but complementary philosophies of mathematics, synthetic and analytic; that is, a system is admitted into the body of mathematics only if it can be put into correspondence with a system already established as consistent, or all mathematics is a formalism without meaning beyond that implied by the postulates defining the formalism.

He was the first European mathematician who recognized the value of relating geometry and algebra. In his *Practica Geometriae* (1220) he uses algebra in solving geometric problems relating to the area of a triangle.

More important was his work with cubic equations. Magister Johannes, a scholar from Palermo, proposed to him the problem of finding a cube which, with two squares and ten roots, should be equal to 20. In modern symbolism, the problem is to solve the equation

$$x^3 + 2x^2 + 10x = 20$$

He proceeded as follows

$$10(x + 1/10x^3 + 1/5x^2) = 20$$

$$x + 1/10x^3 + 1/5x^2 = 2$$

so that

$$x < 2$$

But

$$1 + 2 + 10 = 13 < 20$$

and so

$$x > 1$$

But x is not fractional; for if $x = a/b$, then

$$\frac{a}{b} + \frac{a^2}{10b^3} + \frac{a^5}{5b^5}$$

cannot be integral, and so x must be irrational. Further, x cannot be the square root of an integer; for, from the given equation

$$x = \frac{20 - 2x^2}{10 + x^2}$$

and if x were equal to the $\sqrt{a}$ we would have

$$\sqrt{a} = \frac{20 - 2a}{10 + a}$$

which is impossible.

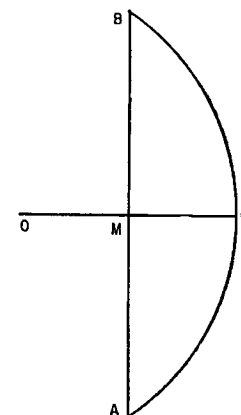
Being unable to give the algebraic solution, he attempted to prove that a geometrical construction of the problem using only a straight edge and compass alone was impossible. He could not succeed with the mathematical technique known to him. Indeed, such proofs did not arrive until the nineteenth century.

Failing in this analysis, Fibonacci simply states the approximate value

$$x = 1^{\circ} 22' 7'' 42''' 33'''' 4^v 40^v$$

which when translated from his sexagesimal fractions to common fractions is only 1/311,040,000,000 too large. We have no idea of how he arrived at the result.

If innovations in arithmetic, algebra, and geometry were not enough, Fibonacci was also active in trigonometry and theory of numbers which pretty well covered the field for his day. He was acquainted with the trigonometry of the Arabs and in his *Practica Geometriae*, applied the subject to surveying. His use of the word and definition of *sine* established it in Europe.



The term he used for *versed sine* is amusing. Early writers, given to fanciful resemblances, spoke of the bow, ACB and string, AB. It was natural for them to speak of the *versed sine* as the arrow. The Arab word *sahem* (arrow) passed over into Latin as *sagitta*, the term used by Fibonacci.

And in the theory of numbers he gave $(\frac{1}{2})(2^2)(2^2-1) = 6$, $(\frac{1}{2})(2^3)(2^3-1) = 28$, $(\frac{1}{2})(2^5)(2^5-1) = 496$, as perfect numbers and in general for $(\frac{1}{2})(2^p)(2^p-1)$ where 2^p-1 is prime, a value which holds for all known perfect numbers.

It is ironic, that of all the mathematical firsts introduced by Fibonacci, he is best remembered for a trivial series

$$u_{n+2} = u_{n+1} + u_n, \quad n = 0, 1, \dots, \quad u_0 = 0, u_1 = 1$$

which gives the sequence 0,1,1,2,3,5,8,13,, each number being the sum of the two preceding it.

Fibonacci discovered this series while working on a problem concerning the progeny of rabbits. It keeps recurring in the most unusual places; the Golden Section of ancient Greece, the theory of continued fractions, the spacing of leaves on a plant, the convolutions of sea shells, in Sam Loyd's haunted checkerboard problem, Curry's paradox, and whatnot.

Bear in mind that the man who introduced the Hindu number system to Europe, who first saw the significance of negative numbers, who anticipated analytical geometry was no scholar. He was a merchant, a commercial traveler. At that time any professor of the University of Paris (soon to become the intellectual center of the world) would have scorned the practicality of the *Liber Abaci* and could not have comprehended the reasoning of the *Liber Quadratorum* or the *Flos*. Yet Emperor Fredric II traveled to Pisa to watch this man perform in a mathematical contest.

In addition to his other names, he sometimes signed his works Leonardo Bigollo. Translated as "traveler" it may simply mean that he traveled a great deal. It also means "blockhead". It has been suggested that he was so-called by the professors of his day because he was not a product of their schools. He thereupon adopted the name to show the professors what a blockhead could do.

In any event, we cannot explain this out-of-time genius because, for genius, there is no explanation.

"Why are you taking that math paper to the gym?"

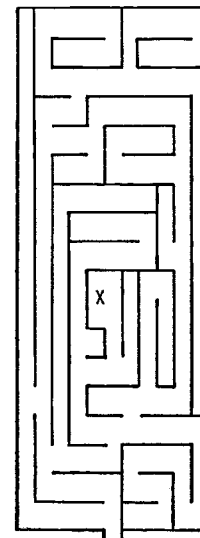
"I gotta reduce these fractions."

* * * * *

Word Calculus

$$\int \frac{d \text{ cabin}}{\text{cabin}} = \log \text{ cabin}$$

Letters to the Editor



Dear Mr. Madachy:

Mr. Golomb gives what *appears* to be an infallible method of escaping from a maze. He should have emphasized that the method is certainly dependent on the construction of the walls. The maze he shows (December 1961 RMM, page 14) is OK, but the maze shown to the left will not yield to the method described, viz., keep one hand, say the right, on a wall until one gets out of the maze. This method will not work if the maze consists of two or more sets of *separated* and *unconnected* walls.

Possibly the technique to employ in such a case is to *mark* the walls being used. Then, when one finds himself back in the middle, he starts over again until he finds *on his left* (if he has used his right hand for the marking) an unmarked wall. Then he should follow *that* wall. If he finds himself back where he started without having left the maze, he may be in a maze of three or more walls and the process just described may have to be repeated until he finds himself outside the maze - or admits he doesn't know when he *was* outside!

U. Clid
Cleveland, Ohio

Dear Mr. Madachy:

I can now add some information to the query I made previously (October 1961 RMM, page 57) concerning prime numbers of the form $(10^n - 1)/9$. I have since found that n must be prime. However, this does not guarantee that the *result* will be a prime.

Robert Brieger of Houston, Texas has informed me that for $n = 43$, the number formed is divisible by 173; for $n = 53$, the number is divisible by 107; for $n = 61$, the number is divisible by 733, 4637 and possibly other numbers; for $n = 79$, the number is divisible by 317, 6163, 10271 and possibly other numbers.

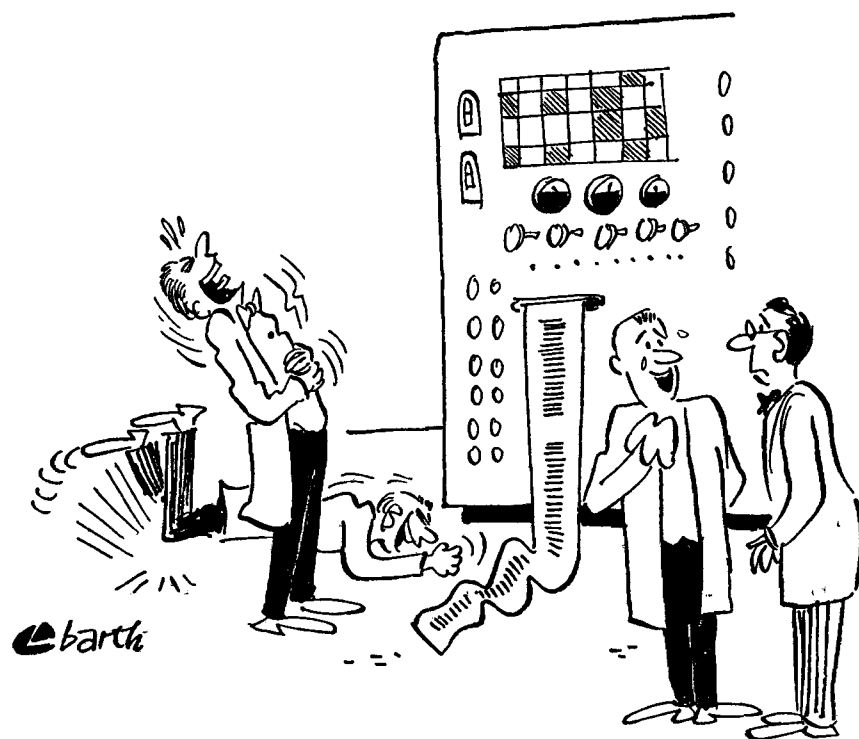
By "brute force", i.e. long division by odd divisors, I have found that for $n = 37$, the number is divisible by 2,028,119 and possibly other numbers.

This leaves but 8 unknowns (for $n = 47, 59, 67, 71, 79, 83, 89$, and 97), 14 composites (for $n = 3, 5, 7, 11, 13, 17, 29, 31, 37, 41, 43, 53, 61$ and 73) and only 3 primes (for $n = 2, 19$, and 23) out of the first 25 primes for n in the above expression: $(10^n - 1)/9$. The three primes found are

11 and 1,111,111,111,111,111,111,111 and 11,111,111,111,111,111,111,111

I hope some other RMM readers can settle the question for some or all of the 8 unknowns and furnish references to this particular problem.

Rudolph Ondrejka
NAFEC
Atlantic City, N. J.



"IT KNOWS THE BEST STORIES!"

Readers' Research Department

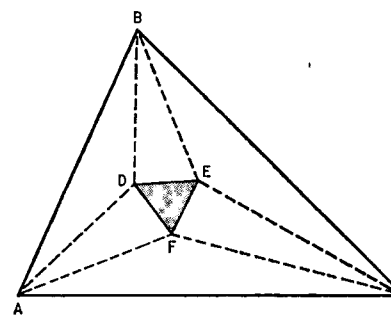
The Research for this issue will deal with prime numbers and then with a famous theorem.

(1) Alan Sutcliffe of England passes on the conjecture that between n^2 and $(n+1)^2$ there is always at least one prime number, where n is an integer greater than zero. Has this conjecture been proven previously?

(2) Arithmetic Progressions of primes have always fascinated recreational mathematicians. No one knows of any method of finding such progressions except by hunting through prime tables. For example, here is an arithmetic progression of 10 primes with a common difference of 210: 199, 409, 619, 829, 1039, 1249, 1459, 1669, 1879 and 2089. Malcolm H. Tallman found the following arithmetic progression of 11 primes with a difference of 210: 10859, 11069, 11279, 11489, 11699, 11909, 12119, 12329, 12539, 12749, and 12959. The longest known arithmetic progression of primes consists only of 12 terms with a difference of 30030: 23143, 53173, 83203, 113233, 143263, 173293, 203323, 233353, 263383, 293413, 323443, and 353473. W. A. Golubiev discovered this last series.

Alas, that is the longest series known and it contains only 12 terms. Are there longer series of primes in arithmetic progression?

(3) Morley's Theorem states that the points of intersection of the trisectors of the integral angles of any triangle form an equilateral triangle. This has been illustrated to the left. There are several proofs known, but as H. S. M. Coxeter states in his *Introduction to Geometry* (Wiley & Sons, 1961) "Much trouble is experienced if we try a direct approach, but the difficulties disappear if we work backwards, beginning with an equilateral triangle and building up a general triangle which is afterwards identified with the given triangle ABC."



Can some RMM reader(s) come up with a simple *direct* proof of Morley's Theorem?

Answers for the December 1961 Issue of *Recreational Mathematics Magazine*

The list of puzzle solvers will be found on Cover II - though it was the intention to have the list follow this Answer Section and appear on Cover III.

WORD GAMES (Page 25 - Dec 61 RMM)

"7" Letter Scramble

Solution A (Add an L)

GLACIER (1)
FRAGILE
ESCALOP
DECANAL (2)
CALIBER
BALDRIC
AURICLE

Solution B

GRIMACE (3)
GIRAFFE (4)
CAPOTES (5)
CADENZA (6)
CARBINE (7)
CARBIDE (8)
SAUCIER (9)

Alternative words to those by Mr. Baker were supplied by RMM readers: (1a) GRACILE, (2a) CANALED, (3a) CADGIER, (3b) CIGARET, (3c) GRECIAN, (4a) FEARING, (4b) FRIGATE, (5a) PEACODS, (5b) PEASCOD, (5c) POACHES, (5d) SCOPATE, (6a) ADVANCE, (7a) ASCRIBE, (7b) CABEIRI, (7c) CARBIDE, (7d) CARIBES, (8a) CATBIRD, (9a) ACQUIRE.

See Cover II for the list of "7" Letter Scramble solvers.

Change a Letter (From HEATING to CONFIRM)

Mr. Baker's answer is shown as Solution A. Other changes given by RMM readers are listed in separate columns.

Solution A

HEATING			
GNATHIC	1a Hearing	1b Tearing (or Ingrate or Granite)	
CHAFING	2a Fearing	2b Grecian	2c Orating
FARCING	3a Farming	3b Fancier	3c Organic 3d Foreign
FORCING	4a Forming	4b Conifer	4c Romanic
CONFIRM			

See Cover II for the list of Change a Letter solvers.

ALPHAMETICS (Page 27 - Dec 61 RMM)

- (1) BE ABLE SIR = 27 9207 341
- (2) HERE'S + MERRY + XMAS = READER
- (2a) $64143 + 74115 + 9783 = 148041$
- (2b) $X212E + 42113 + 549E = 129721$ (Base-12)
- (2c) $52123 + 82116 + 4893 = 129021$ (Base-11)
- (3) DO + SING + BING + SING = SONGS
- (3a) $83 + 1726 + 9726 + 1726 = 13261$
- (3b) $20 + 139(30) + (87)39(30) + 139(30) = 109(30)1$ (Base-89)

(-) And a multitude of solutions in many other bases by Harry L. Nelson and J. A. H. Hunter.

(4) TWO + THREE + SEVEN = TWELVE

(4a) $106 + 19722 + 82524 = 102352$

(4b) $108 + 1E977 + X7374 = 107237$ (Base-12)

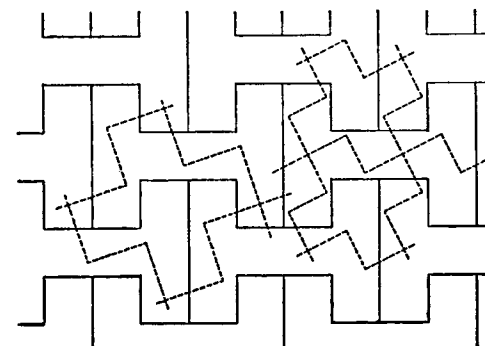
In the above, $X = 10$ and $E = 11$ in the Base-12 system.

See Cover II for the list of Alphametic solvers.

Some late solutions to the TIRED alphametic featured in the October 1961 RMM were supplied by *Howard Cohodas, Cleveland Heights, Ohio* (2); *John Fallows, Negri Sembilan, Malaya* (1); *Herbert Flesch, Osnabrueck, West Germany* (1); *Marvin J. Frey, Goshen, Indiana* (1); *Susan Orwig, Fostoria, Ohio* (1).

No more solutions will be accepted for the free year offer. A total of 74 free 1-year subscriptions (or extensions) and 2 Life Subscriptions to RMM have been entered for those who submitted solutions to the TIRED alphametic.

* * *



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H^2 PROBLEM (Page 38 - Dec 61 RMM)

Dissections of the H into one Greek cross and into two Greek crosses are illustrated. Just so much of the Greek-cross tessellations is drawn as will show how each figure is dissected.

Solved only by the proposer, *Harry Lindgren, Canberra, Australia*.

PUZZLES AND PROBLEMS (Pages 46 to 48 - Dec 61 RMM)

In response to many requests we are presenting the full solutions to all but the most trivial puzzles and problems given in the December 1961 RMM.

1. *Eight Stamps*: Prisoner B in the second round is the first to be able to state correctly the color of the stamps on his forehead. His reasoning is as follows:

B says to himself, when his turn comes up in round two; "I see two reds and two greens, therefore I can have either red-green or green-green. If I have green-green, A would have said to himself in round two:

"I see three greens and one red, so I can have red-red, red-green or green-green. However, if I had red-red, B would have known in round one that he (B) had green-green, as all the reds would have been visible to him. So I (A) can't have red-red. Supposing I (A) had green-green. Then C in the first round would have seen four (4) greens, and known that he (C) could only have had red-green or red-red. So C would have said to himself:

"If I (C) had red-red, B would *know* in round one that he (B) has green-green from A's round one answer that he (A) didn't know. But B didn't know he has green-green, so I (C) can't have red-red. Therefore I (C) must have red-green."

"That is, if I (A) had green-green, C would have known in round one that he had red-green. Therefore I (A) have red-green."

However, (this is B thinking) A did not know at the beginning of round two that he had red-green, therefore I (B) can't have green-green; I must have red-green."

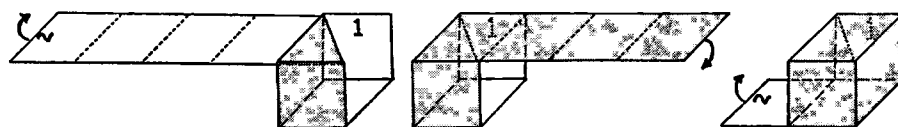
(And in payment for the above vast mental effort, our aged logician is set free.)

2. *Cube Cutting*: The table below gives the answers to the puzzle. Line 1 gives the number of cuts required to produce 1-inch cubes; line 2 - number of cubes obtained; line 3 - cubes black on one side; line 4 - cubes black on two sides; line 5 - cubes black on three sides; line 6 - cubes black on four sides; and line 7 - cubes all white. In the general formula, n is the unit-dimension of the large cube.

	2"	3"	4"	5"	6"	7"	8"	9"	10"	Formula General
1	3	6	9	12	15	18	21	24	27	$3(n-1)$
2	8	27	64	125	216	343	512	729	1000	n^3
3	0	6	24	54	96	150	216	294	384	$6(n-2)^2$
4	0	12	24	36	48	60	72	84	96	$12(n-2)$
5	8	8	8	8	8	8	8	8	8	8
6	0	0	0	0	0	0	0	0	0	0
7	0	1	8	27	64	125	216	343	512	$(n-2)^3$

3. *More Cube Cutting*: By referring to the above table we find that both a 2-inch or a 4-inch cube, when cut into 1-inch cubes, produce as many cubes with paint on one side as with paint on two sides. Likewise, both a 2-inch and an 8-inch cube, when cut into 1-inch cubes, produce as many cubes with no paint as with paint on one side.

4. *Cube Formation*: The Editor forgot to stipulate that the resulting black cube should be a one-inch cube. The solution is shown below which shows that no more than eight inches of paper are necessary to form an all-black cube.

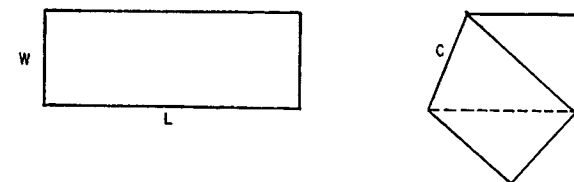


If we do not assume a 1-inch cube as a result, but are allowed to form smaller cubes, it is easy to see that a very small strip of paper only one inch wide will do! For example, a piece of paper 1 inch by $\frac{1}{8}$ inch will form a $\frac{1}{8}$ -inch cube by folding in a manner similar to that shown above. In fact a 1-inch piece of paper of *nearly zero* length will suffice!

5. *Problem in Analysis*: C approaches W as L becomes larger; C approaches W which approaches 0 (zero) as W becomes smaller. The formula given,

$$C = (W/L)(\sqrt{L^2 + W^2})$$

gives the length of a crease, C , when a rectangular piece of paper, W by L , is folded so that opposite corners meet.



As the paper gets longer (L approaches infinity) relative to the width, W , the length of the crease approaches W . If we decrease W , then C will approach W , which approaches zero.

6. *There's Always a Way*: Many readers tripped up on this puzzle when they assumed that a single weighing of, say, three coins would indicate whether they were or were not equal to the weight of three genuine coins. *No weights* were given in the problem, hence a single weighing could tell us nothing about the expected weights. The solution is given below.

Mark the coins A, B, C, D, E, F.

1st Weighing: $A + B + C + D = w_1$ say

2nd Weighing: $C + D + E = w_2$ say

3rd Weighing:

If $3w_1 = 4w_2$. The fake is F. Weigh F alone.

If $3w_1 \neq 4w_2$. Weigh $A + C = w_3$, leading to:

(1) $w_1 = 2w_3$: Fake is E, weight = $(w_1 - w_3)$

(2) $2(w_1 - w_2) = w_3$: Fake is D, weight = $(w_2 - w_3)$

(3) $w_1 + w_3 = 2w_2$: Fake is C weight = $(2w_3 - w_2)$

(4) $2w_2 = 3w_3$: Fake is B, weight = $(w_1 - w_2)$

(5) $3(w_1 - w_3) = 2w_2$: Fake is A, weight = $(w_1 - w_2)$

In each of the third steps, the weight of a genuine coin can be calculated easily when the weight and identity of the fake has been ascertained.

7. *A Playground Problem*: The diameters of the circles are 300, 450 and 550 feet. The solution involves a simple algebraic set-up once a diagram is drawn.

8. *Paper-Covering Problem*: The proposer, Jack H. Halliburton, was able to cover 99 15/16 square inches of an 8 1/2" by 12 1/2" piece of paper with seven 3" by 5" cards. Some readers stated they covered more, but few indicated *how* they did it. The best solution will be diagrammed in the April 1962 issue of RMM. A high figure of 102 square inches was turned in - but no set-up.

9. *What? No Dog Sled?* Mr. Smith travelled 4 hours on the bus at 35 miles per hour (mph), 4 hours on the train at 70 mph and 2 hours on the plane at 280 mph. This is a simple problem in algebra. Let T_p = time on the plane, T_t = time on the train, and T_b = time on the bus. The problem tells us that $T_t = 2T_p = T_b$ and so

$$T_p + T_t + T_b = T_p + 2T_p + 2T_p = 5T_p = 10. \quad \text{i.e. } T_p = 2 \text{ hours.}$$

from which the other answers can be derived.

See Cover II for *Puzzles and Problems* solvers.

* * *

COIN-GAME COFFEE-WINNERS (Pages 48 and 64 - Dec 61 RMM)

Stack 'em Up: Number the coins 1 through 10. The solutions are given below.

1. Put 7 on 10
2. Put 5 on 2
3. Put 3 on 8
4. Put 1 on 4
5. Put 9 on 6

or

1. Put 4 on 1
2. Put 6 on 9
3. Put 8 on 3
4. Put 10 on 7
5. Put 2 on 5

Considering a stack of two coins as one:

1. Put 7 on 10
2. Put 5 on 2
3. Put 3 on 8
4. Put 1 on 6
5. Put 9 on 4

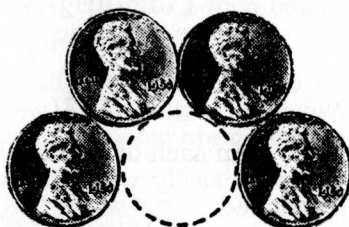
or

1. Put 4 on 1
2. Put 3 on 8
3. Put 1 on 6
4. Put 10 on 5
5. Put 2 on 7

Shifty: In position I, move the upper left coin to the lower right, touching two coins. Then move the middle coin in the bottom row to the position just vacated in the first move.



I



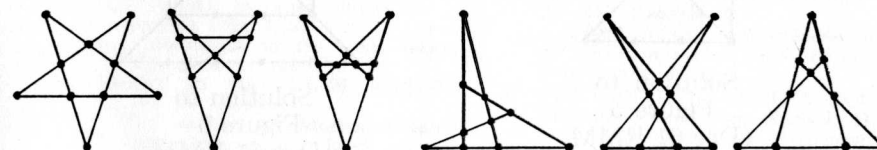
II

See Cover II for *Coin-Game Coffee-Winners* solvers.

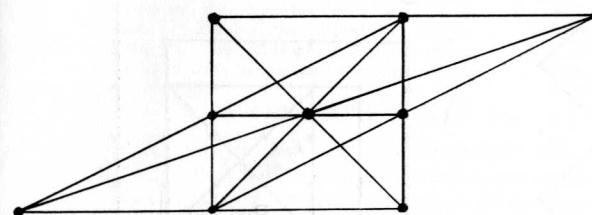
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DOTS AND LINES (Pages 51 to 55 - Dec 61 RMM)

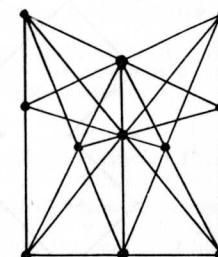
Solutions are below and on the next page. See H. E. Dudeney's *Canterbury Puzzles* (Dover Publications), puzzle 85, for an interesting tale behind the problem "12 dots in 7 rows of 4 each."



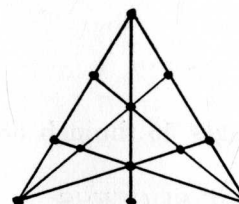
Six solutions to "10 guys in 5 rows of 4 men each"



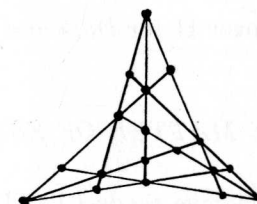
9 trees in 10 rows of 3 each



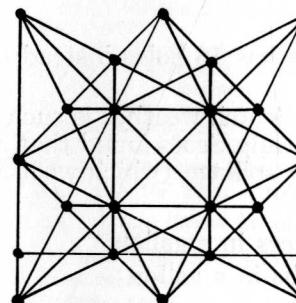
11 dots in 16 rows of 3 each



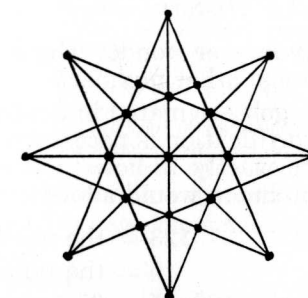
12 dots in 7 rows of 4 each



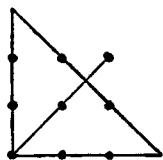
18 dots in 9 rows of 5 each



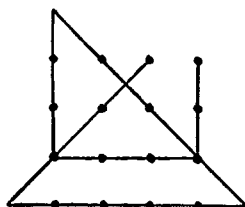
22 dots in 21 rows of 4 each



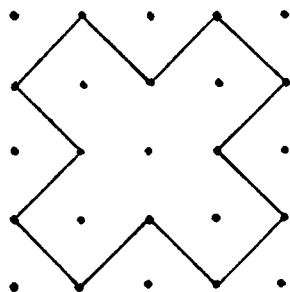
21 dots in 12 rows of 5 each



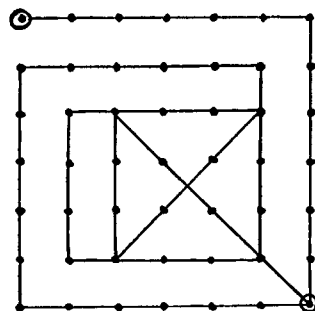
Solution to
Figure 5
(Dec 61 RMM
page 53)



Solution to
Figure 6
(Dec 61 RMM
page 53)



Solution to Figure 7
(Dec 61 RMM, page 53)



Solution to Figure 8
(Dec 61 RMM, page 53)

See Cover II for *Dots and Lines* solvers.

* * *

A *LITTLE MATTER OF ESPIONAGE* (Pages 56 through 58 - Dec 61 RMM)

The message reads **LITTLE CHANCE OF SUCCESS.**

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A *Wee Bit o' History*

Did you ever wonder why a golf course has 18 holes instead of 16 or 20 or some other number?

Well, golf originated in Scotland. The Scotch weather is such that internal fortification is necessary. And the canny Scots found that a jug would last exactly 18 holes, not allowing for spillage (which we'll allow as no Scotchman would allow!).

123 = the number of ounces in a gallon

5 = the number of fifths in a gallon

$128 \div 5 = 25.6$ ounces in a fifth

1.4 = the number of ounces in an average drink

$25.6 \div 1.4 = 18$ drinks in one fifth

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