RECREATIONAL MATHEMATICS magazine

ISSUE NO. 2 APRIL 1961

— In This Issue —

And Other Articles, Puzzles, Brainteasers, Word Games, Number Oddities, Etc.

65¢

RECREATIONAL MATHEMATICS magazine

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APRIL 1961 ISSUE NUMBER 2

PUBLISHED AND EDITED BY JOSEPH S. MADACHY

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RECREATIONAL MATHEMATICS MAGAZINE published monthly by Joseph S. Madachy at The Falls Printing Co., Idaho Falls, Idaho. Second-class postage paid at Idaho Falls, Idaho. Subscription rates: $3.00 per year for teachers, students and libraries; $3.50 per year for the general public. Reprints of any material must be requested within the month of publication - reprint costs are 5c per page. All correspondence concerning changes of address, subscriptions, reprints, and manuscripts should be sent to The Editor, Recreational Mathematics Magazine, Box 1876, Idaho Falls, Idaho.

From the Editor

This second issue of *Recreational Mathematics Magazine* follows a virtually unanimous acceptance of the first issue. The editor could hardly be happier.

Due to last-minute changes and several other factors a few errors found their unwanted way into the first number. Some errors which do not change any meaning at all will not be corrected (e.g. a missing parenthesis where it's obvious) while the others have been corrected in the various departments of the magazine in which they appeared. Mention should be made of some errors: the figure on page 22 of the February issue was, somehow, reversed (how many of you noticed it?); on page 23 there were several errors: Mq in line 3 should read MQ and the third MP in line 3 should read MQ. The third line of the third paragraph (page 23) should read " . . . such that OQ_aP is a right . . ." Needless to say all the Q^a should read Q_a. The editor takes full blame for all the errors which appeared and he hopes that this issue will be entirely free of blemish.

The Readers' Research Department proved so popular that we are including two problems this time - in addition to an analysis of the February problem. S. Baker's Word Games proved popular, also, and those enthusiasts of such entertainment will find another page to keep them busy.

Some praise and some criticism were extended to most pages of issue No. 1, but there is no need to go into all the details. An overall review might be in order at the end of the first year to see what was and what was not acceptable to most of the readers. So if all of you will be patient and tolerant of the items here and there which do not meet with your approval, you'll eventually find them changed or dropped (unless you happen to be the lone critic). Popular items will, of course, take over more space in the magazine.

Now a bit of forecasting. The June issue of RMM will introduce a cross-number puzzle; Maxey Brooke (of the Afghan Bands in the February issue) will describe some haunted checkerboards that forced their attention on him; Leo Moser, of the University of Alberta, will introduce a new game called HOT; F. Emerson Andrews, who introduces this issue with *Some Sorting Tricks*, will explain the method of Counting by Dozens; and there will be more puzzles and problems, Research problems and Word Games, more fun and entertainment in the mathematical manner.

1 April 1961 J.S.M.

*Some Sorting Tricks**

by F. Emerson Andrews

Sometimes it is necessary to arrange a great number of items, such as address cards, in a given order. The usual way to do this is to make as many piles as there are groups. That is, if the cards are to be arranged by states, 50 piles are needed, plus one for the District of Columbia. If alphabetized, then 26 piles, with doubt there will be anything in X or possibly Z. This takes a lot of room and is a tedious job.

Numbers can do a job of this kind by using only two piles and sorting several times over. And of course states, or letters of the alphabet, can be assigned consecutive numbers, and then number-sorting can be used. This is in fact the way big machines in offices do their complicated sorting jobs with many thousands of items at a time. It seems magical the way all cards fall into the right places, but it is really quite simple. By using what is called the binary system, we can devise our own sorting system, for that is precisely the way the big machines do their job.

The key to such sorting lies in the fact that every number that exists can be expressed as the sum of 1 (if it is an odd number) and one or more of the powers of 2, no particular power used more than once. You can see that this would be so by looking at the right-hand column in the table. There 15, for example, is made up of 1 and three different powers of 2; 16 is just one power of 2. All the numbers (up to 16 in the table, but this is true of *all* numbers) use any power of 2 no more than once.

TABLE OF NUMBERS EXPRESSED AS POWERS OF TWO

1	=					1
2	=				2	
3	=				2	1
4	=			4		
5	=			4		1
6	=			4	2	
7	=			4	2	1
8	=		8			
9	=		8			1
10	=		8		2	
11	=		8		2	1
12	=		8	4		
13	=		8	4		1
14	=		8	4	2	
15	=		8	4	2	1
16	=	16				

This article has been adapted from the book *Numbers Please* by Mr. Andrews to be published in July by Little, Brown & Company.

Therefore, if we sort first for 1, and then for the successive powers of 2, something extraordinary may happen. Let's see.

We begin with a small sample, arranging eight categories, 0 through 7, in quite random order, as in Step One, below. Then we take out all the odd numbers (marked with a period below them, in Step Two), and put them behind the remaining numbers (Step Three).

(1)	5 4 6 0 2 1 7 3	Random order
(2)	. . . .	Sort for 1
(3)	4 6 0 2 5 1 7 3	Put together again
(4)	. . . .	Sort for 2
(5)	4 0 5 1 6 2 7 3	Put together again
(6)	. . . .	Sort for 4
(7)	0 1 2 3 4 5 6 7	Put together

Now we sort for the first power of 2, taking out (Step Four) those numbers that have 2 in them in the right-hand column of our binary table. We put them behind the remaining numbers in Step Five.

Now we sort for 4 (Step 6), taking out all those numbers which include the second power of 2 (4). These are, of course, 4, 5, 6, and 7.

When these are placed behind the remaining numbers (Step 7), quite magically the numbers all fall into their proper order!

This is not an accident. It has to happen. If we had begun with two 3's, eight 5's, and four 6's, in the original cards, scattered at various places, after the three sortings all the 3's would be together, all the 5's, and all the 6's, and in their proper order.

And this sorting system is much more efficient for larger numbers. As we have seen, it takes three sortings for 8 categories (0 through 7); one additional sorting will take care of twice as many categories (0 through 15); one more sorting, 32 categories; and so on, doubling each time.

A deck of ordinary playing cards, which nearly everyone has handy, is fine for experiments in sorting. With practice the following can be done fast, and will startle most observers.

Use the whole deck, adding the jokers (regarded as 0), so that you have 54 cards. For the first few times, it may be necessary to make your own sorting table, or use the one on the next page.

Joker				0
Ace				1
2			2	
3			2	1
4		4		
5		4		1
6		4	2	
7		4	2	1
8	8			
9	8			1
10	8		2	
Jack	8		2	1
Queen	8	4		
King	8	4		1

Shuffle the deck thoroughly. Now sort for 1's, placing *face downward* in one pile all the cards you retain, and face down in the second pile all the odd-numbered cards (including aces, jacks, and kings). Pick them up with the second pile behind the first. Sort for 2's; as the above table indicates, these are 2 and 3, 6 and 7, 10 and jack. Put the pile sorted for 2's behind the first pile, and sort for 4's (4 through 7, queen and king). Put these in the back, and sort for 8's (8 and all higher cards). Place them behind the retained cards, and —

Presto! The whole deck of 54 cards is arranged in perfect order, beginning with the jokers, followed by four aces, four 2's, and so on!

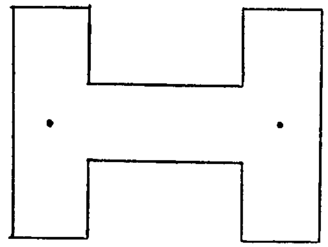
Such sorting tricks are fun to do, and they will amaze most people, who have no idea how or why they work. It all comes from knowing a little bit about numbers and how they are made up.

Geometric Dissections

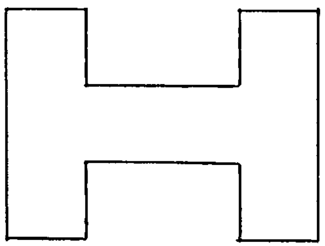
Here are four dissection puzzles that should keep most of you busy 'til the next issue - for which you will have to wait to see the answers.

A 4-H Problem

Can you cut this H into four identical pieces (same shape and area) and with all the cutting lines passing through or touching the two points shown? (Gerard Mosler)



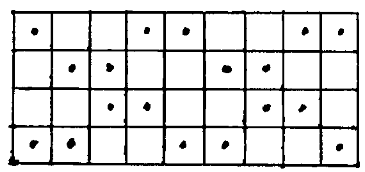
H²



Let's take that same H, with the two points removed and transform it into a perfect square with a minimum number of cuts. No pieces should be turned over (they can be rotated in the plane of the paper, of course) to form the square. (J.S.M.)

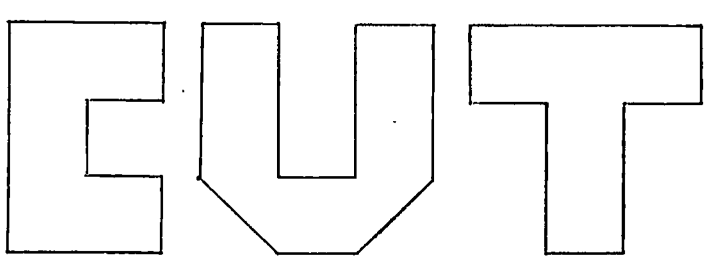
A Problem in Multiple Division

This dissection has a whole range of possibilities. Use the figure to the right and (1) divide (cut) it into two identical pieces; (2) that was simple, now divide it into three identical pieces; (3) into four; (4) into six; (5) into nine; (6) into twelve; and, finally, into eighteen pieces. All pieces for a given dissection must be *identical*, i.e. even the dots must be in the same positions. If some of the dissections are impossible, show which ones are and why. (Gerard Mosler and J.S.M.)



Really Cutting Up, Now!

While we're at it, let's see what we can do with the word CUT. See if you can cut CUT below into no more than five pieces with no more than two cuts and then rearrange the pieces into a perfect square - with no flipping over of any pieces. (J.S.M.)



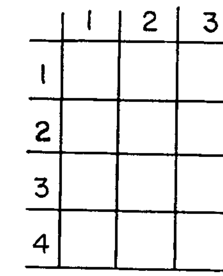
Word Games

by S. Baker

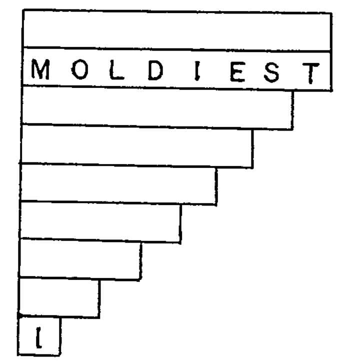
WORDS ALL WAYS

To solve this puzzle, words must be formed in all ways, i.e., 1 to 4 across, forward and backward. 1, 2 and 3 *down* words must be formed also *up* except "2", which only forms a word down. The following four clues should be sufficient to solve this little puzzle:

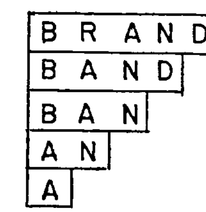
1. UP - Ejected.
2. DOWN - Ten-toed, four-footed animal.
3. ACROSS (Backward) - Farthing or less.
4. ACROSS - Usually all wet.



WORDS BETWEEN



To solve this little word game all you have to do is fill in the "words between". The rules are simple: if you build up from "I" you must add one letter from the top word and keep carrying letters and adding a new one each time, forming a new word each time. Conversely, if you break-down from "MOLDIEST" you must drop one letter at a time. Example:

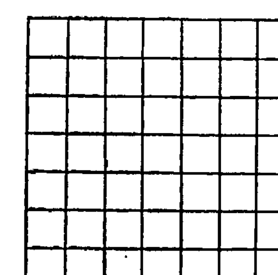


"7" LETTER SCRAMBLE

Fill in each row of the 7x7 diagram with a word formed by rescrambling each set of seven letters next to the diagram. If you work diligently you will see that you have an *earnest* answer.

To test your skill further, you must now reform all 7 letters to 7 completely new words. As a final step, form a 3rd seven letter word for lines 1, 3, 5, 6 and 7.

ILSGENT
EEARHTT
ELSSTRU
ARDPNEO
EEGIMSR
ARITESL
AEIRNGT



Any readers who find more 7 letter words can be consideres "LEE'S" (Lexicographic English Experts).

CHANGE A LETTER

1.	C L A I M E D
2.	
3.	
4.	
5.	
6.	
7.	R A T A B L E

A real teaser. You must go from CLAIMED to RATABLE in exactly six changes. Each change must consist of a new word which differs from the one above by only one letter change. If we started with START we would change the "R" to "E" to make STATE, then change a "T" to 'D" to make DATES, and so on, making only one letter change at a time.

Mr. Baker would like to have the answers to his word games sent directly to him at 265 Vitre Street, West; Montreal 1, Quebec, Canada. He will reply to all letters.

Here are the answers to Mr. Baker's Word Games from the February issue of RMM.

BUILD A WORD - It seems that 13 words using E, S and T was not a perfect score. Here's a list of 14 modern words: SET, SETS, SETTEE, SETTEES, TEES, TEST, TESTS, TSETSE, TESTES, STET, STETS, SESTET, SESTETS; and an additional 15 obsolete and archaic words: EST, ESTS, SEET, SETT, SETTS, SETEE, SETEES, STEE, STEES, TEEST, TEESTS, TEETEES, TES, TESTE and TETES.

WORDS BETWEEN - The other eight-letter word formed by rescrambling DEMEANOR is ENAMORED. To go from DEMEANOR to A: DEMEANOR, MEANDER, REMAND, NAMED, MANE (or NAME), MAN, AM, A.

"7" LETTERSCRAMBLE - The diagram shows the rescrambled words and you can read the clue, *obvious*, in one of the diagonals.

O	U	T	D	O	O	R
A	*B*	Y	S	M	A	L
E	N	*V*	I	R	O	N
C	R	U	*/*	S	E	D
A	B	S	C	*O*	N	D
H	A	L	I	B	*U*	T
I	M	P	E	T	U	*S*

CHANGE A LETTER - It was asked to go from CLEANED to ABOUNDS in six changes. Mr. Baker gives CLEANED, CADENCE, ENCASED, DEACONS, ABSCOND, BANDOG, ABOUNDS. However, W. W. Bell (Fremont, California) does it in the minimum four changes: CLEANED, CANDLES, UNLADES, UNLOADS, ABOUNDS.

NOTE: Mr. Baker relays the following communication to all word game enthusiasts:

"To all correspondents who submitted answers to 'Change a Letter', only the answer CLEANED, CANDLES, DEACONS, ABSCOND, ABOUNDS can be considered as *nearly* 100% correct — six changes were required. The following words which were used in many of the solutions are not considered acceptable since not all are listed in at least two recognized collegiate standard dictionaries: unlades, unbaled, uneased, uncased, unscald, unbased and other 'un' words."

The Art of Solving Multiplication Type Alphametics

by Sidney Kravitz

For the last 20 years solving multiplication-type alphametics has been a constant hobby with me. Through these years I have endeavored to solve these alphametics by logic alone and to reduce trial and error to a minimum. I believe that I have developed the rudiments of a systematic procedure which I would like to present here.

I have chosen to consider in detail the solution of the multiplication of a four digit multiplicand with a four digit multiplier to form a product and as many as four sub-products. The remarks made here however, can be easily modified to alphametics of other sizes.

Figure 1 shows a general 4x4 multiplication-type alphametic. The notation used serves to identify the position of every unknown symbol. For example, D identifies the final digit of the multiplicand and E_1, E_2, E_3, and E_4 identify each digit of the multiplier. Later in the text, the subscript X will be used to indicate the subscript 1, 2, 3, or 4 and Y will be used to indicate another numerical subscript not identical to X.

$$
\begin{array}{cccccccc}
 & & & & A & B & C & D \\
 & & & & E_1 & E_2 & E_3 & E_4 \\
\hline
 & & & F_4 & G_4 & H_4 & J_4 & K_4 \\
 & & F_3 & G_3 & H_3 & J_3 & K_3 & \\
 & F_2 & G_2 & H_2 & J_2 & K_2 & & \\
F_1 & G_1 & H_1 & J_1 & K_1 & & & \\
\hline
L_1 & L_2 & L_3 & L_4 & L_5 & L_6 & L_7 & L_8
\end{array}
$$

Figure 1

The solution of multiplication alphametics will be discussed under the following sections.

I — The Identity of Zero

II — Parity

III — The Inequalities

IV — The Identity of Nine

V — Special Relations of the Final Digits of the Sub-Products

VI — Special Relations of the Leading Digits of the Sub-Products

VII — Other Hints

I — The Identity of Zero

The identity of zero can usually be found or narrowed to two or three possibilities by writing down those symbols which cannot be zero. Referring to Figure 1 we see that the symbols occupying positions A and E_1 cannot be zero. Further, if four subproducts are listed then the other digits of the multiplier (occupying positions E_2, E_3, and E_4) cannot be zero. For five digit subproducts the F's cannot be zero, and for four digit subproducts the G's cannot be zero. In addition, if $J_4 \neq L_7$ then $K_3 \neq 0$ and $D \neq 0$. If $K_3 \neq L_7$ then $J_4 \neq 0$. For eight digit products $L_1 \neq 0$ and for seven digit products $L_2 \neq 0$. Finally it may be stated that if $L_1 = F_1$, and if there is a symbol in position F_2, then $L_2 \neq 0$.

As an example, let us refer to Figure 2*.

```
      P M W S
      T U Q V
    ---------
    W T S N R
  M R W U W
Q Q Q Q P
S V U S M
---------------
T W T V N U V R
```

Figure 2

Following the above rules in the order mentioned we see that the leading digit of the multiplicand $P \neq 0$; that each digit of the multiplier, T, U, Q, and $V \neq 0$; and that the leading digit of each subproduct S, Q, M, and $W \neq 0$.

Further since $N \neq V$ then W and $S \neq 0$; and since $W \neq V$ then $N \neq 0$. Finally $T \neq 0$. Nine of the ten unknown symbols have been shown to be different from zero so that the tenth symbol, namely R, must be equal to zero.

```
        V M P T
        U S R W
      ---------
      M M N M S
    W Q R W R
  W T V R W
R V Q S M
-----------------
S M R T U P V S
```

Figure 3

Admittedly, Figure 2 was deliberately concocted to illustrate the rules stated in this section.

For example, in Figure 3, following these rules, we can conclude merely that either N or P or Q is equal to zero. We will see in the next section, however, that this information will lead to another interesting conclusion.

**Solutions to all alphametics presented in the text are given at the end.*

II — Parity

The rules presented under this topic endeavor to determine which of the symbols are even digits and which are odd.

One of the most important symbols in an alphametic is the one occupying position D (Figure 1). If this symbol is an odd number, the following method will usually ascertain it.

Assume that D is an even number. Then K_1, K_2, K_3, and K_4 are even numbers. Note that with K_3 an even number, then J_4 and L_7 are either both odd or both even. Also if $L_1 \neq F_1$, then $L_1 = F_1 + 1$ and one of L_1 and F_1 is even and the other odd. In addition write down the possibilities for the identity of zero found in the previous section. Since there are only five even digits, a contradiction will usually result if D is an odd digit.

For example, in Figure 3, if we assume that T is an even number then the final digits M, W, R, and S are also even numbers. Since M and R are even numbers then V is an even number. In addition, we found in the previous section that either N, P, or Q is equal to zero (also an even digit). Thus T, M, W, R, S, V, and at least one of N, P, or Q are even. There can be no more than five even digits; therefore T is an odd number. We note further that $S \neq R$ so that $S = R + 1$ and both S and R cannot be even.

On the other hand if we know positively that D (Figure 1) is an even digit we will be able to identify the parity of many of the symbols including K_1, K_2, K_3 and K_4.

There is one rule for determining if D is an even digit. It depends on a set of conditions which occurs with some frequency. If all the multipliers, E_1, E_2, E_3, and E_4 are different from each other, and if two and only two of the final digits K_1, K_2, K_3, and K_4 are equal and the other two different from each other then the following conclusions may be stated. 1) D, K_1, K_2, K_3, and K_4 are even digits 2) If K_x and K_y denote the two K's which are equal then $E_x = E_y \pm 5$ 3) Hence E_x and E_y are of opposite parity.

For example, in Figure 4, $P \neq Q \neq N \neq M$, but two of the final digits, T, are equal, and two are different, $Q \neq W$. Then U, W, Q, and T are even digits. Since U and Q are even then N is even. Thus U, T, Q, W, and N are even so that M, P, R, and V must be odd digits.

```
      M Q Q U
      P Q N M
    ---------
    V V T N T
  V P P U Q
M V W P T
V R N V W
---------------
U U T Q U P U T
```

Figure 4

III — Inequalities

Although the application of the rules under this section does not usually enable us to identify any symbols, it nevertheless yields valuable information when combined with other clues.

The main method here rests on the fact that when one of the multiplying digits E_x multiplies ABCD, Fig. 1, to form a five digit subproduct $F_xG_xH_xJ_xK_x$ then $A \geqq F_x$ and $E_x > F_x$, but if a four digit subproduct is formed $(G_xH_xJ_xK_x)$ then $G_x \geqq E_x$ and $G_x \geqq A$. Other rules are as follows 1) If $L_1 = F_1$, then $L_2 \geqq F_2$ and $L_2 \geqq G_1$, but if $L_1 \neq F_1$, then $L_2 \leqq F_2$ and $L_2 \leqq G_1$. 2) If $F_x > F_y$ then $E_x > E_y$ and vice versa. 3) Multipliers which form five digit subproducts are greater than multipliers which form four digit subproducts 4) If $X > Y$ and $Y > Z$ then $X > Z$ 5) If one of J_4 or K_3 is greater or less than L_7 so is the other. 6) If $Y = Z + 1$ and $X > Z$ then $X > Y$. 7) If $A = F_x$ but $A \neq F_y$ then $E_x > E_y > F_y$.

For example, in Figure 5, only five digit subproducts are formed. Hence W>S, P>W, V>N, N>M. Likewise U>S, W, N, and M. Since M≠S then M=S+1.

Using Rules 4) and 6) above we conclude that V>N>M>S and that P>W>M>S. From Rule 2) since W>M>S as leading digits then P>N>W as multipliers, i.e. P>N>W>M>S. Finally since N>W as leading digits, it follows that V>P as multipliers. Our final conclusion is that V>P>N>W>M>S. (This information combined with the fact that M(=R+R) is an even digit,** with V, R, and N also even, soon leads to a solution).

```
         U R Q M
         W P V N
     -----------
       M M P R V
     N U U W R
   W Q V N N
 S P T P R
 ---------------
 M S U W V U M V
```

Figure 5

IV — The Identity of Nine

Whenever a five digit subproduct is formed the leading digit, F_1, F_2, F_3, or F_4 cannot equal nine. This follows from the fact that the largest five digit subproduct which can be formed is 9(9999)=89991. Further, if D and E_x are not equal to each other, and not equal to zero or one, then the corresponding K_x is not equal to nine. This follows from the fact that only 3x3 or 7x7 can form a nine. Other rules to eliminate nine are 1) A symbol known to be even cannot equal nine 2) If X>Y then Y≠9. 3) If $E_x + K_x \neq 10$ then $D \neq 9$ (This would occur if E_x and K_x are of opposite parity) 4) If $D + K_x \neq 10$ then $E_x \neq 9$.

***See also section on parity.*

In Figure 6, for example, we see that the leading digits of the subproducts, T, N, W, and R are not equal to nine. The final digits of the subproducts U, P, S and V are not nine. From the rules on the identity of zero we deduce that M=0. Therefore Q=9 by elimination.

```
         Q W V T
         S V P U
     -----------
       R R Q U V
     W T W M S
   N S U R P
 T P U N U
 -------------
 T U Q N M P P V
```

Figure 6

V — Special Relations on the Final Digits of the Subproducts

The following rules pertain to the final digits of the subproducts. (It is assumed that neither D or E_x equals zero or one).

1) If $K_x = 0$ then one of D and E_x equals five and the other is an even digit.
2) If $K_x = 1$ then one of D and E_x equals 3 and the other equals 7.
3) If $K_x = 3$ then one of D and E_x equals 7; the other is 9.
4) If $K_x = 7$ then one of D and E_x equals 9; the other is 3.
5) If $K_x = E_x$ then one of the following two possibilities hold a) $E_x = 5$ and D=3, 7, or 9. b) D=6 and $E_x = 2$, 4 or 8.
6) If $D = K_x$ then either a) D=5, $E_x = 3$, 7, or 9 or b) $E_x = 6$, D=2, 4, or 8.
7) If $E_x = K_y$ and $E_y = K_x$ and $E_x \neq E_y$ then either a) D=9 or b) D=4 and one of E_x, E_y is 2 and the other 8.
8) If $E_x = D = K_x$ then D=5 or 6.
9) If $K_x = K_y$ but $E_x \neq E_y$, see the next to the last paragraph in Section II, (Parity).

VI — Special Relations on the Leading Digits of the Subproducts

In those cases where a five digit subproduct is formed a) If $F_x = 5$, 6, 7, or 8 then $A + E_x - F_x = 9$ or 10 b) If $F_x = 3$ or 4 then $A + E_x - F_x = 8$, 9, or 10 c) If $A = E_x$ then $E_x - F_x = 1$, 2, or 3.

VII — Other Hints

1) If D=9 then $E_1 + K_1 = 10$, $E_2 + K_2 = 10$, $E_3 + K_3 = 10$, and $E_4 + K_4 = 10$.

2) If $E_x = 9$ then $D + K_x = 10$ and $F_x + G_x + H_x + J_x + K_x$ is evenly divisible by 9. If F_x, G_x, H_x, J_x and K_x are all different from each other then the sum of the other five digits is also divisible by 9. (The sum of the ten digits is equal to 45).

3) If $L_1 \neq F_1$, then $L_1 = F_1 + 1$.

4) If $E_x = E_y + 1$ then we may write the following addition problem to assist us.

$$\begin{array}{r} F_yG_yH_yJ_yK_y \\ +\quad A\,B\,C\,D \\ \hline F_xG_xH_xJ_xK_x \end{array}$$

Solving multiplication-type alphametics is really an art. Skill comes only with lots of practice. Many alphametics present their own special combinations which may be exploited but which are not sufficiently general to be worth listing. The experienced will readily spot these combinations however.

The reader will find the solutions to the alphametics given in the text by turning the page upside down.

Figure	M	N	P	Q	R	S	T	U	V	W
2	6	1	8	7	0	2	3	9	5	4
3	3	2	0	1	5	6	9	7	8	4
4	3	4	5	8	9	7	6	2	1	0
5	2	4	7	9	6	1	0	5	8	3
6	0	1	4	9	5	8	7	6	2	3

Sidney Kravitz is a mechanical engineer at the Picatinny Arsenal in Dover, New Jersey who has had considerable experience in ballistics research and stress analysis. He is married, has two children and lives in Dover, New Jersey.

It seems appropriate to follow Mr. Kravitz' article with the regular Alphametics department. Following the general procedures outlined by Mr. Kravitz in his article should help you solve the following Alphametics.

Alphametics

In May 1931 the use of the word *Cryptarithmie* (Cryptarithmetic) appeared in the magazine *Sphinx*. A *cryptarithm* is a mathematics problem involving addition, subtraction, multiplication, or division in which the digits have been replaced by letters of the alphabet or some other symbols. The term *Alphametics*, originated by J. A. H. Hunter, is used to refer specifically to those *cryptarithms* in which the combinations of letters make sense.

Here we offer some *alphametics* for your fun. At the end we give the answers to the *alphametics* given in the February issue on page 35.

```
A L L ) F O O L S ( D A Y
        G x x
        -----
        x A x x
        x x M x
        -------
          x x E x
          x x x S
          -------
          - - - -
```

Young and old, they all enjoy these!

Each letter here stands for a different figure, and the little crosses indicate figures about which we're told nothing.

What do you make of the FOOLS? (J. A. H. Hunter)

```
      R M M
      S L Y
    -------
    x x x x
  x x R x
x x M
---------
G A M E S
```

In keeping with the reputation of the magazine, as implied by the wording, this is quite a tricky alphametic.

The same rules apply as explained above.

What do you make of the GAMES? (J. A. H. Hunter)

A little game of poker was played the other night and the originator of the multiplication alphametic below tells us that the card filling the winning hand is represented by the letter "A". (Mrs. Rae C. Nelson).

(A) (SPADE) = FLUSH

The answers to this month's *Alphametics* will be found in the June issue.

Here are the answers to the February *Alphametics*.

```
R M M        155
O U R        241
R M M   =    155
O U R        241
R M M        155
-----        ---
F U N        947.
```

```
T H E ) F I R S T ( R M M
        x x x
        -----
        x x x x
        T O U R
        -------
          x x x x
          x x x x
          -------
              x M
```

Answer on the next page.

HAM		932		763		
EGGS	=	1447	or	2884	etc.	
MASH		2379		3647		

DOG		123		125		134	
CAT	=	689	or	687	or	579	etc.
ADO		812		812		713	

BEAVER		251453
TIGER	=	60753
RABBIT		312206

The second *Alphametic* in the February issue was, unfortunately, misprinted and should have read as you see it on page 15 of this issue. For those of you who would still like to tackle it - in its proper form - we have printed the answer below, upside down and in a not visually obvious manner. In spite of the error the problem was solved correctly by F. L. Miksa (Illinois) and by D. Murdoch (Ontario).

$R=M-H$; $S=4$; $T=5-3-1$; $U=23-19-3-1$

$E=19-11$; $F=7-2$; $H=3+4$; $I=3$; $M=3^2$; $O=1+5$

How We Obtained the Word *Algebra*

Muhammad ibn Musa al-Khwarizmi, an associate of Harun Er-Rashid, the Caliph of the Arabian Nights, wrote a mathematics book in which the Hindu numerals were used. About 830 A.D. he also wrote a treatise *Ilm-al-jabr wa'l muqabalah* meaning, probably, transposition and removal of terms in an equation. It was later quoted in shortened form as *Al-jabr* which eventually ended up as Algebra.

(H. V. Gosling)

Minor Puzzles - A Selection

The answers to the following puzzles will be found, upside down, on page 37. Try solving before peeking.

1. Declaration Data

Everyone knows 13 Colonies were represented in the Second Continental Congress when the Declaration of Independence was adopted in July 1776. As the number of signers of that Document of Liberty was 56, it is evident the Colonies were not represented equally. Though representation was not based on population, the smallest Colony had the smallest representation. That Colony was Rhode Island.

How many representatives did each Colony have if 4 Colonies each had 1 more than the least number of Signers, 4 Colonies each had 2 more than the least number of Signers, 2 Colonies had 3 more than the least number of Signers, 1 Colony had 5 more than Rhode Island, and 1 Colony had 2 more than that? (Bertha Newhoff).

2. Outguessing the Piggy-Bank

Whoever of the 6 children would come closest to guessing the number of coins contained in a piggy-bank was promised the whole contents by their father. The 6 guesses were 50, 47, 40, 53, 37 and 30. Of these, one child was 9 out, another 4, and the others 1, 6, 11 and 12. Can you tell from this information how many coins there must have been in the bank? (G. Mosler).

3. A Honey of a Puzzle!

Nectar as gathered by bees from flowers consists of various solids, mostly different sugars, in water. The nectar usually averages about 70% water and the bees evaporate the water until the resulting honey contains only about 17% water. How much nectar is used to produce one pound of honey? (H. J. Down).

4. Miss-Matched Twins

A metropolitan newspaper ran a twin-matching contest as a circulation booster. Over a period of weeks they printed pictures of 100 people and offered sizeable prizes for the contestants who could most accurately pair the pictures into sets of twins.

When the contest was over, it was found that 98.1% had four or more sets wrongly matched. The number of entries with 3 sets wrong was 18 more than the number that had 1 set wrong. The number with 2 sets wrong was the same as the number that submitted perfect answers.

Now if we tell you that the circulation of the paper was less than 4000 you should be able to figure out the number of entries in the contest. (Mel Stover).

5. Bees A-Buzzin'

The square root of half the number of bees in a swarm has flown out on a jasmine bush; 8/9 of the whole swarm has remained behind; one female bee flies about a male that is buzzing within a lotus flower into which he was lured in the night by its sweet odor, but is now imprisoned in it. How many bees were in the swarm? (H. J. Down).

6. The Impractical Mathematical Entomologist

An insect collector put an equal number of beetles and spiders in a small bottle and then proceeded to count the total number of legs in the bottle. There were 140 legs altogether so how many insects did the collector have? (Hint: beetles have 6 legs and spiders have 8 legs.) (H. V. Gosling).

7. The Enigmatic Milestones

A car moves along a highway, its speed remaining constant throughout, when it reaches a milestone showing a number of two figures. After exactly one hour, the car reaches another milestone showing the former figures in reverse position. After still another hour, the car arrives at a milestone whose three figure number consists of the figures on the first milestone with a zero between them. Can you tell the numerical inscription of all three milestones and the speed at which the car was moving? (G. Mosler).

Answers to Puzzles in February Issue

We shall give the answers to all the problems posed in the last issue of RMM except for the *Word Games*, *Alphametics* and *Readers' Research Department*. The answers to the problems in these departments will be found on their respective pages in this issue.

SOME INFERENTIAL PROBLEMS - by J. A. H. Hunter (pages 5 and 6 in the February issue).

1. Joe's son's name was Jim and he was 21 years old.

Let Jack = a, Jim = b, Not Jack = c, 18 years old = d, 21 years old = e, and 25 years old = f. Then Sam said "ae", Gwen said "bd", and Ann said "cf". Since each made one true and one false statement:

$$ae = bd = cf = 0 \quad \text{and } a+e=1, \quad b+d=1, \quad c+f=1$$

Since Joe's son cannot have two names nor two ages ab, ac, de, ef and df have zero value if they occur in subsequent calculations. Combine

$$(a+e)(b+d)=1 \text{ to yield } ab+ad+be+ed=1$$

Dropping out the zero terms we have: $ad+be=1$

Now form the equation

$$(c+f)(ad+be)=1 \text{ to yield } acd+bce+adf+bef=1$$

By dropping the zero terms we are left with only $bce=1$. Since none of the terms can be zero (i.e. false) they must have a value of 1 (i.e. true). Jim (b) is 21 years old (e). Not Jack (c) has the value 1 since the name Jim is Not Jack.

2. The order of finishing was Rialto, Queenie, Prefect and Satan last. Zebra was not in the race.

Let Prefect = P, Queenie = Q, Rialto = R, Satan = S, Zebra = Z, Not Prefect = P_1. Let 1st place = a, 2nd = b, 3rd = c, 4th = d. Then the statements made by Stan were

Pa, Qb, Rc and Sd

those made by Steve were P_1a, Qb, Zc and Rd

and those made by Bob were Qa, Zb, Pc and Sd.

Since each made two mistakes, they must also have made two correct statements. We can set up equations using Stan's statements and Steve's statements as follows:

$$\text{Stan:} \quad Pa\cdot Qb+Pa\cdot Rc+Pa\cdot Sd+Qb\cdot Rc+Qb\cdot Sd+Rc\cdot Sd = 1$$
$$\text{Steve:} \quad P_1a\cdot Qb+P_1a\,Zc+P_1a\cdot Rd+Qb\cdot Zc+Qb\cdot Rd+Zc\,Rd = 1$$

Multiply these together, omitting zero terms and impossible terms such as $P_1a\cdot Pa\cdot Qb$, $Qb\cdot Rc\cdot Zc$, etc. Also note that the product of any three terms in either's statement must equal zero. We get:

$$P_1a\cdot Qb\cdot Rc + P_1a\cdot Qb\,Sd + P_1a\cdot Qb\cdot Rc\cdot Sd + P_1a\cdot Qb\cdot Zc\cdot Sd + Pa\cdot Qb\cdot Zc + Pa\cdot Qb\cdot Zc\cdot Sd + Qb\cdot Zc\cdot Sd + Pa\cdot Qb\cdot Rd + Pa\cdot Qb\cdot Zc\cdot Rd = 1$$

Now set up an equation using Bob's statements:

$$Qa\cdot Zb+Qa\cdot Pc+Qa\cdot Sd+Zb\cdot Pc+Zb\cdot Sd+Pc\cdot Sd = 1$$

If we multiply this equation by the one preceding it and omit the zero and impossible terms we are left with

$$P_1a\cdot Qb\cdot Pc\cdot Sd = 1$$

Since Rialto was given as one of the four and $Qb=1$, $Pc=1$ and $Sd=1$, then $Ra=1$.

MINOR PUZZLES - A SELECTION (pages 7 and 8 in the February issue).

1. There are only three men involved: a grandfather, a father and a son, all related. Grandfather gave his son $90.00. He, in turn,

gave his son $80.00. When father and son counted their money they had, total, only the $90.00 grandpa had handed out.

2. The first player can guarantee winning if, on his first move, he removes two coins from the row containing three. Thereafter he plays so as to leave either (1) three rows with 1, 2 and 3 coins in each row, or (2) the same number of coins in each row.

3. Bedtime is 9:54 p.m. The watch requires 20 turns daily. The 12 turns for the daytime running of the watch is 3/5 of 20 and, also, 3/5 of 24 hours. 3/5 of 24 hours is 14 hours and 24 minutes which is added onto 7:30 a.m. to give 9:54 p.m.

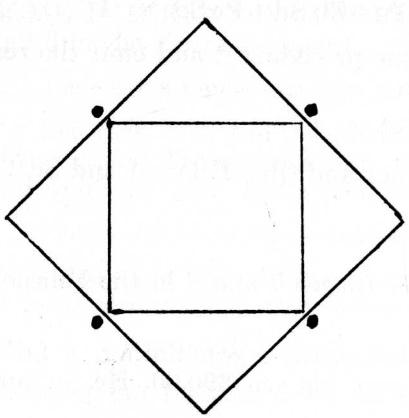
4. A red hat. Let the three persons A, B and C be in chairs 1, 2 and 3 respectively. C sees A and B. Now if he saw two black hats he would know his hat was red since there were only two black hats available. Since he says he doesn't know then he must see either two red hats or a red and a black hat. B now knows that C sees either two red hats or a red and a black hat. If B saw A with a black hat then he would know his hat was red. Since he says he doesn't know then he must see a red hat on A. A, who has been following the same lines of reasoning, can state that his hat is red.

5. Joshua and Joseph were both 110 years old, Moses 120, Sarah 127, Jacob 147, Abraham 175 and Isaac 180.

6. The figure below, to the left, shows the old walls and the new walls.

7. Two drink all, one drinks nothing.

8. The figure below, to the right, is the solution to the Magic Square.



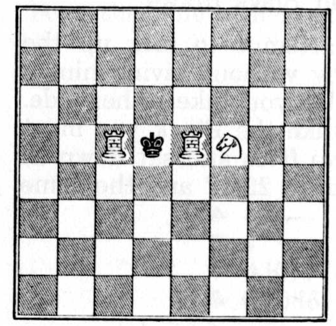
50	1	20
4	10	25
5	100	2

WAGER PROBLEMS - OLD AND NEW - by Mel Stover

(Pages 14-16 in the February issue)

1. The solution to this five-mover:

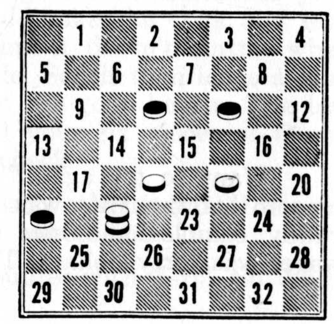
	White	Black
1.	P-QN4 (threatening R-Q5 or KB5)	R-B4ch.
2.	PxR (threatening R-N1 mate)	P-R7
3.	P-B6 (threat R-Q5 or KB5)	B-B2
4.	PxP	any
5.	PxN=Q mate	



2. Loyd announced in one of his columns that he had discovered this unusual mate. It bothered a number of his readers until he followed with the chimerical solution shown.

3. To follow the different possibilities in this clever production, let's visit the "Mythical Chess and Checker Club". Joe Kalyika was absorbed in a game of checkers with his friend Sam Palooka. Except for a lone kibitzer the club was empty.

Sherwin Betts, a stranger to the checker players, was well known to the pool hall gentry as a shrewd operator who made a good living out of gambling. He had already classified the two as mediocre palyers and he had noticed that Joe Kalyika had a large measure of that ostentatious smugness known to grifters the world over as the mark of the mark.



Sam had moved 7-10 and Joe claimed the game. (See diagram) "I go here and you go there and I go here and you go either there or there and I go here and win.

(translation) 22-17 21-25
17-13 25-29 or 30
18-14 and white wins

Betts interrupted this demonstration, "I know it's none of my business but Black should draw that game every day in the week."

"Are you out of your cotton-picking mind? That is a clear cut win for White," shouted Joe.

"I can see that a beginner might think so," said Betts in a tone calculated to irritate.

"Beginner, am I. Put up some money and take the Black and I'll demonstrate it move by move."

Betts placed a five dollar bill on the table and occupied Palooka's empty chair. This time the game proceeded:

22-17	21-25
17-13	10-14
18-9	25-30
9-6	30-26
6-2	26-23 and Black draws

Kalyika was silent and Betts made no attempt to pick up the money. "I don't like to take a person's money without giving him a chance to win it back. So for ten dollars I'll let you take either side. If you take the White you must win, If you take the Black you must draw." Kalyika placed a tenspot beside the two fives and sat down at the seat that Betts had just vacated. Betts moved 22-17 and the game went:

	21-25
17-21	25-30
18-14	10-17
21-14	30-26
14-18	and White wins

Betts' smile infuriated Joe. Kalyika replaced the checkers and sat down at the white side of the board. "I guess it is my turn at the White," said Kalyika as he placed twenty dollars on the table. Betts, still smiling, again took the black men.

22-17	21-25
17-21	10-14
18-9	25-30
9-6	30-26
6-2	26-23 and Black draws

"Want to try again?" asked Betts. Kalyika said nothing but a clue to his thoughts was visible in the puce tone which tinged his ruddy complexion. Once again they changed places. By now the wager was forty dollars. The moves:

22-17	21-25
17-21	10-14
18-9	25-30
21-25	30-21
9-6	21-17
6-2	17-14
2-7	and White wins

"Perhaps if you took the Black and we bet eighty dollars," said Betts as he turned around but Kalyika had already gone.

Stuffing the bills into his wallet Sherwin Betts sadly shook his head and slowly walked away.

4. The ones that castled lose immediately. Since the white corner square is at the left the board must be turned 90°. If you turn the board to the right the Black King would be in check so the board must be turned to the left. Now the move becomes P-K8=N mate. The solvers who were on their toes would of course run into an argument about the correct move being White castles. Some days you just can't win.

5. Note: Figure 9 in the February issue had a six of hearts which should not have been included, and a ten of hearts which should have been the nine of hearts. The complete, and correct, deal was:

North:
♠ AK
♥ KQJ10986
♦ — —
♣ AK42

West:
♠ — —
♥ 4
♦ AKJ97532
♣ Q1086

East:
♠ J97532
♥ A7532
♦ — —
♣ J9

South:
♠ Q10864
♥ — —
♦ Q10864
♣ 753

If you were naive enough to take a large bet on this hand, your partner would have led a club. North would take the trick and lead hearts. When you covered with the Ace, and it doesn't matter when it is done, South trumps with his low spade. South then leads a club and discards a club and three diamonds on the good hearts. The last six tricks were made by cross ruffing and your six pieces of trump were rendered as useless as a deadbeat's check.

6. Since white Queens never attack each other it would depend on the size of the board and how high you pile them!

7. If White moves R-N7 Black will counter with K-B3. So White must make a drastic move:

1. R-Q8ch. KxR
2. P-N7

Now White looks like a sure thing. But after

	R-QN5
3. KxR	P-B4ch

White looks dead because if he takes the pawn Black will move his King to B2 and catch the Pawn. So White moves:

4.	K-N5	K-B2
5.	K-R6	K-N1 (forced)
6.	K-N6	P-B5
7.	P-R4	P-B6
8.	P-R5	P-B7
9.	P-R6	P=Q
10.	P-R7 mate	

8. Where was Black's last move? An examination will show that there is no square from which he could have moved so we are forced to the conclusion that it is now Black's move. After any Black move White mates.

9. Such a hand occurs once in every 1828 deals so the correct odds are 1827 to 1.

10. White plays and loses his wager.

11. The trick is done this way:

1.	N-Q7ch.	K-N2
2.	R-B8	K-N3
3.	N-K6	PxN
4.	Q-B7ch.	K-N4
5.	N-K5	PxN
6.	B-K4	PxB
7.	B-K3	PxB
8.	Q-K7ch.	K-N3
9.	K-R2	P-R6
10.	P-N3	P-R5
11.	P-N4	P-R4
12.	P-N5	P-R3
13.	Q-B6ch.	K-R2
14.	P-N6 mate	

12. See next page.

13. The Bridge problem looks simple enough. Just cover the Queen and trump East's Ace. Then draw trumps ending on the table and lead a small spade. If East goes up with the Ace South will take the next lead in his own hand, play the Jack of Spades, get to dummy with the Jack of clubs and discard diamonds on the good heart and the two high spades. If East doesn't play his Ace of spades South takes the trick with the Jack and discards the other spade on dummy's good heart. Now he only has a diamond loser and he brings home the slam.

But East can toss a large monkey wrench into the works by refusing to put up his Ace of hearts at the first trick. Now there is no way the contract can be made.

The superplay is not to cover the Queen of hearts. Instead trump with the Ace of clubs and lead a small club to the eight. Now lead a

small spade. If East plays his Ace a heart can be established and the three diamonds discarded. If East doesn't play the Ace Dummy is reentered and the King of hearts is led. This is covered by the Ace and trumped high. Now Dummy is entered with the last trump and a small spade is thrown on the heart Jack.

12. It takes a little checker skill to negotiate this coup successfully. The game shown will demonstrate the system. Care must be taken in the final stages to give up the penultimate man on a square which will leave White with the move.

	White	*Black*		*White*	*Black*
1.	23-19	7-10	29.	5- 1	15-10
2.	19-16	10- 7	30.	1- 5	10-15
3.	16-11	7-16	31.	5- 9	15-10
4.	24-20	16-11	32.	9-13	10-15
5.	20-16	11-20	33.	13-17	15-18
6.	28-24	20-16	34.	17-21	18-15
7.	24-20	16-11	35.	21-25	15-18
8.	20-16	11-20	36.	25-30	18-15
9.	27-23	20-16	37.	29-25	15-18
10.	23-18	16-11	38.	25-21	18-15
11.	18-14	11- 7	39.	21-17	15-18
12.	14-10	7-14	40.	17-13	18-15
13.	21-17	14-21	41.	13- 9	15-10
14.	22-18	21-17	42.	9- 5	10-15
15.	18-14	17-10	43.	30-25	15-18
16.	26-23	10-15	44.	25-21	18-15
17.	23-19	15-24	45.	21-17	15-18
18.	32-27	24-19	46.	17-13	18-15
19.	27-24	19-28	47.	13- 9	15-18
20.	25-22	28-24	48.	9- 6	18-23
21.	31-27	24-31	49.	5- 1	23-18
22.	30-25	31-26	50.	1- 5	18-23
23.	22-18	26-22	51.	5- 9	23-19
24.	25-21	22-15	52.	9-14	19-24
25.	21-17	15-18	53.	6-10	24-27
26.	17-13	18-15	54.	10-15	27-32
27.	13- 9	15-10	55.	14-18	32-28
28.	9- 5	10-15	56.	18-23	resigns

FUN FOR FIGUREHEADS - by Gerard Mosler

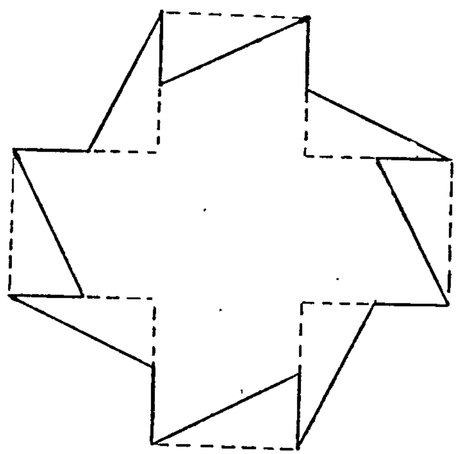
(page 25 in the February issue)

$9-3\div 6=1$	$9+7\div 8=2$
$7+2-8=1$	$6-5+1=2$
$5-4\times 1=1$	$2\times 3-4=2$
$9-6\div 1=3$	$9+7\div 4=4$
$7+5\div 4=3$	$8\times 1\div 2=4$
$8-3-2=3$	$6-5+3=4$
$6\times 2-7=5$	$7-6+5=6$
$8\div 4+3=5$	$8\div 4\times 3=6$
$9+1-5=5$	$9-2-1=6$
$9\div 3+4=7$	$8\div 2+4=8$
$6+8-7=7$	$1\times 3+5=8$
$1\times 2+5=7$	$9-7+6=8$
$8\times 2-7=9$	$8+4-2=10$
$9\div 3+6=9$	$9\div 3+7=10$
$1\times 5+4=9$	$6-1+5=10$

GEOMETRIC DISSECTIONS (Page 28 in the February issue)

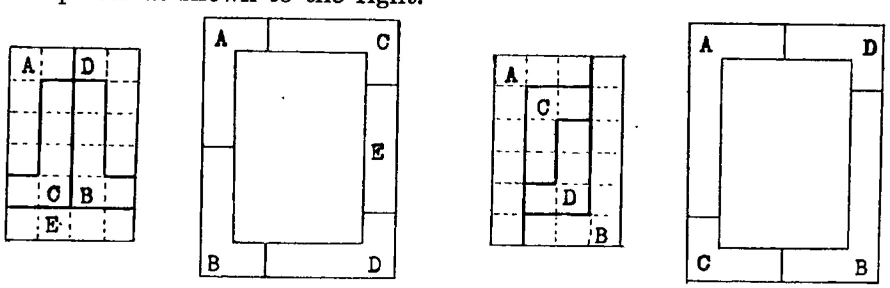
A Greek Cross

The solution is shown in the diagram. The dashed lines indicate the cuts and the small triangles can be turned over to complete the arms of the Greek Cross.



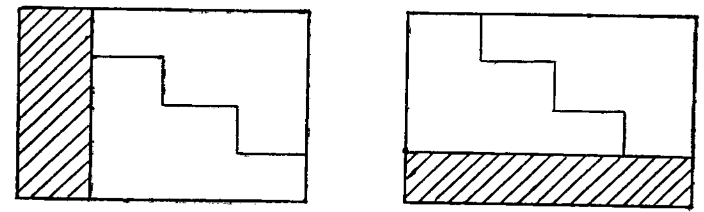
"Frame-Up"

The author's solution is shown below to the left, indicating the five pieces rearranged to form a picture frame. However, J. B. Fyffe of Massachusetts shows that the same frame can be formed with only four pieces as shown to the right.



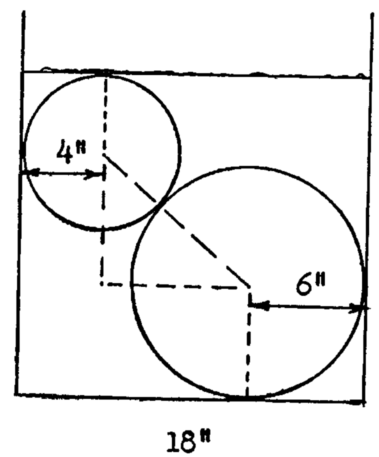
Let's Cut Papers!

As you can see, the two pieces shown below fit interchangeably to form either of the two rectangular sheets of paper.



SOME SIMPLE PUZZLES (Page 33 in the February issue)

1. The largest number which can be written using *only four* ones (*no* mathematical symbols) is 11^{11} or 285,311,670,611.



2. We can calculate the volume of water required to just cover the spheres by subtracting the volume of the spheres from the volume of the cylinder up to the level of the water. By simple geometry we find the height of the water to be 16 inches; by using the formulas for calculating the volumes of cylinders and spheres

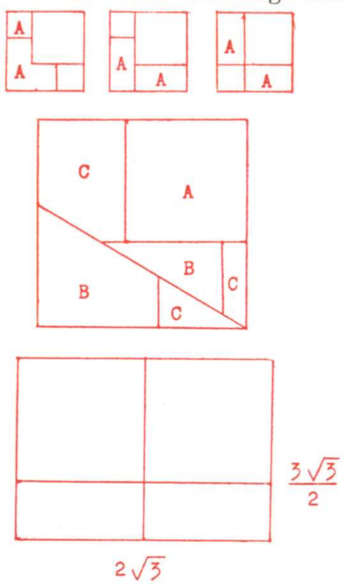
$$V=R^2\pi H=\text{volume of a cylinder}$$

$$V=\frac{4\pi R^3}{3}=\text{volume of a sphere}$$

we find the total volume occupied by the two spheres to be equal to 1173 cubic inches and the volume of an 18 inch cylinder 16 inches high to be 4072 cubic inches. The difference, 2899 cubic inches, is the volume of water required to just cover the spheres.

3. Unfortunately an error in printing occurred in this problem and, as stated, there are two answers - 56 or 72. However, the third line of the problem should have read "the number of my grandsons lies between 50 and 80. . . .". Then there could only have been a perfect square number of grandsons. The only perfect square between 50 and 80 is 64 and the number of sons is 8 - for a total of 72 sons and grandsons.

4. The drawings show how the sisters could have cut the rug in three ways to form three square rugs of two different sizes (the parts labelled A are sewed together to form a square rug). Also shown is the method the sisters used to cut the rug to form three equal sized rugs. A is one rug, parts B form another, and parts C form the third rug.



The rug that the brothers had was stated as being nine square feet - but nothing was said about the dimensions being the same as the sister's rug (3 feet by 3 feet). Their rug was of the dimensions shown and the two smaller pieces were sewn together to form a square rug the same size as the other two.

THE COMMONER'S DILEMMA (Page 34 in the February issue)

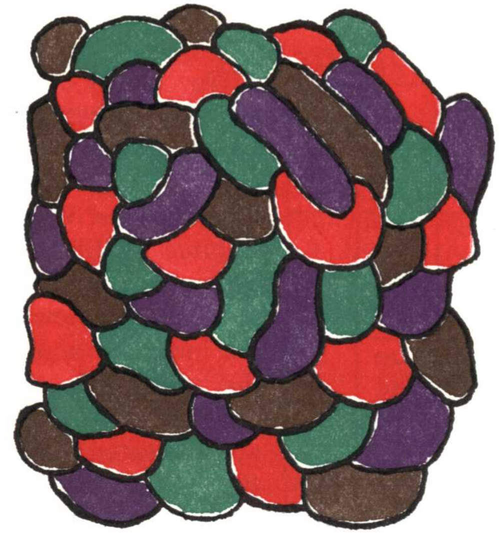
The solution to his dilemma was simple. He reached into the bag, closed his fist around a pea and swallowed it quickly before anyone could see what color it was. Since everyone could examine the bag and discover a black pea the conclusion was that he swallowed a white pea. The King could not very well admit he put in two black peas so the commoner did marry the King's daughter after all.

A LOGIC PUZZLE

Mr. Carpenter, Sr. is a tailor and his son is a butcher who married Miss Baker. Mr. Baker, Sr. is a butcher and his son is a tailor who married Miss Carpenter. Mr. Tailor, Sr. is a baker and his son is a carpenter who married Miss Butcher. Mr. Butcher, Sr. is a carpenter and his son is a baker who married Miss Tailor.

MAJOR PUZZLES - A SELECTION (Page 44 in the February issue)

Next page.



1. An Unlucky Map-Coloring Problem.

One answer to the map-maker's problem is shown here in full color.

2. Some Number Toughies

These two problems can be solved in a surprisingly simple manner:

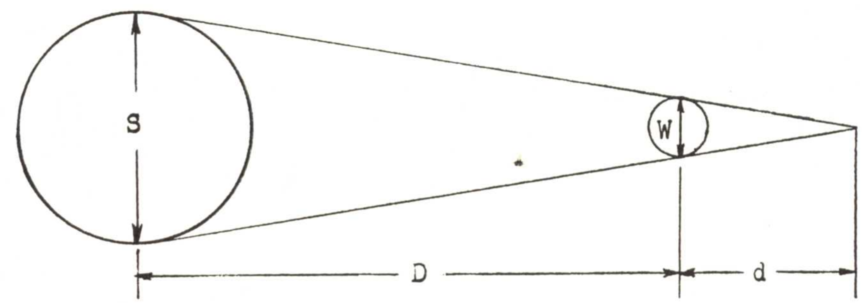
$1{,}000{,}000{,}000 = 10^9 = (2 \cdot 5)^9 = 2^9 \cdot 5^9 = (512)(1{,}953{,}125)$

and $1{,}000{,}000{,}000{,}000{,}000{,}000 = 10^{18} = 2^{18} \cdot 5^{18} = (262{,}144)(3{,}814{,}697{,}265{,}625)$

Neither pair of factors contain any zeros.

3. Shadows

The solution to this problem requires a knowledge of the distance to the sun and the diameter of the sun - information which can be found readily enough from various references. Let the distance to the sun, D, be 93,000,000 miles, let the sun's diameter, S, be 866,400 miles, the diameter of the wire, W, is 4 inches and we'll indicate the length of the wire's shadow by d. Then from the diagram:



$$\frac{D+d}{S} = \frac{d}{W} \text{ or } d = \frac{DW}{S-W}$$

Since W (4 inches), compared to S (866,400 miles), is negligible we can drop it from the denominator to form:

$$d = \frac{DW}{S} = \frac{93{,}000{,}000W}{866{,}400} = 107W = 428 \text{ inches}$$

$$= 35 \text{ feet, 8 inches} = \text{the length of the perfect shadow.}$$

The insignificant length of a perfect shadow explains why it is not always seen on the ground or on the walls of buildings. The weak streaks that one does see are not shadows but penumbra.

4. The Fouled Baseball Teams

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Sun	Tue Sat		Mon Wed				Thu Fri
Mon		Wed Fri		Tue Thu		Sun Sat	
Tue	Mon Fri		Sun Thu		Wed Sat		
Wed		Thu Sat		Sun Fri			Mon Tue
Thu			Fri Sat		Sun Mon	Tue Wed	
Fri	Wed Thu	Sun Tue		Mon Sat			
Sat					Tue Fri	Mon Thu	Sun Wed

The teams playing each other are shown in the diagram. The x's of Professor Gee Whiz' diagram are filled in with the opposing teams - the towns are indicated across the top and the days of the week are indicated down the side of the diagram.

Book Review (Page 53 in the February issue)

Just to make the record complete, the answers to the two problems quoted in the review of Mr. Hunter's little booklet are:

Fanny Flinders and father had 85 flamingo feathers.

The Alphametic puzzle answer is written out completely:

```
45)954(21
   90
   --
    54
    45
    --
     9
```


Major Puzzles - A Selection

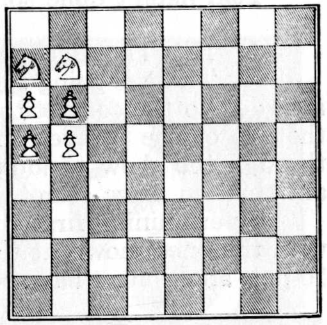
The answers to these problems will be found in the June issue of RECREATIONAL MATHEMATICS MAGAZINE.

1. Lecture Attendance

In a lecture room at a veterans hospital there were 35 people. Some were doctors, some nursing sisters and some paratroop casualties. The doctors outnumbered the women 5 to 1. There were more nurses than paratroopers. How many of each were in the room? (Mel Stover).

2. Self Protection

How can you place the 16 white chessmen on a chessboard so that each man protects only one other piece and if Black captures he, in turn, can be captured by only one white piece? (White moves up the board).



In the diagram to the right we've placed 2 white knights and 4 pawns in such a way as to satisfy these conditions. Now work out the positions of the remaining 10 men or try working out a different arrangement completely. (Fred Galvin - Minneapolis Chess Club).

3. The Farmer's Financial Finagling

A farmer who sold eggs for a living had 90 eggs to be sold one day. He sent his three sons to the market square giving 10 eggs to the oldest, 30 to the next youngest and 50 to the youngest. He told his sons, "Now, I want you three to agree on the price you're going to ask for the eggs. I want no price wars within our own family. But I want all three of you to bring home 90c each for the sale of your eggs."

Later that day, when the market square was cleared of peddlers, the sons returned home and dutifully handed over their 90c each to a well-pleased father. Just how did the sons sell their eggs and still hold to the conditions demanded by their father? (H. V. Gosling).

4. Moonshine Sharing

Three illegal distillers wanted to share their production of 6 gallons of moonshine. The only measuring containers they had were three cylindrical measuring cans holding 10, 11, and 13 quarts respectively, and the barrel holding the 6 gallons of corn. Feuding was imminent unless they could figure out how each of them could get his share of 8 quarts. Let's stop the possible bloodshed and tell them how to divide their liquor using the available containers - of course the cans are only marked for full content and not graduated in quarts.

5. The Jewel Box

"Oh, it's beautiful!" Olive exclaimed, taking the jewel box from its wrappings. "A lovely gift."

It was indeed a work of art, rectangular and sleek in rich shagreen. "It's empty, you know." Tom laughed. "All the gems are outside."

His wife smiled happily. "I know," she told him. "Fancy having my initial set on the lid in diamonds!"

"They're small, but real. It was a problem to decide exactly where the initial should be," chuckled Tom. "But I agreed to have its center nine inches from one corner, two from the opposite corner, and six from the corner between."

"Horribly technical." Olive stroked the shiny surface. "Don't tell me you figured out the distance from the fourth corner too."

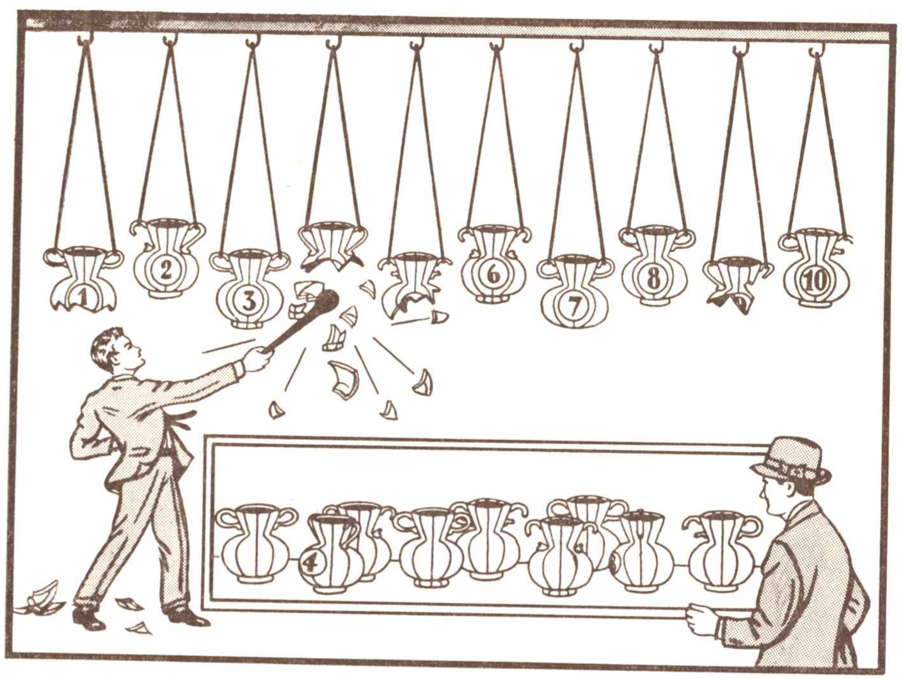
Tom hadn't done so. Maybe you could, however.

(J. A. H. Hunter)

6. Hit The Jackpot - But how?

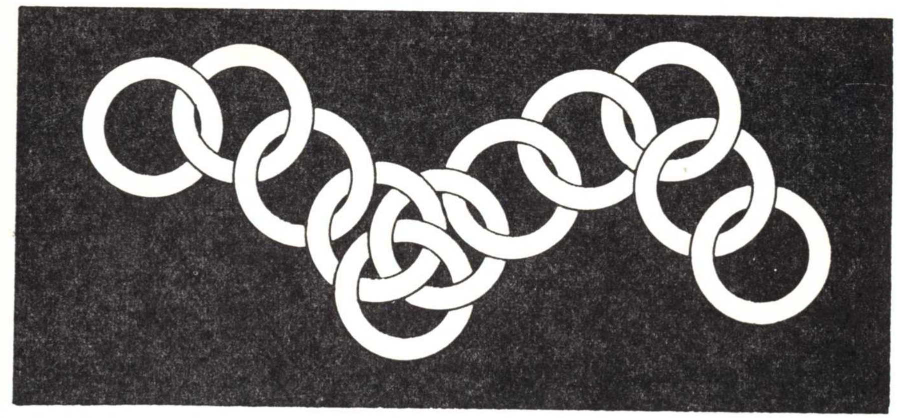
There is going to be a big prize hidden in one of the pots to be awarded to the competitor who hits the correct pot with the stick. On the eve of the competition a supersmartie tries to examine the pots through the show window but the organizers of the contest, in wise anticipation, have secreted the pot with the prize.

By examining first the pots at bottom and then comparing them with the ones shown at the competition, can you deduce which of the 10 pots above must have been the one containing the prize? (G. Mosler)



7. The Divorcee's Dilemma

A faded movie Queen, between husbands and down to her last mink, was stranded in a fashionable hotel in Cannes with no immediately available liquid assets. The management had long since put her on a cash advance basis and to make the trust mutual she had responded by making daily payments in advance. She was expecting some alimony checks in eleven days and persuaded the hotel to accept a link of her gold belt for a day's rent. The belt is pictured in the diagram below. Here is her problem. She wants to give the hotel a link a day and because she expects to redeem the belt after eleven days are up she does not want to cut up the belt any more than necessary. What is the minimum number of cuts required? (Mel Stover).



The Sun Dial

by Norvin Pallas

The death of young Harley Jamison, while hardly mourned, left a few unexplained tangents which had to be cleared up. Inspector Warren of homicide laid the problem before Professor Eric Mansion for his consideration.

"The cause of death, as we now know, was that Harley Jamison dug beneath the sun dial, expecting to find the fortune left by his late father. As you know, the elder Mr. Jamison had two sons, David, of whom he was very fond, and Harley, who was a considerable blackguard. When Harley dug beneath the sun dial he touched off an explosion which killed him."

"Isn't it clear that Mr. Jamison merely set up a boobytrap for him?"

"Yes, that was probably it, but it leaves several other questions unanswered. First, how did the explosion go off?"

"Wasn't it a question of contact?"

"Very likely. But there is the additional point that the fortune *really was there!* Mr. Jamison, as is well known, developed a highly suspicious nature in his later years. He had no faith in banks, lawyers, wills, trustees, or anything at all. Now what did he intend? Did he want the explosion to kill *anyone* who came in search of his treasure? That seems hardly likely, for it might just as easily have been his favorite, David, who did the digging. It was mere chance that David happened to be abroad with the army at the time of his father's death. In that case it would have been David who got blown up seeking the fortune."

"Did David have any advance indication, or warning, concerning the treasure?" asked Professor Mansion.

"He claims that he didn't, and we're inclined to believe him. He seems to be a substantial person, totally different from either his father or his brother."

"How did Harley know where to dig?" questioned the professor. His manner appeared indolent, and only those who knew him well were aware of that razor-sharp mind.

"His father left a message in the nature of a code. David had followed his father's enthusiasm for the subject of cryptography, but Harley also knew something about cryptograms — possibly because he wanted to be certain nothing was put over on him. At any rate, the message which the father had probably intended for David fell into Harley's hands."

The inspector drew a sheet of paper from his pocket and smoothed it out. It read:

GX AI XJ IM IF KT JV DO KJ OT LT KI LJ ZO UJ UO TH OX

After studying the paper for a moment, the professor asked, "What do your cryptanalysts make of it?"

"Well, it didn't turn out to be a very difficult problem. The most obvious point of attack, where the letters are divided into bigrams, is to suspect an amateur form of the Playfair, which either brother would be able to break. You are aware of the great weakness of the Playfair, the strong inter-relationships between letters. You have this block of twenty-five letters (Q was probably omitted in this case), and encipher two letters at a time by imagining them to form two corners of a rectangle within the block, and substituting the other two corners. If the two letters are found on the same line, or in the same column, you substitute the letters following, or the ones below, in which case a different type of relationship exists. How many bigrams do you have to identify correctly to reconstruct the diagram? Two or three should do it, and often an educated guess will be good enough to get the thing started.

"However, our men were unable to break the message as a Playfair, and so they scouted around for something else. You will notice that the message is fairly brief, which rules out any system of great difficulty, for in that case David would have been unable to break it either."

"Unless there was some form of pre-arrangement," suggested the professor mildly.

"Which David denies."

"Has David seen this message?"

"No, he hasn't returned home yet, but we've spoken to him by transatlantic phone. Of course we could have waited for him to see if he had anything to suggest, but as it turned out we didn't need his help. Our men naturally turned to the Vigenere, and this, as it turned out, easily solved their problem."

"Which form of Vigenere table was used?"

"The original table, using a straight alphabet on each line, and each subsequent line moved over one position to the left. Of course the Vigenere is usually used with a key word, so that a key letter will indicate which line of the table should be used to make each substitution. But in a case where no key word has been agreed upon in advance — as was the case here — the table can be used in a different way. Two letters in the cipher message will equal one letter in the original message. Simply take your Vigenere table, find the row and column headed by the first two letters of the message, and the point where they meet will give you the first letter of the original message. In this manner the message was easily read. It goes:

DIG UNDER THE SUN DIAL

"Now this appears to be a legitimate message. The treasure really was buried under the sun dial, buried very deeply with the explosive planted above it. Here's what we want to know: How did this message, which gave the location of the treasure to both sons, kill Harley while at the same time it must have contained a warning which would have been clear to David?"

"Have you considered the possibility of an underlying secondary message, such as 'danger, disconnect explosive,' spelled out by letters at certain intervals, or something along that line?"

"Yes, but our men were unable to find any such thing. In fact, they felt it would be difficult to do such a thing successfully in such a short message."

The professor laughed. This was the sort of problem which touched his fancy. He kept the paper and worked on it until the early morning hours. Somewhere about three o'clock he found the answer, so simple that he had to laugh again. He thought of calling the inspector, but decided that his friend didn't have a sufficient sense of humor to risk it. But he did give him the answer the next morning.

(*But before you read on, perhaps you'd like to try your hand at solving the puzzle.*)

"I believe if you will examine the remains of the sun dial you may find a rather curious mechanism. While the sun is shining a circuit is broken, which disconnects the explosive, and digging can be carried on in complete safety. Harley, of course, carried on his nefarious work in the darkest hours, and was blown up.

"Any two-for-one substitution allows the possibility for the concealment of a secondary message. Simply take every second letter as follows:

X I J M F T V O J T T I J O J O H X

"This is a simple Caesar's substitution. Reading the previous letter in the alphabet for each letter of the above, we get:

W H I L E S U N I S S H I N I N G W

"Harley got only half the message. David, with his greater knowledge of cryptography and his father's eccentricities, would have found the whole message before he went fooling around with that sun dial!"

Answers to the Minor Problems on Page 17

1. Declaration Data

Rhode Island had 2 Signers, New Hampshire 3, Georgia 3, North Carolina 3, Delaware 3, Maryland 4, South Carolina 4, Connecticut 4, New York 4, Massachusetts 5, New Jersey 5, Virginia 7 and Pennsylvania 9.

2. Outguessing the Piggy Bank

The range of guesses was 30 to 53. Since the worst guess was 12 off, that means that the number of coins was either $30+12=42$ or $53-12=41$. Only 41 coins checks with the other information given.

3. A Honey of a Puzzle

One pound of honey contains 0.83 lbs. solid and 0.17 lbs. water. One pound of nectar contains 0.30 lbs. solid and 0.70 lbs. water. Therefore, the 0.83 lbs. of solids contained in 1 lb. of honey were contained in $^{83}/_{30}$ or $2^3/_4$ lbs. of nectar.

4. Miss-Matched Twins

The main point of this puzzle is that it is impossible to get only one set of twins mismatched - 49 sets correctly matched automatically matches the remaining set. This means that exactly 18 people had 3 sets wrong. Since the number of perfect answers was the same as the number with 2 wrong we have the following equation:

$$18+2x=\frac{1.9}{100}\text{ of the number of entries.}$$

As the number of people who entered cannot be a fraction, x must be 10 or 29 or some multiple of 19 added to 10. If x were 29, the number of entries would be 4000. However, we were told that the circulation of the paper was less than 4000, so $x=10$ and the number of entries was 2000.

5. Bees A-Buzzin'

The number of bees was 72.

6. The Impractical Mathematical Entomologist

There were 10 beetles and 10 spiders

7. The Enigmatic Milestones

The inscription on the first milestone was 16, on the second milestone it was 61, on the third 106 and the speed of the car was 45 miles per hour.

Readers' Research Department

From the volume of mail received it appears that this department is one of the more popular in RMM. After a discussion of the first Research Problem we'll suggest two more problems for the readers.

Five complete, and apparently correct, analyses were received by the editor and all agreed on the formula eventually arrived at. Let's restate the problem and then give an analysis derived by one of the readers.

> A parallelepiped is built up from regular cubes. If an imaginary straight line, the main diagonal, is constructed from one corner to the opposite corner through how many of the component cubes will this line pass? The answer should be expressible in terms of the three dimensions of the parallelepiped.

The following analysis was worked out by E. O. Buchman of Altadena, California.

In a lattice formed by the cubes there exist three structures of importance: faces, edges and corners of the component cubes. Each time the diagonal passes through a corner, an edge or a face it leaves one cube and enters a new cube. Thus we must count the corners, edges and faces that the diagonal crosses.

Let the dimensions of the solid be m, n and p. Each corner of the lattice which lies on the diagonal represents a division of each m, n and p into two integral parts, each in the same ratio. The number of points for which this should be true should be one less that the greatest common factor of m, n and p. [Hereafter, by (n, p) we mean the greatest common factor of n and p.]

We have

Number of corners on the diagonal $=(m,n,p)-1=C$

The number of edges in the lattice in the m-direction that the diagonal will pass through is

$$(n, p)-1$$

(This may be seen by considering the projection of the diagonal on an $n \times p$ face.)

The total number of edges crossed by the diagonal will be

$$(m, n)-1+(m, p)-1+(n, p)-1$$

But since with each corner that the diagonal crosses three edges were already counted, the total number of effective edge crossings is

$$(m, n)-1+(m, p)-1+(n, p)-1-3[(m, n, p)-1]=E$$

$$\text{or } E=(m, n)+(m, p)+(n, p)-3(m, n, p)$$

The number of interface crossings made by the diagonal will be

$$(m+n+p)-3=(m-1)+(n-1)+(p-1)$$

But since each time the diagonal crossed a corner three interfaces were accounted for, and each time an edge was crossed two interfaces were accounted for, the number of total effective interface crossings is

$$(m+n+p)-3-3[(m, n, p)-1]-2[(m, n)+(m, p)+(n, p)-3(m, n, p)]$$

$$I=(m+n+p)+3(m, n, p)-2[(m, n)+(m, p)+(n, p)]$$

Since the number of cubes, K, passed through is one greater than the total number of corners, effective edges and effective interfaces,

$$K+1=C+E+I$$

$$K=(m+n+p)-[(m, n)+(m, p)+(n, p)]+(m, n, p)$$

If the dimensions of the solid are such that the greatest common factor for all three dimensions is 1, the formula reduces to

$$K=m+n+p-2$$

Analyses and correct formula also given by Marlow Sholander (Western Reserve University), R. O. Hughes (Ontario), Kenneth K. Hickin (14 years old, New Orleans) and W. W. Horner (Pittsburgh). Mr. Sholander also derived a formula for a structure built up of N-dimensional blocks.

For the Research in this issue we'd like to give two essentially different problems.

1. The partitioning of a line, a plane or a space by points, lines or planes, respectively, is described, analytically, by G. Polya in Volume I of his *Mathematics and Plausable Reasoning* (Princeton University Press). We will leave the analyses to those readers who wish to attempt them, or wish to refer to Polya's work. For the benefit of those who wish to know the respective formulae, here they are:

The number of divisions of a line by n points $= n+1$

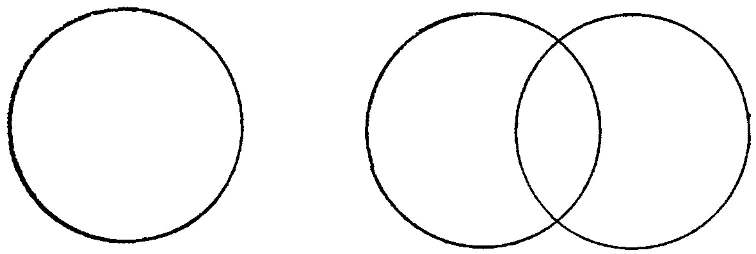
The number of divisions of a plane by n lines $= \dfrac{n(n+1)}{2}+1$

The number of divisions of space by N planes $= \dfrac{N(N^2+5)}{6}+1$

The problem suggested by a reader of RMM is to determine the number of regions into which a plane can be divided by M circles. A general problem would be to determine the number of regions of space formed by S spheres.

In the entire discussion above, we are dealing with the *maximum* number of divisions or regions formed.

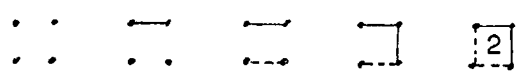
The two diagrams below show that one circle divides a plane into two regions, an inside and an outside; two circles divide a plane into four regions. We leave the spheres to the readers.



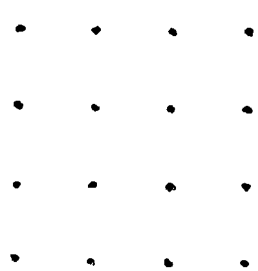
2. The second Research problem is to determine the best strategy for a children's game. Martin Gardner suggests that we try to determine the best strategy for the old game of Dots and Squares (variously called the French Polytechnic School's Game, Square It, and probably other things).

A square array of dots is to be used and the idea is for two players playing alternately, to draw single lines horizontally or vertically between adjacent dots. A player completing a square scores a point and the player with the most points after the array of dots is completely filled to form a grid wins the game.

Where only four dots are used, it is inevitable that the second player will win



If the number of squares which can eventually be formed is an even number then the game may end in a draw. Here is the smallest field (except for the one drawn above) in which the game cannot end in a draw



What is the best strategy for a player to use to win this game? And which player, the first or the second, has the advantage? If someone devises a suitable strategy for this simple array, he may try

larger arrays.

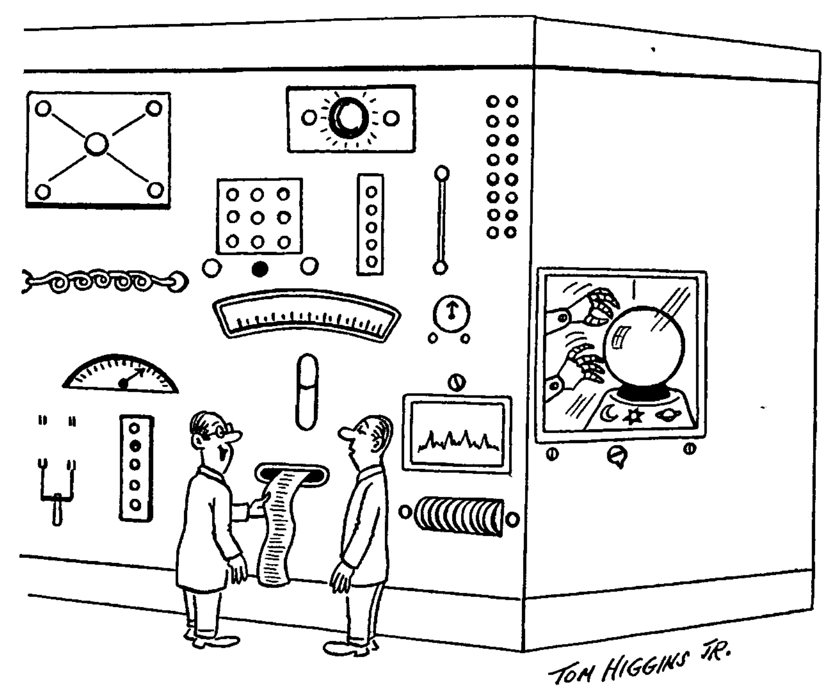
We might settle the question of the existence of two sets of rules known by the editor.

1. Players play alternately throughout the game, one stroke each time.

2. Players play alternately unless a player completes a square, at which time he is allowed an extra stroke as long as he completes squares.

The best strategy may depend on which set of rules is followed, so keep this in mind.

We may add a note here. The game can also be played using a triangular array of points, or in many other forms.



"IT'S THE LATEST IN ELECTRONIC BRAINS AND ACCURATE TO THE N[th] DEGREE."

Pascal and Fibonacci

by Walter W. Horner

The purpose of this note is to point out an interesting relationship which exists between Pascal's Triangle and Fibonacci's series. Fibonacci arrived at his series in solving a problem something like this. If a man starts with a pair of newborn rabbits, how many will he have in n months? It is assumed that a pair of rabbits will mature in two months at which time they will produce a new pair and will continue to produce a new pair each month thereafter. Now suppose we generalize the problem by making the time of maturity t months and leave the other assumptions as stated above. Then solving for $t = 1, 2, 3, 4$, we get the following series respectively.

(1)	1	2	4	8	16	32	64	128	- -
(2)	1	1	2	3	5	8	13	21	- -
(3)	1	1	1	2	3	4	6	9	- -
(4)	1	1	1	1	2	3	4	5	- -

Now the relationship between series (1) and Pascal's Triangle is well known and is shown in the following table.

$$\binom{n}{m} = \frac{n!}{(n-m)!\, m!} \text{ for integers m, n, where } 0 \leqq m \leqq n.$$

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$		SUM
0	1						1
1	1	1					2
2	1	2	1				4
3	1	3	3	1			8
4	1	4	6	4	1		16
		etc.					etc.

Pascal's triangle is written in several forms but the one above is used in handbooks for finding binomial coefficients. For convenience we shall call this the "one step" form since each column begins one space lower than the preceding column. Note that the terms of the Triangle are the parts into which the terms of the series can be partitioned and that these parts can be expressed in the form $\binom{n}{m}$.

Thus $16=1+4+6+4+1=\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}$

Now consider series (2). To get this series we arrange the columns of Pascal's Triangle in a "two step" form i.e. each column begins two spaces lower than the preceding column. This is shown in the following table.

TRIANGLE				SUM
1				1
1				1
1	1			2
1	2			3
1	3	1		5
1	4	3		8
1	5	6	1	13
1	6	10	4	21
	etc.			etc.

This shows how the terms of Fibonacci's series can be partitioned into parts which can be expressed in the form $\binom{n}{m}$.

Thus $21=1+6+10+4=\binom{7}{0}+\binom{6}{1}+\binom{5}{2}+\binom{4}{3}$

In a like manner we can construct a table for series (3) by constructing a triangle of "three step" form and repeat the process above. Likewise for series (4). This of course can be carried on indefinitely for various values of t.

From these tables above we can express the recursion relation of each series as well as term p and the sum of p terms. These are summarized as follows:

SERIES	RECURSION RELATION
(1)	$a_p = a_{p-1} + a_{p-1}$
(2)	$a_p = a_{p-1} + a_{p-2}$
(3)	$a_p = a_{p-1} + a_{p-3}$
(4)	$a_p = a_{p-1} + a_{p-4}$

SERIES	TERM p
(1)	$a_p = \binom{p-1}{0} + \binom{p-1}{1} + \binom{p-1}{2} + \binom{p-1}{3} + - - -$
(2)	$a_p = \binom{p-1}{0} + \binom{p-2}{1} + \binom{p-3}{2} + \binom{p-4}{3} + - - -$
(3)	$a_p = \binom{p-1}{0} + \binom{p-3}{1} + \binom{p-5}{2} + \binom{p-7}{3} + - - -$
(4)	$a_p = \binom{p-1}{0} + \binom{p-4}{1} + \binom{p-7}{2} + \binom{p-10}{3} + - - -$

SERIES	SUM OF p TERMS
(1)	$\Sigma a_p = a_{p+1} - 1 = \binom{p}{1} + \binom{p}{2} + \binom{p}{3} + \binom{p}{4} + \binom{p}{5} + - - -$
(2)	$\Sigma a_p = a_{p+2} - 1 = \binom{p}{1} + \binom{p-1}{2} + \binom{p-2}{3} + \binom{p-3}{4} + \binom{p-4}{5} + - - -$
(3)	$\Sigma a_p = a_{p+3} - 1 = \binom{p}{1} + \binom{p-2}{2} + \binom{p-4}{3} + \binom{p-6}{4} + \binom{p-8}{5} + - - -$
(4)	$\Sigma a_p = a_{p+4} - 1 = \binom{p}{1} + \binom{p-3}{2} + \binom{p-6}{3} + \binom{p-9}{4} + \binom{p-12}{5} + - - -$

Numbers, Numbers, Numbers

Again we delve into the realm of numbers and their properties. In the February issue some space was devoted to prime numbers.

Now it so happens that prime numbers of certain types are related to perfect numbers. A prime number is an integer which has no integral divisors except itself and 1. A perfect number is an integer in which the sum of all its divisors, except itself, is equal to itself. For example, 6, 28 and 496 are perfect numbers.

$$1+2+3=6$$

$$1+2+4+7+14=28$$

$$1+2+4+8+16+31+62+124+248=496$$

The first four perfect numbers, 6, 28, 496 and 8128, have been known for about 2,000 years but it wasn't until 1460 that some anonymous person recorded the fifth perfect number 33,550,336. Then in 1644 Father Marin Mersenne announced his discovery of the next six perfect numbers. Unfortunately, two of Mersenne's numbers have been shown not to be perfect.

All the known perfect numbers, there are 18 to date, are even numbers and all are of the form

$$2^{p-1}(2^p-1)$$

where p *and* 2^p-1 are prime. The determination of the primality of 2^p-1 has always been foremost among students of prime and perfect numbers. Mersenne himself stated that 2^p-1, now referred to as M_p in his honor, is prime for

$$p=2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257$$

Depending on the historian, Mersennes is either praised for his discoveries, in spite of his errors, or derided for making such assertions. As a matter of fact Mersenne erred in five ways: he included 67 and 257 which do *not* yield primes and excluded 61, 89 and 107 which *do* yield primes. When D. H. Lehmer proved that M_{257} was not prime the list of perfect numbers stood at 12 with M_{127} as the largest prime number known at that time. In 1952 SWAC (National Bureau of Standards' Western Automatic Computer) was properly programmed and found five more prime numbers of the Mersenne form: for p=521, 607, 1279, 2203 and 2281. And, finally, on November 17, 1959 the IBM 709 at the National Facilities Experimental Center at Atlantic City, New Jersey took five minutes of its time to announce the next Mersenne prime: p=3217.

This last Mersenne prime forms the largest perfect number known

$$2^{3216}(2^{3217}-1)$$

containing 1,937 digits.

No one has found any odd perfect numbers nor has anyone proven whether or not odd perfect numbers exist. Leonhard Euler proved, in 1750, that all even perfect numbers are of the form known to Euclid, viz.:

$$2^{p-1}(2^p-1)$$

Here is a complete list of the 18 known perfect numbers and the full value for the first 8. By convention the perfect numbers are referred to as V_n.

$V_1=2^1(2^2-1)=6$
$V_2=2^2(2^3-1)=28$
$V_3=2^4(2^5-1)=496$
$V_4=2^6(2^7-1)=8{,}128$
$V_5=2^{12}(2^{13}-1)=33{,}550{,}336$
$V_6=2^{16}(2^{17}-1)=8{,}589{,}869{,}056$
$V_7=2^{18}(2^{19}-1)=137{,}438{,}691{,}328$
$V_8=2^{30}(2^{31}-1)=2{,}305{,}843{,}008{,}139{,}952{,}128$
$V_9=2^{60}(2^{61}-1)$ $V_{14}=2^{606}(2^{607}-1)$
$V_{10}=2^{88}(2^{89}-1)$ $V_{15}=2^{1278}(2^{1279}-1)$
$V_{11}=2^{106}(2^{107}-1)$ $V_{16}=2^{2202}(2^{2203}-1)$
$V_{12}=2^{126}(2^{127}-1)$ $V_{17}=2^{2280}(2^{2281}-1)$
$V_{13}=2^{520}(2^{521}-1)$ $V_{18}=2^{3216}(2^{3217}-1)$

The size of the last 10 perfect numbers is such that the editor will publish the complete values only upon overwhelming demand by the readers of RMM.

Charles W. Trigg of Los Angeles City College furnishes us with several choice items in the realm of pure mathematical recreations.

POWER PLAYS IN 1961

1. $1961=(-19+61)^2+196+1=(-19+61)^2-19+6^3(1)$
2. $1961=2^0+2^3+2^5+2^7+2^8+2^9+2^{10}$
3. $1961=5^2+44^2=(10)(14)^2+1^2=10^3+31^2$
4. $1961=1^3+2^3+3^3+5^3+6^3+7^3+8^3+9^3$
5. $1961=1^3+2^3+2^3+3^3+4^3+5^3+12^3$
6. $1961=13^3-6^3-3^3+2^3-1^3$

PLAYING WITH THE DIGITS OF 1961

1. $1961=(37)(53)=(1+9\sqrt{16})(9{\cdot}6-1{\cdot}1)$
2. $1+9+6+1=17$, a prime
3. $1+9+6+1=17=1+6+9+1$
   $19+96+61+11=187=11+16+69+91$
   $196+961+611+119=1887=911+116+169+691$
   $1961+9611+6119+1196=18887=6911+9116+1169+1691$
4. $19^2+96^2+61^2+11^2=13419=11^2+16^2+69^2+91^2$
   $196^2+961^2+611^2+119^2=1349419=911^2+116^2+169^2+691^2$
   $1961^2+9611^2+6119^2+1196^2=135089419=6911^2+9116^2+1169^2+1691^2$

INTEGER SOLUTIONS OF $\frac{ABC}{A+B+C}$

There are 179 three-digit integers which are exactly divisible by the sums of their digits. Every integer, $11 \leq I \leq 61$, and sixteen integers >61 can be represented by $\frac{ABC}{A+B+C}$, some in more than one way.

ABC/(A+B+C)	ABC	ABC/(A+B+C)	ABC	ABC/(A+B+C)	ABC
11	198	34	102 204 306	54	972
12	108		408	55	715 825 935
13	117 156 195	35	315		AA0
14	126	36	324 648	56	504
15	135	37	370 407 481	57	513
16	144 192 288		518 592 629	58	870 522
17	153		AAA	59	531
18	162	38	342 684	60	540
19	114 133 152	39	351	61	732 915
	171 190 209	40	120 240 360		
	228 247 266		480	64	320 512 640
	285 399	41	738		704 832 960
20	180	42	756	67	201 402 603
21	378	43	516 645 774		804
22	132 264 396	44	792	68	612
23	207	45	405	69	621
24	216	46	230 322 414	70	210 420 630
25	150 225 375		460 506 552		840
26	234 468		644 690 736	73	511 730 803
27	243 486		782 828 874	76	912
28	112 140 224		966	78	702
	252 280 308	47	423 846	79	711
	336 364 392	48	432 864	80	720
	448 476 588	49	441 735 882	82	410 820 902
29	261	50	450	85	510
30	270	51	918	89	801
31	372 465 558	52	312 624 780	90	810
32	576		936	91	910
33	594	53	954	100	A00

A few errors slipped into the February issue *Numbers, Numbers, Numbers* department.

Page 37: $p=127$ inadvertently omitted.
$2^{2281}-1$ contains 687 digits (not 685 as given)

Page 38: The second column of Dr. Gosling's curious equivalents beginning with $1^2=1^3+0^3$ were all misprinted: the plus signs should be minus signs. Otherwise, they are curious indeed.

Page 43: The number 2013, listed as prime, should be omitted and 2017 substituted in its place.

The Next 478 Prime Numbers - 5701 to 9973

The February issue of RMM contained the 750 prime numbers from 2 to 5693. One error was noticed - 2013 should read 2017. Enough readers expressed their thanks for publishing that table that the editor is adding the following list of prime numbers to complete the values up to 9973 (the next prime number is greater than 10,000). Some readers may wish to examine the two tables for possible oddities such as the permutable primes given on page 36 of the February issue. The following table was supplied to the editor by Maxey Brooke and we would both like to know if there are any errors.

5701	6067	6373	6763	7109	7507	7841	8221	8599	8933	9311	9649
5711	6073	6379	6779	7121	7517	7853	8231	8609	8941	9319	9661
5717	6079	6389	6781	7127	7523	7867	8233	8623	8951	9323	9677
5737	6089	6397	6791	7129	7529	7873	8237	8627	8963	9337	9679
5741	6091	6421	6793	7151	7537	7877	8243	8629	8969	9341	9689
5743	6101	6427	6803	7159	7541	7879	8263	8641	8971	9343	9697
5749	6113	6449	6823	7177	7547	7883	8269	8647	8999	9349	9719
5779	6121	6451	6827	7187	7549	7901	8273	8663	9001	9371	9721
5783	6131	6469	6829	7193	7559	7907	8287	8669	9007	9377	9733
5791	6133	6473	6833	7207	7561	7917	8291	8677	9011	9391	9739
5801	6143	6481	6841	7211	7573	7927	8293	8681	9013	9399	9743
5807	6151	6491	6857	7213	7577	7933	8297	8689	9027	9403	9749
5813	6163	6521	6863	7219	7583	7937	8311	8693	9041	9413	9767
5821	6173	6529	6869	7229	7589	7949	8317	8699	9043	9419	9769
5827	6197	6547	6871	7237	7591	7951	8329	8707	9059	9421	9781
5839	6199	6551	6883	7243	7603	7963	8353	8713	9067	9431	9787
5843	6203	6553	6899	7247	7607	7993	8363	8719	9091	9433	9791
5849	6211	6563	6907	7253	7621	8009	8369	8731	9103	9437	9803
5851	6217	6569	6911	7283	7639	8011	8377	8737	9109	9439	9811
5857	6221	6571	6917	7297	7643	8017	8387	8741	9127	9461	9817
5861	6229	6577	6947	7307	7649	8039	8389	8747	9133	9463	9829
5867	6247	6581	6949	7309	7669	8053	8419	8753	9137	9467	9833
5869	6257	6599	6959	7321	7673	8059	8423	8761	9151	9473	9839
5879	6263	6607	6961	7331	7681	8069	8429	8779	9157	9479	9851
5881	6269	6619	6967	7333	7687	8081	8431	8783	9161	9491	9857
5897	6271	6637	6971	7349	7691	8087	8443	8803	9173	9497	9859
5903	6277	6653	6977	7351	7699	8089	8447	8807	9181	9511	9871
5923	6287	6659	6983	7369	7703	8093	8461	8819	9187	9521	9883
5927	6299	6661	6991	7393	7717	8101	8467	8821	9199	9533	9887
5939	6301	6673	6997	7411	7723	8111	8501	8831	9203	9539	9901
5953	6311	6679	7001	7417	7727	8117	8513	8837	9209	9547	9907
5981	6317	6689	7013	7433	7741	8123	8521	8839	9221	9551	9923
5987	6323	6691	7019	7451	7753	8147	8527	8849	9227	9587	9929
6007	6329	6701	7027	7457	7757	8161	8537	8861	9239	9601	9931
6011	6337	6703	7039	7459	7759	8167	8539	8863	9241	9613	9941
6029	6343	6709	7043	7477	7789	8171	8543	8867	9257	9619	9949
6037	6353	6719	7057	7481	7793	8179	8563	8887	9277	9623	9967
6043	6359	6733	7069	7487	7817	8191	8573	8893	9281	9629	9973
6047	6361	6737	7079	7489	7823	8209	8581	8923	9283	9631	
6053	6367	6761	7103	7499	7829	8219	8597	8929	9293	9643	

Sums of Cubes

by William R. Ransom

Do these look like extraordinary coincidences, or is there some underlying law about them:

$$1^3=1=(1)^2$$
$$1^3+2^3=9=(1+2)^2$$
$$1^3+2^3+3^3=36=(1+2+3)^2$$
$$1^3+2^3+3^3+4^3=100=(1+2+3+4)^2$$

If such a relation is true for the numbers up to N - and it has just been seen that it is true for N=1, 2, 3, 4 - let us see what happens when the next cube is added.

Now if N=4 we know, from the above, that

$$1^3+2^3+\cdots+N^3=(1+2+\cdots+N)^2$$

and using the formula for the sum of the arithmetical progression

$$1+2+\cdots+N=\frac{N(N+1)}{2}$$

The next cube can be written in various ways as follows:

$$(N+1)^3=(N+1)(N+1)^2=N(N+1)^2+(N+1)^2$$
$$=2\frac{N(N+1)}{2}(N+1)+(N+1)^2$$
$$=2(1+2+\cdots+N)(N+1)+(N+1)^2$$

and if we add these two equal expressions to the sides of the previous equation, we see that the right hand member presents the three terms of a perfect square, thus:

$$1^3+2^3+\cdots+N^3+(N+1)^3=$$
$$(1+2+\cdots+N)^2+2(1+2+\cdots+N)(N+1)+(N+1)^2=$$
$$[(1+2+\cdots+N)+(N+1)]^2$$

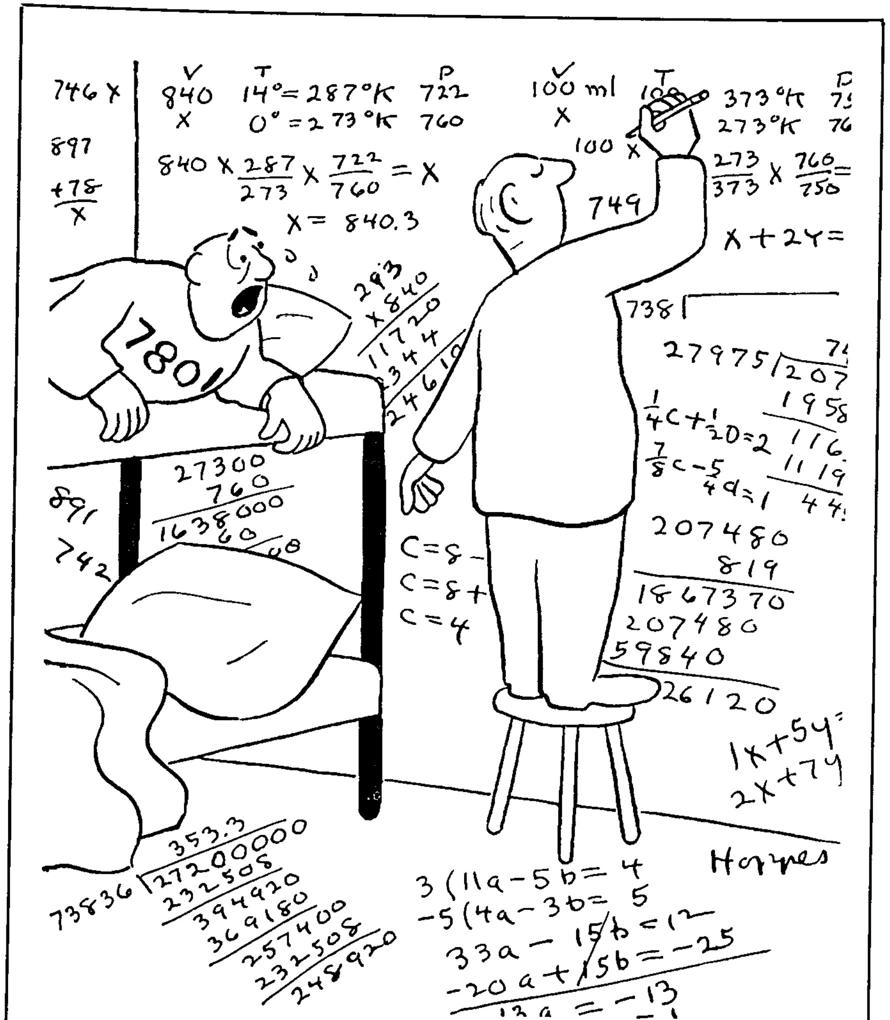
So the relation is true for the next counting number, and for the next one, and so on for any succession of counting numbers from one up as far as you like.

Another way of expressing the sum of cubes from 1^3 to N^3 appears if we use the formula for the sum of the arithmetic progression above.

By squaring, rearranging and using the above proof:

$$(1+2+3+\cdots+N)^2=\frac{[N(N+1)]^2}{4}=1^3+2^3+3^3+\cdots+N^3$$

which shows that the sum of successive cubes to N is ¼ the square of N(N+1).



"I HOPE YOU'RE NOT A LIFER! I CANT STAND MUCH MORE OF THIS!"

3-D in 2-D

by Joseph S. Madachy

The problem of translating our three-dimensional world onto flat surfaces, where it is more conveniently portrayed, can sometimes be a vexing one. Artists have utilized techniques for many years to give the illusion of depth and, in general, they've been very successful.

But what does the draftsman do when his *portrayals* of object are confined to blueprints and drawings? Simple, he merely takes three head-on views of the object and just draws what he would see from a given head-on view. If there are cuts or projections on the other side of the object, invisible from his present point of view, he represents them by dashed lines. It may not always be possible to visualize, easily, the object drawn in this fashion but it serves the purpose demanded in the profession. An isometric view, or some overall portrayal of the object, may be given to show what the object looks like.

In this little article we'd like to present a few objects in which only certain views are shown and you are to figure out what the object looks like. To ensure that we all have the same interpretation of drawings let's make a few sketches and indicate how they are to be "read".

Below you see a sketch of a cube with the faces labeled and next to it you see the three head-on views of the same cube.

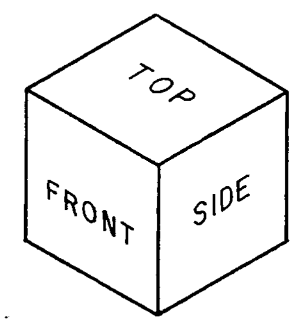


Figure 1

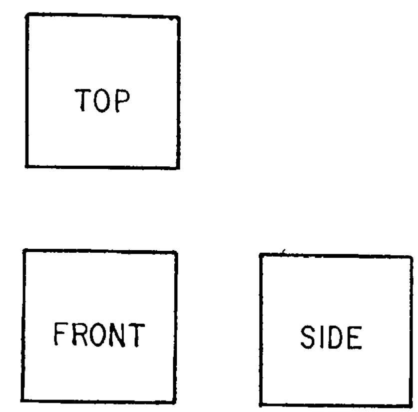


Figure 2

The next set of figures shows, first, an object in an isometric view and then, in the conventional manner, the three head-on views corresponding to the top, front and side views similar to the cube above. The dotted line in the top view indicates the indentation which would be invisible from the top when actually viewed.

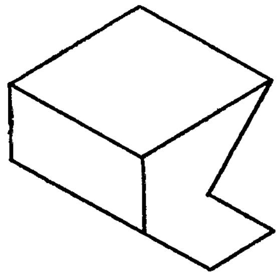


Figure 3

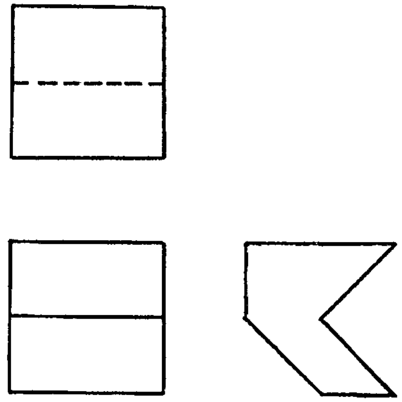


Figure 4

The rest of the drawings are puzzles for the readers - the answers will appear in the June issue of RECREATIONAL MATHEMATICS MAGAZINE. Some figures show all three views, in which case you are to determine the object and draw an isometric view. Others show only two views or shadow views. Just fill in the missing information for all the figures and you can score yourself high on the Visual Imagination scale. Except for figures 9 and 10, all sketches are complete, i.e. no lines, solid or dashed, have been omitted if they are supposed to be shown.

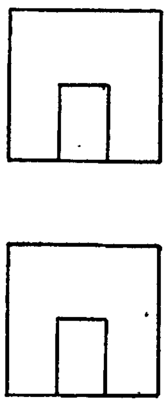


Figure 5

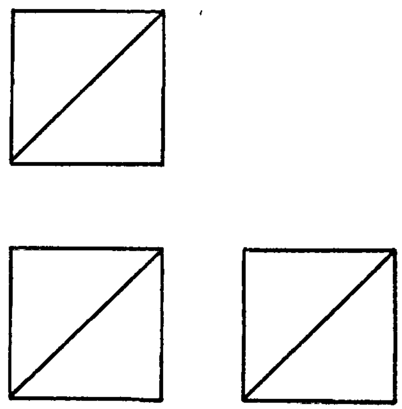


Figure 6

Figure 5 may be known to some of you. Draw the missing side view and isometric. Figure 6 may look impossible — but it isn't! Draw the isometric.

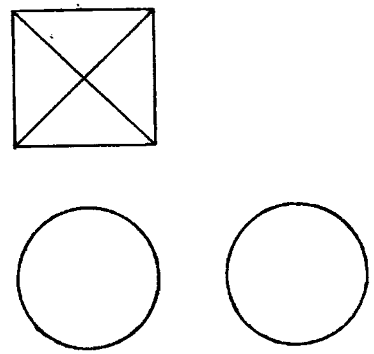


Figure 7

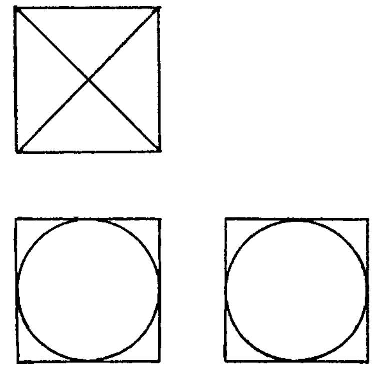


Figure 8

No foolin', all the information is given in figure 7 — but what's the thing look like? Figure 8 looks like figure 7 — almost!

The next two puzzles are just a bit different. The three views shown were made by projecting a bright light behind the object — the drawings are the shadows produced. The corresponding top, front and side views were made by a light held below, behind and on the other side of the object. Careful — there are no solid or dashed lines to help you out.

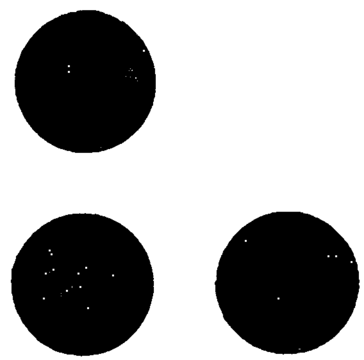


Figure 9

(Hint (?): NOT a sphere.)

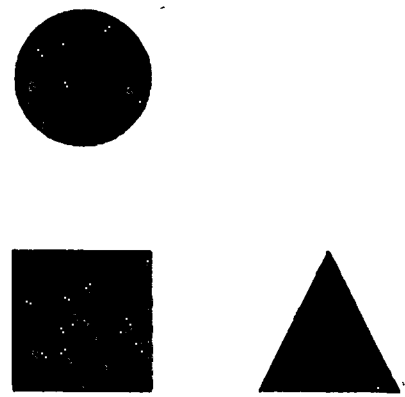


Figure 10

(Actually, a fairly common object!)

Letters to the Editor

Dear Sir:

To those magic squares of prime numbers given in your *Numbers, Numbers, Numbers* department (February) you may add the following:

1459	1669	619
409	1249	2089
1879	829	1039

3747

23	-73	29
-1	-7	-13
-43	59	-37

21

-23	31	13
43	7	-29
1	-17	37

21

11	-37	29
23	-1	-19
-31	41	-7

3

The first square is made up of primes with a common difference of 210 and another one could be made by starting with —11 and adding 210 each time.

Malcolm H. Tallman
Brooklyn 22, N. Y.

— — —

Dear Mr. Madachy:

Where can I find the solution to the geometry theorem which states "If the bisectors of the base angles of a triangle are equal then the triangle is isosceles"? Also, where can I obtain a copy of *Mathematical Essays and Recreations* by W. W. Rouse Ball? Is it still being published? Where can copies of *Fairy Chess Review* and *Games Digest* be found?

Bob Underwood
University of N. C.
Chapel Hill, N. C.

Here are a few references giving proof of Lehmus' Theorem which reads: A triangle which has two equal *interior* angle-bisectors is isosceles. (The proposition is not valid for two equal *external* bisectors).

Altshiller-Court, Nathan COLLEGE GEOMETRY Published by Barnes & Noble, Inc., N. Y., 1925 (reprinted in 1950), pages 65-66.

Henderson, Archibald SCRIPTA MATHEMATICA Vol. 21 (1955), pages 223-232.

Langer, Barbara MATHEMATICS STUDENT JOURNAL Feb. 1955, page 2.

Thebault, Victor MATHEMATICS TEACHER Feb. 1955, pages 97-98.

Concerning this theorem it's interesting to note that Jerome S. Meyer, in his book *Fun with Mathematics,* states the problem and then says "The proof is extremely long and far too involved to go into here. Take it to your mathematics professor and let him struggle with it." Miss Langer's and Mr. Altshiller-Court's proofs take up a half-page or less of space!

Last year The MacMillan Company published a revision of Ball's *Mathematical Essays and Recreations* by H. S. M. Coxeter, of the University of Toronto. The result is delightful. As for the magazines you'd

like to locate, Bob, I wish I could help. If some of the readers know where these can be found please contact either Mr. Underwood or the Editor.

— — —

Dear Mr. Madachy:

Where can I find the proof that an angle cannot be trisected?

John R. Tyndall
Chapel Hill, N. C.

This was once a frustrating question to me, too, John. If you pile all the books which say "It's been proven that, in general, an angle cannot be trisected with a ruler and compass" on top of each other I'm sure they'd reach from here to anywhere. The books which actually *show* the proof are almost non-existent. However, here's one:

Young, J. W. A. (Editor) MONOGRAPHS ON TOPICS OF MODERN MATHEMATICS, Dover Publications, Inc., N. Y., 1955 Chapter VIII, specifically pages 353-365 (which, by the way, also prove the impossibility of the construction of a cube twice the volume of a given cube and of a square whose area is equal to that of a given circle).

— — —

In answer to several readers who wondered just how far one can go with the problem of using the nine digits (see 113 ways of writing 100 on pages 39-42 of the February issue) the editor submits the following:

$$-(1234) \div (5-6-7+8)(9) = -\infty$$
$$(1234) \times (5-6-7+8)(9) = 0$$
$$(1234) \div (5-6-7+8)(9) = \infty$$

Dear Sir:

Add to Mr. Ogilvy's "Layman's Glossary of Mathematical Terms":

Normal Subgroup - Not equipped with Polaris.
Closed Set - Idle dentures.
Determinant Zero - Find the insect's idol.

C I A N
M A T I
―――――
M A T H E

Dear Sirs:

More for Mr. Ogilvy:

Locus - An insect causing great crop damage.
Law of the Excluded Middle - You can't fit into last year's clothes.
Transitive Law - Money is always changing hands.
Associative Law - You are known by the company you keep.
Commutative Law - You are going to miss your train.
Distributive Law - Your pay is quickly dispersed by your family.

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Bibliography

Some readers may wish to delve further into some of the ideas presented in some of the articles and departments in this issue of RMM. We give a brief list of suggested references below.

The Sun Dial

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