

RECREATIONAL MATHEMATICS magazine

ISSUE NO. 1

FEBRUARY 1961

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J. A. H. HUNTER - Some Inferential Problems

MEL STOVER - Wager Problems - Old and New

C. STANLEY OGILVY - Conics by Paper-folding

*Other Articles and
Puzzles - Problems - Brainteasers*

65¢

RECREATIONAL MATHEMATICS magazine

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FEBRUARY 1961

ISSUE NUMBER 1

PUBLISHED AND EDITED BY JOSEPH S. MADACHY

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From the Editor

For those many people who placed their subscription orders long before *Recreational Mathematics Magazine* was scheduled to appear the receipt of this issue must be accompanied by some such thought as "It's about time!" The editor is in complete accord with that same thought. The task has been long and hard, but deeply satisfying.

RMM is a magazine intended to fulfill the desire of many for a unique periodical entirely devoted to the strictly lighter side of mathematics. Here you will find no advanced calculus or number theory requiring a Ph.D. in mathematics. But you will find such things as number curiosities and tricks, paper-folding creations, chess and checker brainteasers, articles about mathematics and mathematicians, discussions of certain aspects of higher mathematics and their application to everyday life and to puzzle solving. You will find word games, geometric dissections, magic squares, map-coloring problems, cryptography, and many other topics generally included in the fields of puzzles and recreational mathematics.

It is hoped that some of the effects of reading RMM is the stimulation of an interest in mathematics in general, the acquiring of a deeper appreciation of the values and beauties of mathematics and the promotion of at least a certain degree of logical thinking. It is hoped, also, that both teachers and students will find material aplenty to augment formal mathematical education.

RMM could not have reached its present stage without the help of many people. At the head of the list are the many subscribers who expressed confidence and hope enough to subscribe before seeing even a review of the magazine itself. Thanks and deep appreciation are also extended to: Martin Gardner, who conducts the "Mathematical Games" section in *Scientific American* and who has spread the word of RMM throughout the world and given me aid beyond measure; to Jim Hunter of Toronto who contributed the first article in this issue and who saw to it that many of his fellow Canadians were contacted; to Dr. H. V. Gosling of Kingston, Ontario, whose material has been most welcome and whose many letters gave much-appreciated encouragement and moral support; to Bob Underwood of the University of North Carolina who suggested, among other things, the idea of a Readers' Research Department; and to many, many others who must remain nameless due to lack of space.

It seems to be the custom in dedicatory editorials and prefaces to extend unending accolades to the one person most closely allied to the editor - his wife. I cannot break custom. Without her continued ego-bolstering encouragement, her hours of monotonous typing and the tolerance generally extended to all puzzle nuts, it is doubtful that RMM could have come into existence this soon, if at all.

1 February, 1961

J.S.M.

Some Inferential Problems

by J. A. H. Hunter

Many "inferential problems" - problems in logic - may be solved very neatly by the methods of Boolean Algebra, which is the basis of the mathematics of logic. Only a bare outline of the elements is possible here, but it may be of interest.

We adopt the convention that something "true" has a value 1, and something "false" the value 0. Using code symbols for the "somethings", we can then form expressions and equations which may be treated much the same as those in normal algebra. A very simple example will show how these ideas are applied.

Say we have two conflicting statements about the name of a boy, and we know that each statement contains one mistake. One said "Jack Dibble", the other "John Dibble".

Obviously, Dibble was his name, and his first name was neither Jack nor John. But let us see how this would be handled by Boolean Algebra.

We have only the two numerical values, 0 and 1. There's nothing more true than "true"; if, in the course of the working, we arrive at any number greater than unity we must represent it as unity.

Let A stand for Jack, B for John, and C for Dibble. Then we can represent each statement in two ways:

Multiplication — If both A and C were equal to 1 (i.e. true), the product $AC = 1$. But if either A or C has value 0 (i.e. false), the product $AC = 0$.

Addition — If either A or C (or both) has value 1 (i.e. true), then $A + C = 1$.

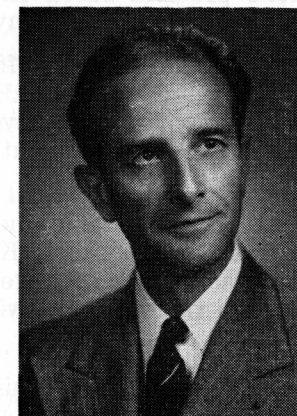
Now, on the basis of the two statements we can say

$$(A + C)(B + C) = (1)(1) = 1$$

whence

$$AB + AC + BC + C^2 = 1$$

But we know that each statement contains one mistake, so $AC = 0$ and $BC = 0$. Obviously $AB = 0$, hence we are left with $C^2 = 1$, i.e. $C = 1$, which tells us that the boy's surname is Dibble. Furthermore, as $C = 1$, and $AC = 0$ and $BC = 0$, we see that $A = 0$ and $B = 0$. This confirms that his first name was neither Jack nor John.



J. A. H. Hunter

That was very simple! So now we'll use the method to solve a little problem based on the peculiar customs of Kalota, that island paradise which so many tourists have sought in vain!

I found it very difficult to understand the Kalotan women when visiting there recently. They conform rigidly to the strange custom that a woman must never make two consecutive true or untrue statements: if one statement is true, then her next must be a lie, and vice versa.

The hospitable merchant with whom I was staying had four attractive daughters: Kassa, Kessa, Kissa, and Kossa. I did try to find out their respective ages, but the girls got me really tangled, even though there are no twins at all in Kalota.

Kassa started it: "Kissa is twenty-two, and Kessa twenty-one." Kessa's version was quite different: "Kossa is nineteen," she told me, "and Kissa is twenty-one."

I appealed to Kissa, a real beauty, but she wasn't very helpful. "Kassa is twenty-one, and Kossa eighteen," she assured me.

I did know for sure that one of the four girls was eighteen, and after some scribbling on a scrap of paper I managed to figure out their actual ages. Here is an outline of the method I used.

Let our code be:

KISSA-I	18-a
KESSA-E	19-b
KOSSA-R	21-c
KASSA-A	22-d

In code, then:

Kassa said: Id, Ec
Kessa said: Rb, Ic
Kissa said: Ac, Ra

Since one, and only one, of the statements made by each of the girls is true we have the equations:

$$\begin{aligned} Id + Ec &= 1 \text{ and } Id \cdot Ec = 0 \\ Rb + Ic &= 1 \text{ and } Rb \cdot Ic = 0 \\ Ac + Ra &= 1 \text{ and } Ac \cdot Ra = 0 \end{aligned}$$

There are no twins, so any term of the form $Ec \cdot Ic$ must have zero value. No girl can have two ages, so terms of the form $Id \cdot Ic$ must have zero value. Now combine as

$$(Id + Ec)(Rb + Ic) = 1$$

whence $Id \cdot Rb + Id \cdot Ic + Ec \cdot Rb + Ec \cdot Ic = 1$

Striking out terms with zero value, this becomes

$$Id \cdot Rb + Ec \cdot Rb = 1$$

Now bring in the third equation as

$$(Ac + Ra)(Id \cdot Rb + Ec \cdot Rb) = 1$$

whence $Ac \cdot Id \cdot Rb + Ac \cdot Ec \cdot Rb + Ra \cdot Id \cdot Rb + Ra \cdot Ec \cdot Rb = 1$

Striking out terms with zero value, we are left with

$$Ac \cdot Id \cdot Rb = 1$$

So none of these terms can be zero, hence $Ac=1$, $Id=1$, $Rb=1$. So the respective ages were:

Kassa - 21
Kissa - 22
Kossa - 19
and Kessa - 18

Now you may like to try a couple of problems which lend themselves to the Boolean treatment. The first is easier than the second.

The answers will be given in the next (April) issue of RECREATIONAL MATHEMATICS MAGAZINE.

* * *

"There's a Jack Brent got married down in Dallas," remarked Sam, looking up from his paper. "That must be Joe's son. Same name and he's twenty-one."

Gwen shook her head. "It's quite a while and you've forgotten, dear," she told her husband. "His son is Jim, and he'd be eighteen now."

Ann had never met the Brents, but she'd heard plenty about them. "His name certainly wasn't Jack," she informed her mother. "Anyway he's at least twenty-five by now."

Of course all three were wrong one way or another, but each had made a correct statement about either the age or the name.

So how old was Joe's son, and what was his name? (Hint: Jack a, Jim b, 'not Jack' c)

* * *

A length and a half back as they wheeled down on the wire, No. 3 came on then to carry the crimson silks to a popular triumph. A great race and a great finish, but there was an argument about it only that very evening.

"I tell you Prefect carried No. 3, and Queenie beat Rialto by a short lead for second place," insisted Stan. "And Satan was lengths behind."

Steve shook his head. "Prefect didn't win anyway," he said, "but you're right about Queenie finishing second."

That brought Bob to his feet. "So none of us bet on that race, but I was right there on the rails halfway down the stretch and I saw Queenie win," he declared. "Zebra came in second, with Prefect third and Satan way behind like he wasn't interested."

The other two laughed at his vehemence. "Well!" Steve chuckled. "I saw Zebra finish third with Rialto a close fourth."

So the argument went on. The funny thing was that each of them had made precisely two mistakes, although Rialto was indeed one of the four horses in that race.

Now figure out the order of finishing. (Hint: If Prefect is P, let 'not Prefect' be P1).

The author thanks Oxford University Press, Toronto and New York, for their courtesy in permitting use of the Kalota story from his book Fun with Figures published by them.

Mr. Hunter saw active service in World War I with the Royal Navy and later saw service throughout the world including two years commanding a river-gunboat in China. Retiring at the end of World War II with the rank of Commander he took a civilian job with the Occupation forces in Germany. As a result of a conversation in the Allied Officers' Club Bar one day he provided teasers, called **Fun With Figures**, for the British Occupation Forces. He came to Canada in 1952 and started his syndicated newspaper series. Mr. Hunter has had published two collections of teasers, **Fun With Figures** and **Figurets** (Oxford University Press, Toronto and New York). He is a member of the British Mathematical Association and of the Mathematical Association of America.

Minor Puzzles - A Selection

1. Two fathers gave their two sons some money. One father gave his son \$90.00 and the other father gave his son \$80.00. When the two sons counted their wealth they found that together they had become wealthier by only \$90.00! What happened to all that money?

(H. V. Gosling)

2. The problem in the following game is to determine the strategy which will enable one of the players to force the other to lose. Place twelve pennies in the following arrangement of three rows:



Two players move alternately and on each play a player may remove as many pennies as desired, provided they are from the same rows. The player taking the last coin loses the game. Which player, the first or second, can guarantee himself a win and by what strategy? (Lillian S. Graham; Minneapolis).

3. How long-winded are you? Suppose you wind your watch daily to the full extent, once at 7:30 a.m. and again when retiring for the night. When eight turns are required for the winding in the morning and twelve turns at night can you tell exactly when you go to bed? (Gerard Mosler).

4. The Mad Hatted!

There are five hats, three red and two black, which are to be placed on the heads of two other persons and you. All three of you know how many hats there are and their colors. Three chairs are placed in a single file, all facing the same. You are in the front seat and the other two persons are sitting behind you in the other two chairs. All of you are blindfolded and a hat placed on your heads. The two remaining hats are kept out of sight and when the blindfolds are removed no one is to turn around to look behind him.

The person in the third chair is asked the color of his own hat but he doesn't know. The second person is also asked the color of his own hat but he, too, doesn't know.

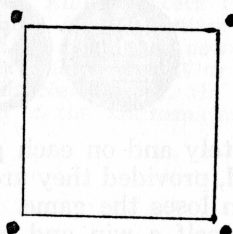
What color hat do you have on and how do you know?

5. Here's an age old problem. Now everyone knows Methuselah lived longer than anyone else in the Bible. His 969 years make the exact total of the life spans of seven other people in the Bible, each of whom lived to be over a hundred. Can you tell how long each of the following lived?

Joseph and Joshua lived the same length of time. Moses lived ten years longer than either. Sarah lived seven years more than Moses, but twenty years less than Jacob. Abraham lived forty-eight years longer than his wife, Sarah, but five years less than his son, Isaac. (Bertha Newhoff; Versailles, Kentucky).

6. Doubling Up

The small circles in the drawing below are immovable iron pillars and the lines indicate the walls of a square room. Can you double the area of the room by tearing down the old walls and erecting new one? However, the new room must also be square and the pillars must still be outside the room. (Gerard Mosler).



7. How Spirited Are You?

Of three members of a club, two drink wine, two liquor and two beer. The one that does not drink beer does not touch liquor, and the one that does not drink liquor does not drink wine. Which of the three beverages do they drink, respectively? (G. M.).

8. A Magically Productive Magic Square

Can you fill out the empty spaces with whole numbers - none greater than 100 - so that the product of all numbers in all rows, all columns and the two main diagonals is 1000? The middle square is filled in but it might be interesting to see how many answers come from the readers if we leave the nine spaces completely blank and fill them in with whole numbers, again less 101, so that the row, column and main diagonal products are equal. (Smith).

	10	

Wager Problems - Old and New

by Mel Stover

"Son, no matter how far you travel or how smart you get, always remember this: Someday, somewhere, a guy is going to come to you and show you a nice brand-new deck of cards on which the seal is never broken and this guy is going to offer to bet you that the Jack of Spades will jump out of this deck and squirt cider in your ear. But, son, do not bet him for as sure as you do you are going to get an ear full of cider."

from *The Idyll of Miss Sarah Brown*
- Damon Runyan



Mel Stover

Since the earliest times wagers and games have been associated. In fact gambling and gaming are two different forms of the same Anglo-Saxon word. Some games such as Craps and its more sophisticated cousin, Poker, are in essence a series of bets on the outcome of a shuffle. Because of this basic simplicity interest in the games is dictated largely by the size of the bets. Bridge, Checkers and Chess belong to a class of games which has enough intrinsic strategy to make the stakes unimportant. Indeed, today Checkers and Chess are usually played without stakes. Another factor which enters into this situation is the speed with which the strength of two players becomes apparent. Obviously no one would wager a large sum of money on a game unless he thought he had at least an even chance.

Chess was invented in the East and infected Europe in the tenth century. It soon became the favorite avocation of the leisured classes and held this position until the invention of playing cards a few hundred years later. At this period Chess was always played for a stake and variations in skill were compensated by allowing the weaker player an extra piece or by some other special condition. Itinerant entertainers always carried a chessboard and exploited their skill in the various castles and courts they visited.

The earliest recorded chess problems are to be found in some Arabian manuscripts dating back to the ninth century. In most of these early positions mate is achieved at the end of a series of checks. To ensure that non-checking moves were impossible the winning king was placed in a threat of immediate mate. The number of moves required for the mate was not given and the purpose of the position was most likely didactic.

The medieval European composers made two notable contributions to the problem art. The first was the important idea of limiting the number of moves, thus adding the dimension of time to those of force and position. The second was the invention of the selfmate in which the solver must commit suicide by forcing the opponent to mate him. A third not-so-notable contribution was the invention of the wager problem.

The genesis of the wager problem is unknown. The protagonists were probably potion and philter salesmen, professional gamblers and travelling entertainers. The technique went something like this: A position was set up, a wager was made and the prospective victim was allowed to choose either the attack or the defense. With the attack he was required to move and mate in a stipulated number of moves. If he chose to defend he could win the bet by warding off the mate for the given number of moves. This sounds very straightforward but a clue to the chicanery involved may be gained from the preface to one of the most famous of the medieval chess manuscripts. The following is a free translation of the important points.

Always lose the first wager. This gives the victim a sense of security which may lead him to bet incautiously.

When setting up a position feign uncertainty as to the exact location of the pieces. If the prospect should pick the winning side, sometimes you can change the position of a piece to make the mate impossible. Or you may set up a completely new problem on the pretext that you have forgotten the exact position.

Watch your customer closely. Take note of the tone of his voice, the amount of the bet, how quickly he makes it and whether his interest is limited to certain propositions. These things will indicate whether or not he is familiar with the problem.

Sometimes a ruse is used to find out which side he will choose before you settle the amount of the wager. Explain to the punter that the one who takes the "strong" side must give odds of two to one in order to compensate for the advantage of the position. In this way you can often make a large bet on a sure thing.

The chess manuscripts of the 13th and 14th centuries are studded with problems that have no other purpose but to bilk the unwary. To understand their popularity one must understand the conditions of the time. Magic was greatly in vogue. Authors wrote of goblins and

ghosts, scholars studied Astrology and Alchemy, priests exorcised devils and wrinkled old ladies were burnt at the stake. Dice, merels and chess were the prevalent amusements of the indolent upper classes. Professional gamblers enjoyed a kind of respectability and their profits then, as now, were based on the difference between the skill the opponent professed and the skill the opponent possessed.

Figure 1 is the first problem in a famous collection written about 1450 in Italy by a person who used the pen name, Citizen of Bologna. If you should choose to play the white pieces you must mate in exactly two moves. If you take the black you must prevent the mate in two. When the prospect has satisfied himself that R-B7 check followed by N-N6 checkmate is unbeatable and makes up his mind to take the white pieces, the gambler discovers that he still has the white king in his hand. This is placed at KN5 and when the play is started White is shocked to find that his carefully planned mate has completely vanished because his king has been placed in check with Black's first move. At some future date the same bet is offered to the same gullible gentleman only this time the white king is at KR6. The gentleman shrewdly notices that the white king will again be in check after NxR and places a large bet on the black pieces. He is then made to realize the danger of a little knowledge when the gamester plays RxN and N-N6 mate.

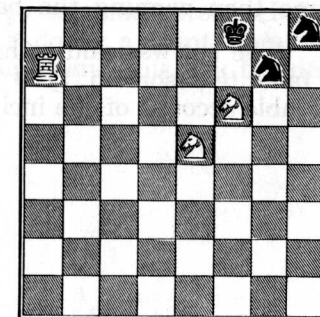


Figure 1

Figure 2 is a problem from a French manuscript dated 1400. You are asked to mate in 13 moves or less with the stipulation that the rook may only move once - to administer the mate. By setting each piece one square to the right the problem becomes impossible.

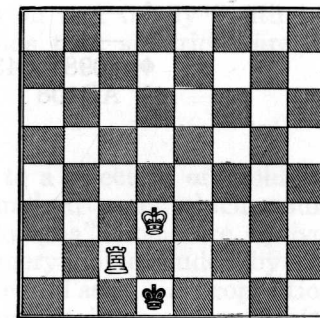


Figure 2

Today's Bridge players are not gamblers. Recently Las Vegas hosted a Bridge tournament. The players came, played Bridge before

the tournament, during the tournament and after the tournament, leaving the slot machines and Crap tables strictly alone. Today's prototype of the medieval sharper prefers Gin Rummy or Poker where guile results in greater profits. But this was not always the case.

Duke William Augustus Cumberland, known not very affectionately as Butcher Cumberland, son of George III of England, once lost \$100,000 on a single hand of whist. This feat won him more lasting fame than winning the battle of Culloden Moor.

The bet was said to have been made at the Whist Room at Bath, a favorite resort in 1750 for high stake card players. The most believable account of the incident is that the Duke was handed 13 cards:

♠ AKQ
♥ AKQJ
♦ AK
♣ KJ97

Figure 3

and asked what lead he would make in a game of Whist with clubs as a trump. He replied that he would lead a trump to cut down the ruffing power of his opponents. When he was told that a trump lead was the worst possible lead and the only one to give him no tricks at all, the wagers began to fly. Finally the cards were laid out in this arrangement:

♠ — —	♠ J109876	♠ 5432
♥ — —	♥ 109876	♥ 5432
♦ 1098765432	♦ QJ	♦ — —
♣ AQ108	♣ — —	♣ 65432

	N	
W		E
	S	

♠ AKQ
♥ AKQJ
♦ AK
♣ KJ97

Figure 4

The club was covered by ^{West}~~East~~ who led a diamond which his partner ruffed. Another club lead, another diamond lead and another trump lead by West taught the Duke that when you play another man's game you must hold ALL the aces.

Another famous deal in Bridge lore is known as the Mississippi Heart hand. It was reputedly used by sharpers on the Mississippi steamboats during the Civil War. When Bridge Whist became popular in the 1890's this hand was a natural for the swindlers of the day. In Bridge Whist the dealer had the privilege of naming the trump suit and the opponents could double if either decided that the declarer could not take seven tricks. This could be in turn redoubled by the declarer's side. Each doubling multiplied by two the value of the tricks and doubling and redoubling could go on indefinitely. Hearts counted eight points a trick and stakes of a dollar a point were not uncommon. A cold deck was secretly introduced and this was the deal that broke up the game:

♠ — —	♠ 10987	♠ 65432
♥ 8765432	♥ — —	♥ — —
♦ — —	♦ J10987	♦ 65432
♣ AKQJ109	♣ 8765	♣ 432

	N	
W		E
	S	

♠ AKQJ
♥ AKQJ109
♦ AKQ
♣ — —

Figure 5

South deals and makes hearts trump. This is doubled by West and redoubled by South. The doubling goes on and finally South becomes a sadder but wiser man when he finds that six tricks are all that the South hand can capture.

The rest of this article will be devoted to a selection of problems culled from the compositions of the last hundred years which would have delighted the anonymous "Citizen of Bologna". Some are equivocations dependent on some piece of pettifoggery not excluded by the terms. Others use the theme of the "Herculean Task" - a proposition that seems so difficult one is inclined to think it impossible. A third type, which would have been highly prized by the early subtlers, shows a seemingly simple win which is converted to a loss by a surprise defensive maneuver. When this can be repeated from the other side you have a first rate betting problem.

The answers to the following problems, except for number 14, will appear in the April issue of RECREATIONAL MATHEMATICS MAGAZINE.

No discussion of wager problems would be complete without a mention of the great Sam Loyd. His impish approach to chess problems coupled with his amazing originality would have made him highly admired by the medieval confidence men. To lead off the problems we present his well-known "Excelsior" and the anecdote concerning it from his *Chess Strategy*.

1. "I made this problem at the Morphy Chess Rooms. It was quite an impromptu to catch old Dennis Julien, the problemist. He used to wager that he could analyze any position so as to tell with which piece the principal mate was accomplished. So I offered to make a problem which he was to analyze and tell which piece did not give the mate. He at once selected the Queen's Knight Pawn as the most improbable piece, but the solution will show which of us paid for the dinner."

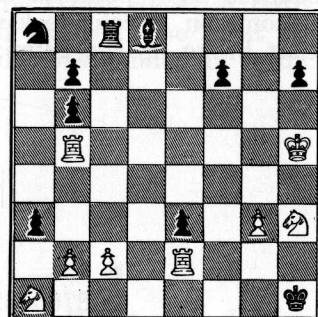


Figure 6

White is to move (see figure 6) and mate in 5 moves. Can YOU mate with the Queen's Knight Pawn?

2. Place the black King in mate in the center of the board using only a Knight and two Rooks.

3. The game is Checkers and White is moving up the board. Take either side. If you choose the white men you must win. If you choose the black side a draw will win the bet.

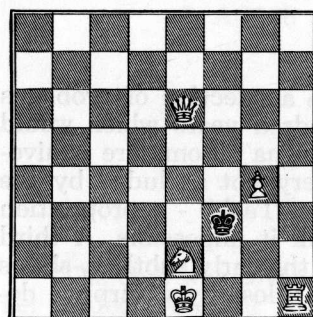


Figure 8

4. White to move and mate in one. Five will get you ten you can't.

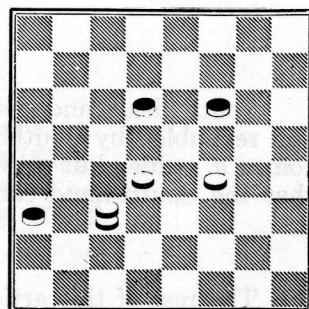


Figure 7

5. It is 2:00 A.M. and you are playing Bridge in a spacious cabin on a transatlantic liner with three prosperous business men that you met on the second day out. You have played every evening and you are ahead almost a hundred dollars. This is the last session as the boat docks at Southampton at noon. Your hand looks like this:

♠ J97532
♥ A76532
♦ — —
♣ J10

Figure 9

Your left hand opponent has bid seven spades which your partner promptly doubled. Your left hand opponent suggests a side bet to your partner of \$5000 because it is getting late. Your partner readily accepts and suggests that you also be interested. As he goes to get a check he accidentally flashes his hand and you see the Ace and King of diamonds and the queen of clubs. What is the complete deal?

6. What is the maximum number of white Queens that may be placed on a regular chessboard with the condition that no Queen may attack another?

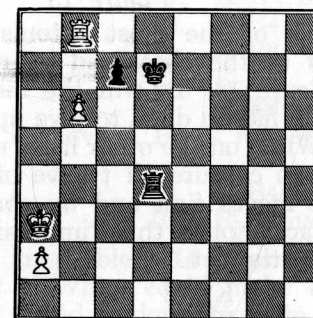


Figure 10

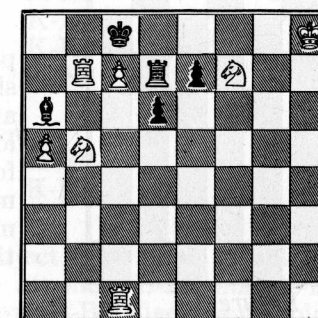


Figure 11

7. Figure 10. Taken either side. White moves first. If you take the white pieces you must win. If you take the black pieces a draw will win the bet.

8. Figure 11. White has to mate in one move. Looks simple but don't bet on it.

9. Lord Yarborough back in the 19th Century used to offer a thousand to one odds on any particular deal that you would not be dealt a hand without an honor. He made this bet so often that today a bridge hand without a ten or a face card is known by all Bridge players as a Yarborough. How good a bet did the old boy have?

10. White wagered that he would NOT win this game. He therefore now plays and loses. Figure 12.

11. A form of hazing traditionally used in some English schools is known as "running the Gauntlet". The initiate is required to run between two rows of boys each of whom is armed with a willow switch. The switches are swung at the gluteal region of the passing boy. The theory is that the faster the boy runs the less the relative speed of the switch.

The problem shown in figure 13 is called "Running the Gauntlet" and the white Pawn must mate the black king after he has passed the eight black Pawn. The black Pawns are immune to capture, but any of them may capture White's Pawn any time a chance occurs. Surely Scylla and Charybdis were no less forboding than these eight black Pawns!

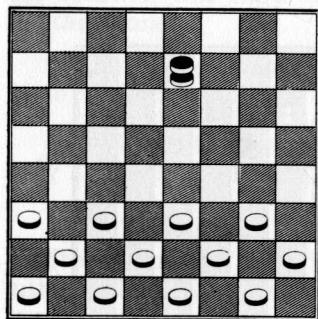


Figure 14

13. South has the bid at a contract of six clubs. Since East had bid Hearts and North bid No Trumps, West opened the play with the lead of the heart Queen. Would you choose to undertake the slam or take over the defense and set it?

♠ 9543
 ♥ Q543
 ♦ 1096
 ♣ 75

	N	
W		E
	S	

♠ A108
 ♥ A109872
 ♦ QJ
 ♣ 42

♠ J6
 ♥ —
 ♦ AK852
 ♣ AKQ1063

Figure 15

S. to make 6 clubs. W. leads QH.

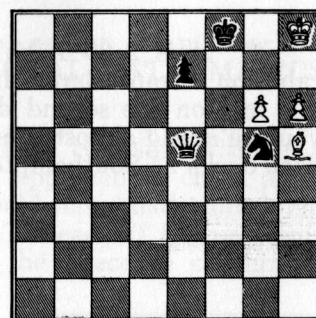


Figure 12

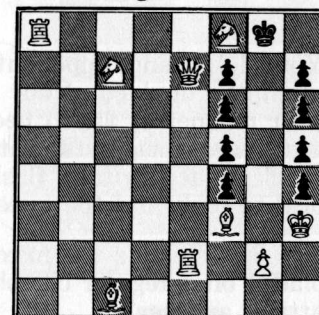


Figure 13

12. One of the most astonishing propositions of the checkered board is shown in figure 14. To win the money all that black has to do is to give up his lone King. White on the other hand must force Black to capture all twelve of his men. White moves first and by correct moves can accomplish this "impossible" task and win the bet.

♠ KQ72
 ♥ KJ6
 ♦ 743
 ♣ J98

14. The last proposition is a suicide. In figure 16 White must force Black to mate him in 9 moves or less. If you should choose to take the black pieces your task would be to avoid mating the black King. Be careful as this one is extremely tricky.

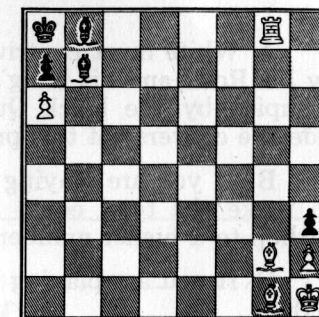


Figure 16

* *

The problem from the medieval manuscript (figure 2) is solved as follows:

- | | | |
|-----|------------|-------|
| 1. | K-B3 | K-K8 |
| 2. | K-N3 | K-Q8 |
| 3. | K-N2 | K-K8 |
| 4. | K-B1 | K-B8 |
| 5. | K-Q1 | K-N8 |
| 6. | K-K1 | K-R8 |
| 7. | K-B2 | K-R7 |
| 8. | K-B3check | K-N8* |
| 9. | K-K4 or B4 | K-R8 |
| 10. | K-B4 or K3 | K-N8 |
| 11. | K-B3 | K-R8 |
| 12. | K-N3 | K-N8 |
| 13. | R-B2mates | |

*If, in move 8, Black replied K-R6 then:

9. K-B4 K-R6
10. R-R2mates K-R5

In problem 14 (figure 16) the object is suicide. It will be seen that if White could make two moves at once, B-B6 and R-R8, the feat would be done. If white were to move B-B6 Black would play BxB and the Rook would have to interpose and there would be no mate. The plot then resolves itself into a duel between the black Bishop and the white Bishop and Rook. The solution involves extremely subtle moves by both Black and White. A complete answer would contain fourteen nine-move variations and many more shorter ones.

The most interesting point about this problem is that while the setting is chessical the mechanics are arithmetical. The numbers shown in figure 17 with the following rules will generate all the variations in short order.

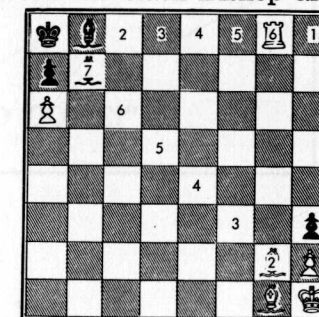


Figure 17

A. White must contrive to keep the total of the squares occupied by his Rook and his King's Bishop equal to the number of the square occupied by the black Queen Bishop. If you are playing the black side the converse of this proposition will prevent the selfmate.

B. If you are playing the white side and have a choice of moves to make the total equal to the black number, always advance the Bishop to a higher number or move the Rook to a lower number.

C. If you are playing black and the white total is over seven play BxB.

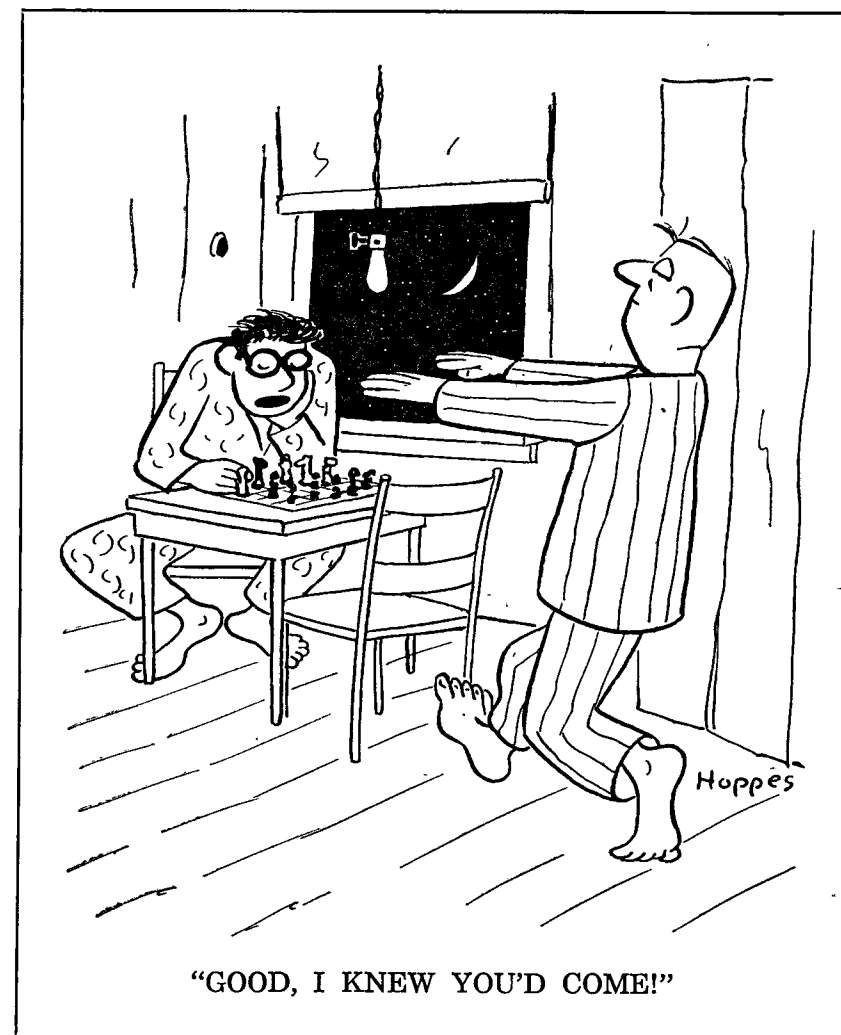
It now becomes clear that the key move for White is R-KB8. This will make a total of seven for White which is the number of the square on which the black bishop is situated. If black moves to 4 then the Rook will move to 2, etc. A shrewd punter with the white pieces would not make the keymove on his first move but would try to confuse things by moving his Rook to 4 or 3 as a starter and hope to make a systematic move on his second or third.

* * *

The composers of some of these problems:

- | | |
|------------------|------------------------|
| 1. Sam Loyd | 7. J. Moravec |
| 2. Sam Loyd | 8. J. C. Wainwright |
| 3. Hugh Byars | 10. T. R. Dawson |
| 4. K. Soltsien | 11. Marshall of Saxony |
| 5. George Coffin | 14. Von Broecker |

Mel Stover lives in Winnipeg, Manitoba. He is employed by the University of Manitoba as a Technician. His interests range over the complete field of puzzeldom. He has written for *Esquire* and *Science and Mechanics* as well as such exotic magazines as *Ibidem* and *Phoenix*. At present he writes a monthly column for the *Canadian Chess Chat*.



"GOOD, I KNEW YOU'D COME!"

Word Games

by S. Baker

BUILD A WORD

The three letters, E, S and T - some of the most often used in English - are to be used at least once in each word you will make. No other letters may be used and each letter may be repeated as often in any word as you like. Plurals acceptable, but no proper nouns. Here are two words to get you started: SET and SETS. Now, eleven more words for a perfect score. Build more and you're a genius!

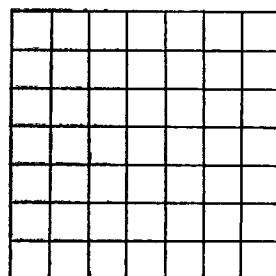
WORDS BETWEEN

To solve this little word game all you have to do is fill in the "words between". The rules are simple: if you build up from "A" you must add one letter from the top word and keep carrying letters and adding a new one each time, forming a new word each time. Conversely, if you break-down from "DEMEANOR" you must drop one letter at a time. Example:

B	R	A	N	D
B	A	N	D	
B	A	N		
A	N			
A				

D	E	M	E	A	N	O	R
A							

For a bull's eye score rescramble DEMEANOR into another common eight-letter word in the top space.



DUOROOT
BALMYAS
NORVINE
DISCURE
BACONDS
THUBIAL
SPETIUM

"7" LETTER SCRAMBLE

Fill in each row of the 7x7 diagram with a word formed by rescrambling each set of seven letters next to the diagram.

As you keep working, the answer will become *obvious*.

1.	C	L	E	A	N	E	D
2.							
3.							
4.							
5.							
6.							
7.	A	B	O	U	N	D	S

CHANGE A LETTER

Now here's a real teaser. You must go from CLEANED to ABOUNDS in six changes. Each change must consist of a new word which differs from the one above by only one letter change. If we had started with the word START we would change the "R" to "E" to make STATE, then change a "T" to "D" to make DATES, and so on, making only one letter change at a time. You have only two months to work it out. Better get started!

A Layman's Glossary of Mathematical Terms*

by C. Stanley Ogilvy

Determinant zero — Lacking in will-power.
Skew field — Needs the fences repaired.
Log tables — Rustic furniture.
Linearly independent — Has a private income.
Normal subgroup — Morons.
Transposed matrix — Mother-in-law.
Hermitian operator — Doctor who lives alone.
Delta x — The mouth of an unknown river.
Closed set — Society clique.
General Case — Civil War hero.
Independent of the path — Likes to roam in the woods.
Group operation — Club picnic.
Beta function — Fraternity dance.
Conic section — The funny papers.

*Reprinted from the January 1959 issue of the *New York State Mathematics Teachers Journal*, the official journal of the Association of Mathematics Teachers of New York State.

Sacred Numbers

by H. V. Gosling

One	Male	The origin of all Numbers.
Two	Female	Regarded as the Symbol of Death
Three	Male	Represents the Holy Trinity and Perfection
Four	Female	The Four Elements. A symbol of Earth
Five	Male	Knowledge. Symbol of Marriage. Union of Numbers Two and Three
Six	Female	Balanced Equality. Two Triangles Base to Base. A perfect number
Seven	Male	Popular Bible Number
Eight	Female	Represents Justice
Nine	Male	Squaring of the Favorable Three
Ten	Female	The Law. The Commandments
Eleven	Male	Represents Evil

Conics by Paper-folding

by C. Stanley Ogilvy
Hamilton College

From a piece of unlined typewriter paper or waxed paper cut out a circular disc four or five inches in diameter. Mark a point P on the interior of the disc. Fold the disc so that the P falls on the circumference, crease the fold, unfold, and repeat the process several times so that P falls on a different point of the circumference each time. The creases envelop an ellipse. Why? Where are the two foci?

Take another piece of paper, cut a new circular disc, and carry out the folding process, but this time fold to a point P outside the circle. (Leave a small tab or projection on the side to contain P.) What curve do you get this time? Why? Some of the creases envelop a piece of a second curve on the tab of paper outside the circle. What is this?

When P was inside the circle you obtained an ellipse; when P was outside the circle a hyperbola appeared. Therefore when P is on the circle you should get a parabola. Wrong! Try it.

To obtain the parabola a different device is needed. Instead of cutting out a circle, simply use a single straight edge of the paper and fold so that a fixed point P on the paper is brought into coincidence with various points along the edge. The creases will envelop a parabola with focus at P and directrix on the edge. Why?

Anyone who likes this sort of thing should actually make the models and try to answer the above questions before reading further.

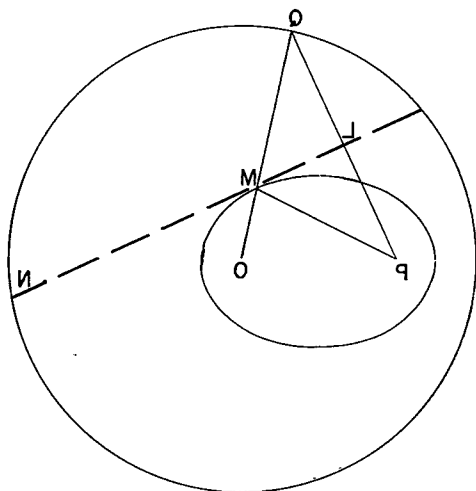


Figure 1

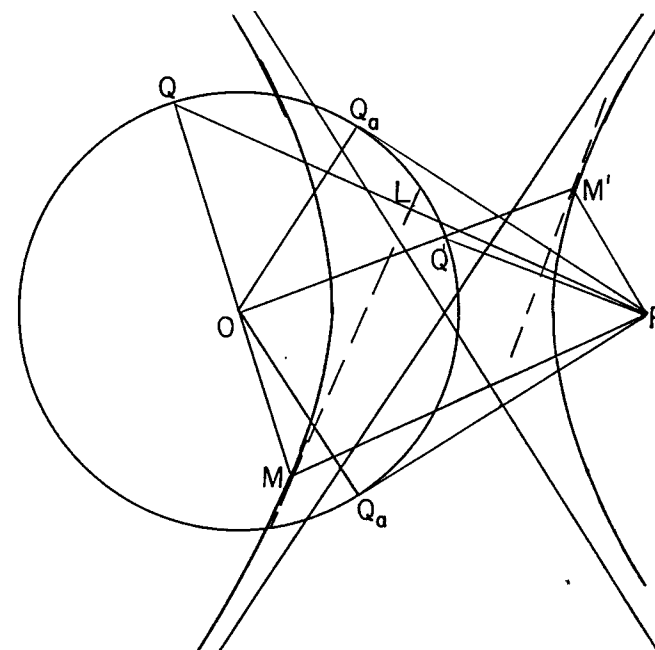


Figure 2

Referring to Fig. 1, OQ is any radius, LN is the fold which brings Q into juxtaposition with P. Then right triangles QLM and PLM are congruent, and $MQ = MP$. Therefore $OM + MP = OM + MQ = OQ = \text{constant}$ for all positions of Q. This says that M lies on an ellipse whose foci are O and P. Further, by the reflection property of the ellipse, LN is indeed tangent to the ellipse: for $\angle PML = \angle QML = \angle OMN$.

The lettering in Fig. 2 corresponds to that in Fig. 1, only this time P is outside the circle. Again M is the point where the fold intersects the radius OQ (extended). $MQ = MP$, and this time it is $MP - MO = MQ - MO = OQ$ which is constant, so that M lies on a hyperbola with foci at O and P. Also, through the property of the hyperbola (perhaps not quite so familiar as that of the ellipse), that radii vectori OM and PM, from the two foci to a point on the curve make equal angles with the curve, ML is tangent to the hyperbola at M.

As we progress around the circle there comes a critical position of Q such that the fold (the perpendicular bisector of PQ) is parallel to OQ. This is the point Q^a such that OQ^a is a right angle; in other words Q^aP is tangent to the circle. The fold is then an asymptote of the hyperbola, failing to intersect it anywhere or to be a tangent. There are of course two such Q^a , giving us both asymptotes. A point Q^1 on the minor arc Q^aQ^a yields a point M^1 on the other branch of the hyperbola. Note that here it is $M^1O - M^1P$ which is constant: the difference of the distances from M^1 to the foci in the opposite order.

The proof for the parabola is even more obvious than the other two. It will fall out incidentally from what follows.

We wish now to show, by a rather free-wheeling use of limits, that the three curves are merely three cases of the same curve; that all three have the same reflecting property; and that one can pass continuously from the ellipse, through the parabola, to the hyperbola. This is the kind of transition brought about by varying the angle of the plane when one sections a cone; and it can probably be made rigorous by anyone wishing to take this trouble.

In Fig. 3, the point P is inside the circle O, outside the circle O'. Qe is a point which, when brought into coincidence with P, yields a crease tangent to the ellipse at Me. Suppose now that circle O increases indefinitely in radius, whereas P and T are held fixed. The center O recedes to the right. Taking always points Qe at the same vertex distance above OT, one sees that Qe approaches Qp as a limiting position. Qp can be attained only when the circle has "infinite radius", that is, when it becomes the straight line. This happens when the second focus O recedes indefinitely far, making QpMp parallel to PO, the axis of the parabola, and proving (by extension from the ellipse) the reflection property of the parabola. Since the "radius to the point of contact" has become the horizontal line QpMp, we know

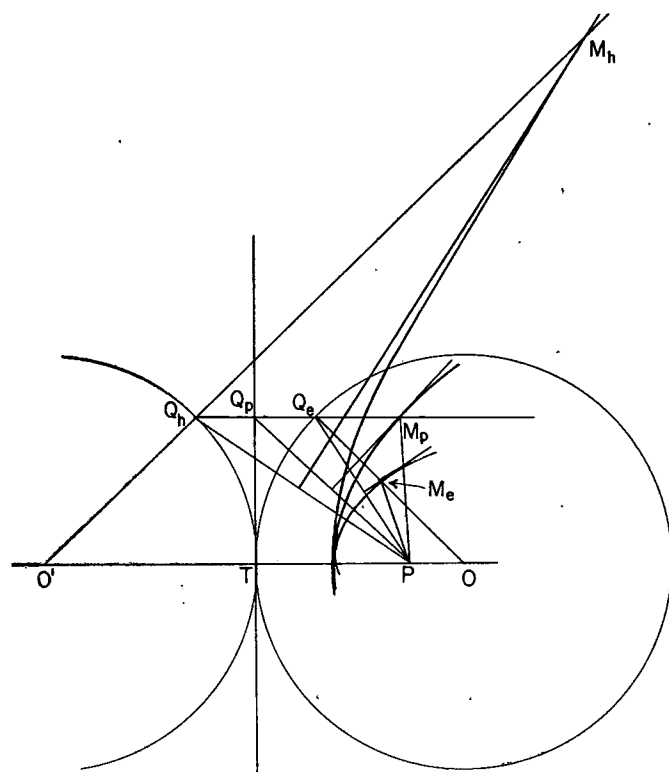


Figure 3

that $QpMp = PMp$, which says that Mp does after all lie on a parabola. If now we make the radius of the circle even bigger - turn it inside out, so to speak - we come in on the other side with center O' and obtain points Mh on the hyperbola.

Dr. Ogilvy is Associate Professor of Mathematics at Hamilton College in Clinton, New York. He received his Ph.D. from Syracuse University and joined the faculty of Hamilton College in 1953. He is the author of over a dozen articles in various journals and of the book *Through The Mathscope* published in 1956 by the Oxford University Press. His interests, besides mathematics, include sailing, skiing, and travel.

Fun For Figureheads

Here is a bit of "figurative" wizardry submitted by Gerard Mosler of Long Island that even the figure shy can attempt without fear. The numbers from 1 thru 9 have to be placed into each group so that all three equations in this particular group result in the same number as indicated on the right side of each equation. The operations (addition, subtraction, multiplication, division) are indicated between the blank spaces. Answers will be given in the April issue.

$$\begin{array}{l} \text{.....} - \text{.....} \div \text{.....} = 1 \\ \text{.....} + \text{.....} - \text{.....} = 1 \\ \text{.....} - \text{.....} \times \text{.....} = 1 \end{array}$$

$$\begin{array}{l} \text{.....} + \text{.....} \div \text{.....} = 2 \\ \text{.....} - \text{.....} + \text{.....} = 2 \\ \text{.....} \times \text{.....} - \text{.....} = 2 \end{array}$$

$$\begin{array}{l} \text{.....} - \text{.....} \div \text{.....} = 3 \\ \text{.....} + \text{.....} \div \text{.....} = 3 \\ \text{.....} - \text{.....} - \text{.....} = 3 \end{array}$$

$$\begin{array}{l} \text{.....} + \text{.....} \div \text{.....} = 4 \\ \text{.....} \times \text{.....} \div \text{.....} = 4 \\ \text{.....} - \text{.....} + \text{.....} = 4 \end{array}$$

$$\begin{array}{l} \text{.....} \times \text{.....} - \text{.....} = 5 \\ \text{.....} \div \text{.....} + \text{.....} = 5 \\ \text{.....} + \text{.....} - \text{.....} = 5 \end{array}$$

$$\begin{array}{l} \text{.....} - \text{.....} + \text{.....} = 6 \\ \text{.....} \div \text{.....} \times \text{.....} = 6 \\ \text{.....} - \text{.....} - \text{.....} = 6 \end{array}$$

$$\begin{array}{l} \text{.....} \div \text{.....} + \text{.....} = 7 \\ \text{.....} + \text{.....} - \text{.....} = 7 \\ \text{.....} \times \text{.....} + \text{.....} = 7 \end{array}$$

$$\begin{array}{l} \text{.....} \div \text{.....} + \text{.....} = 8 \\ \text{.....} \times \text{.....} + \text{.....} = 8 \\ \text{.....} - \text{.....} + \text{.....} = 8 \end{array}$$

$$\begin{array}{l} \text{.....} \times \text{.....} - \text{.....} = 9 \\ \text{.....} \div \text{.....} + \text{.....} = 9 \\ \text{.....} \times \text{.....} + \text{.....} = 9 \end{array}$$

$$\begin{array}{l} \text{.....} + \text{.....} - \text{.....} = 10 \\ \text{.....} \div \text{.....} + \text{.....} = 10 \\ \text{.....} - \text{.....} + \text{.....} = 10 \end{array}$$

Angles On The March

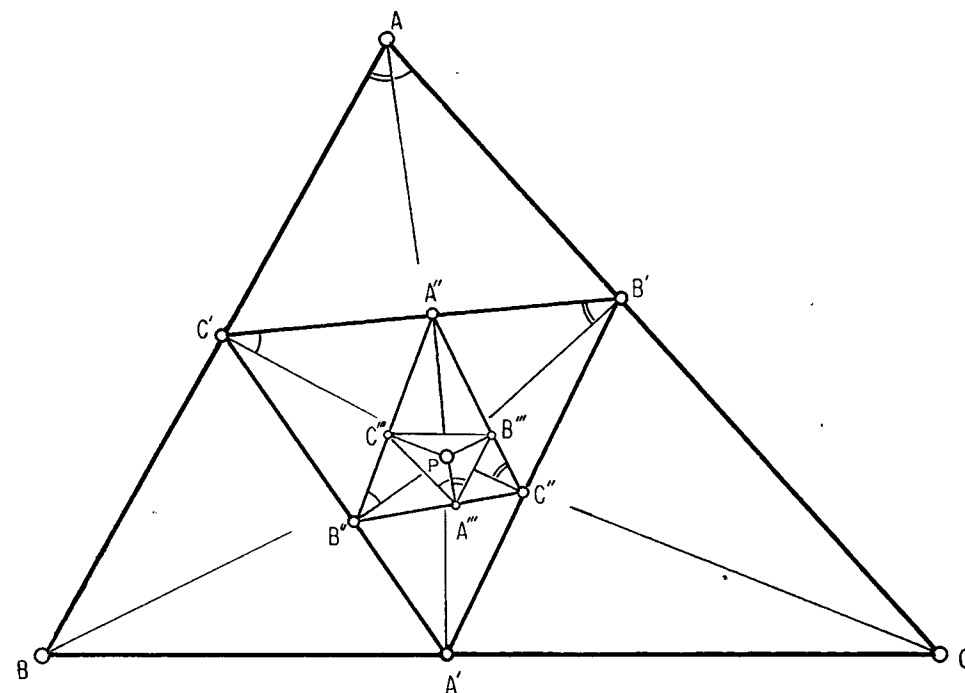
Alexander Oppenheim and Mary Pedoe

Begin with any point called P,
 (That all-too-common name for points),
 Whence, on three-sided ABC
 We drop, to make right-angled joints,
 Three several plumb-lines, whence 'tis clear
 A new triangle should appear.

A ghostly Phoenix on its nest
 Brooding a chick among the ashes,
 ABC bears within it breast
 A younger ABC (with dashes):
 A figure destined, not to burn,
 But to be dropped on in its turn.

By going through these motions thrice
 We fashion two triangles more,
 And call them ABC (dashed twice)
 And thrice bedashed, but now we score
 A chick indeed! Cry gully, gully!
 (One moment! I'll explain more fully).

The fourth triangle ABC,
 Though decadently small in size,
 Presents a form that perfectly
 Resembles, e'en to casual eyes
 Its first progenitor. They are
 In strict proportion similar.



E. H. Neville observed that this property of a triangle was added by Neuberg to the posthumous sixth edition of John Casey, *A Sequel to the First Six Books of the Elements of Euclid* (Hodges Figgis, Dublin, 1892, p. 253). It was rediscovered and generalized by Dr. Oppenheim, who is Vice-Chancellor of the University of Malaya in Singapore. It appears as Ex. 12 on page 16 of H. S. M. Coxeter, *Introduction to Geometry* (Wiley, New York, 1961). Dr. Oppenheim notes that not only is the third "pedal triangle" of a triangle similar to the original triangle, but the n th "pedal n -gon" of an n -gon is similar to the original n -gon. The charming and effective rendering as a poem was contributed by Mrs. Pedoe, whose husband, Professor Daniel Pedoe, is co-author with W. V. D. Hodge of *Methods of Algebraic Geometry* (Cambridge University Press, 1947).

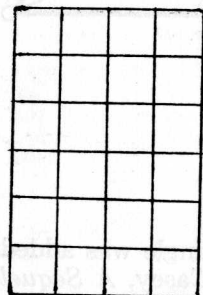
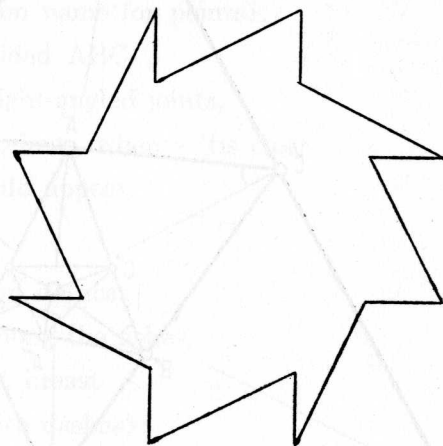
— Notes by H. S. M. Coxeter

Geometric Dissections

The ability to transform one geometric shape into another by appropriate cuts calls for a certain degree of visual imagination. Even if you are not gifted in this way you can get much enjoyment - and sharpen your native ability - by trying some of these rather simple dissections. If you find these a bit too elementary why not try Dude-ney's *The Japanese Ladies and the Carpet* which you will find on page 33.

A GREEK CROSS

Can you transform the buzz saw shown here into a Greek Cross by cutting the figure into no more than five pieces? A Greek Cross is shaped like a large plus sign. (L. S. Graham, M. Wackerbarth)

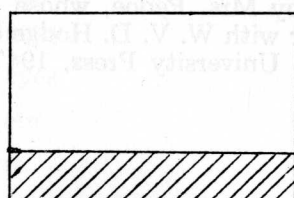
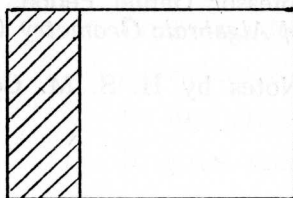


"FRAME-UP".

See if you can cut the diagram to the right into five pieces, along the indicated lines only, and make a picture frame which can hold a picture as large as this diagram. (G.M.)

LET'S CUT PAPERS!

Below are shown two sheets of paper from each of which $\frac{1}{4}$ has been removed, in length and width respectively, as indicated by the lined areas. Can you cut one of the remaining pieces into two parts which, when fitted together, will exactly cover the other piece? (G.M.)



Have Pennies - Will Revel

by Louis Schneider

The national debt of the United States, which we will set at an arbitrary figure of \$280,000,000,000, seems staggering enough to most of us. The distances of interplanetary space and beyond, which we are setting ourselves to invade, escape the grasp of the finite mind. But if you care to toy with an idea that will shrink such sums and distances into sheerest insignificance, imagine gathering to yourself quite a few pennies (which in today's world seem to accomplish little) and a common checkerboard (which is, of course, just a trifling plaything) and call into action a geometrical progression whose factor is a lowly 2. If you will give your mathematical bent free rein you will encounter results that will approach the impossible.

Position your checkerboard. Place one penny on the first of its 64 squares. Then, obeying that factor-2 mandate, spot two pennies on square number 2. Proceed with four pennies on the next square, eight on the next, sixteen on the fifth - and continue doubling the numbers as you go, 32, 64, 128, 256, and so on.

In thickness, pennies stack about 16 to the inch or 192 to the foot. Their diameter is about $\frac{3}{4}$ inch, or 16 to the foot. Their individual official weight is 48 grains avoirdupois, which means there are about 146 of them to the pound (7000 grains).

Having started to stack your pennies according to the factor-2 rule, carry on. By the time the top of the stack on the 12th square is approached the normal person will need a stepladder, for the completed stack, with its total of 2048 pennies, will hit the 10 foot, 8 inch mark. From there on, exercise your imagination and proceed by mathematical methods. Material demonstration will be quite out of the question, both as to physical reach or supply of working material.

Each stack crowned will carry as many pennies as the total of all the preceding stacks plus one more - no matter how far the series is carried. Prove this by testing, say, six squares. The progression here runs 1, 2, 4, 8, 16, 32. Breaking this down, the sum of 1, 2, 4, 8 is 15 - or one short of the next square's total of 16. The sum of 1, 2, 4, 8, 16 is 31 - one short of the next square's total of 32. By the time the top of the first row of 8 squares has been finished you will find that 128 pennies are stacked there. With the 127 on the preceding squares there will be a total of 255 pennies on the first 8 squares. Such calculations hold good, carry the series as far as you will.

The full sequence of 64 squares will work out as shown on the

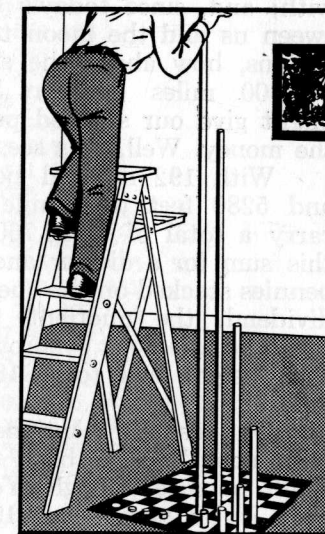


Figure 1

next page where the left-hand column represents the number on each square, the middle column represents the number of the square, and the right-hand column represents the cumulative total of pennies on the board as each square is covered.

Quite a stack on square 64, eh? But how about that cumulative total of pennies on all 64 squares? (The reading starts with quintillions.) That amount, reduced to dollars and cents by pointing off two decimal places, would, of course, stand at \$184,467,440,737,095,516.15 (keep the change). And such a sum, divided by the \$280,000,000,000 debt figure shows that the total sum on the checkerboard would pay off the debt more than 658,812 times.

From here on, let's use round numbers, please, and leave the table on page 31 to those of you who like large exact numbers. The fact is that if your one altruistic aim and intent were retirement of the national debt, by the time you placed the 10,407,813,955,585th coin on square number 45 you could quit and call it a *fait accompli*. At a steady rate of a penny a second, working day and night, no coffee breaks, that would require a mere 887,870 years or so. Proof? At 31, 536,000 seconds a year, *you* work it out! As for a possible urge to stack the final penny on square 64 - have a care! The time required for that works out to over 584,942,000,000 years!!

Considering that last figure, you might decide you would rather travel than sit and count. You may rate the Earth too small to start with; and, since today's satellites have made the 238,000 miles between us and the moon too trifling to bother with while we talk in billions, how about the sun? At first flash you may think the 92,000,000 miles between here and there might give our stacked pennies a run for the money. Well, let's see.

With 192 stacked pennies to a foot and 5280 feet to a mile, each mile will carry a total of 1,013,760 pennies. Using this sum for a divisor and the number of pennies stacked on the checkerboard as the dividend, the quotient (in miles) may make us raise our eyebrows slightly, since that height will total 18,196,362,130,789 miles!

Oh, yes. We mentioned the sun. That faint spot 'w-a-a-y back there may be it. Can't be sure, though. You see, we overshoot it by more than 198,000 times its distance from the earth.

There is this further to be said if you decided that the pennies were to be laid flat and in a straight line for the purpose of going places. To find where that would have taken you, simply multiply that 18,196,362,130,710 (based on the 'stacked' pennies) by 12 (that being the ratio of 192-to-the-foot 'stacked' to 16-to-the-foot

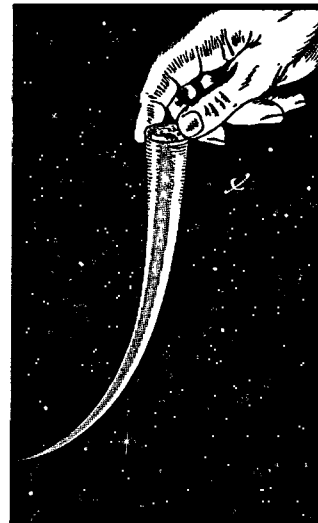


Figure 2

1	-	1	-	1
2	-	2	-	3
4	-	3	-	7
8	-	4	-	15
16	-	5	-	31
32	-	6	-	63
64	-	7	-	127
128	-	8	-	255
256	-	9	-	511
512	-	10	-	1 023
1 024	-	11	-	2 047
2 048	-	12	-	4 095
4 096	-	13	-	8 191
8 192	-	14	-	16 383
16 384	-	15	-	32 767
32 768	-	16	-	65 535
65 536	-	17	-	131 071
131 072	-	18	-	262 143
262 144	-	19	-	524 287
524 288	-	20	-	1 048 575
1 048 576	-	21	-	2 097 151
2 097 152	-	22	-	4 194 303
4 194 304	-	23	-	8 388 607
8 388 608	-	24	-	16 777 215
16 777 216	-	25	-	33 554 431
33 554 432	-	26	-	67 108 863
67 108 864	-	27	-	134 217 727
134 217 728	-	28	-	268 435 455
268 435 456	-	29	-	536 870 911
536 870 912	-	30	-	1 073 741 823
1 073 741 824	-	31	-	2 147 483 647
2 147 483 648	-	32	-	4 294 967 295
4 294 967 296	-	33	-	8 589 934 591
8 589 934 592	-	34	-	17 179 869 183
17 179 869 184	-	35	-	34 359 738 367
34 359 738 368	-	36	-	68 719 476 735
68 719 476 736	-	37	-	137 438 953 471
137 438 953 472	-	38	-	274 877 906 943
274 877 906 944	-	39	-	549 755 813 887
549 755 813 888	-	40	-	1 099 511 627 775
1 099 511 627 776	-	41	-	2 199 023 255 551
2 199 023 255 552	-	42	-	4 398 046 511 103
4 398 046 511 104	-	43	-	8 796 093 022 207
8 796 093 022 208	-	44	-	17 592 186 044 415
17 592 186 044 416	-	45	-	35 184 372 088 831
35 184 372 088 832	-	46	-	70 368 744 177 663
70 368 744 177 664	-	47	-	140 737 488 355 327
140 737 488 355 328	-	48	-	281 474 976 710 655
281 474 976 710 656	-	49	-	562 949 953 421 311
562 949 953 421 312	-	50	-	1 125 899 906 842 623
1 125 899 906 842 624	-	51	-	2 251 799 813 685 247
2 251 799 813 685 248	-	52	-	4 503 599 627 370 495
4 503 599 627 370 496	-	53	-	9 007 199 254 740 991
9 007 199 254 740 992	-	54	-	18 014 398 509 481 983
18 014 398 509 481 984	-	55	-	36 028 797 018 963 967
36 028 797 018 963 968	-	56	-	72 057 594 037 927 935
72 057 594 037 927 936	-	57	-	144 115 188 075 855 871
144 115 188 075 855 872	-	58	-	288 230 376 151 711 743
288 230 376 151 711 744	-	59	-	576 460 752 303 423 487
576 460 752 303 423 488	-	60	-	1 152 921 504 606 846 975
1 152 921 504 606 846 976	-	61	-	2 305 843 009 213 693 951
2 305 843 009 213 693 952	-	62	-	4 611 686 018 427 387 903
4 611 686 018 427 387 904	-	63	-	9 223 372 036 854 775 807
9 223 372 036 854 775 808	-	64	-	18 446 744 073 709 551 615

'flat') to get 218,356,345,569,468 miles from the earth. What's that ahead, there? Another universe?

Getting worried? Needn't be. By the time you actually find yourself that far away, mankind will probably be served by "Light-Year" Jets that will get you back here in a fairly short time, as time is measured in the space age. Light years, you know, are reckoned at six trillion (6,000,000,000,000) miles. So you should be back in a little over 36 years.

Happy landings!

If perhaps you suffer from acrophobia - fear of high places - you might better have chosen the earth's equator for a zone in which to pursue Operation Penny Revel instead of taking off for the skies. Follow the idea a bit. There are 84,480 flat-laid, straight-lined pennies to a mile - so a complete circuit of the globe would consume 2,112,000,000 coins. This latter figure being employed as a divisor and the total of pennies carried by the checkerboard once more used as a dividend, the quotient will tell you that the coins would girdle the earth at the equator over 8,734,250,000 times. Now, laying row beside row for that number of times and keeping the diameters of the coins in both lines laterally and longitudinally until they are all used up, you would form what might be termed a copper tube some 104,000 miles long.

Were you to slit the tube down the side and spread the resulting sheet out flat it would cover an area of 2,600,000,000 square miles. The tube idea would probably appear a bit more intriguing to most people for they could visualize our world loosely sheathed in a tube about thirteen times as long as the earth's diameter (roughly 8,000 miles).

That means it would neatly house thirteen such huge spinning space vehicles - with room to spare.

Concerning the weight of the coins, as mentioned in paragraph 3 a rough calculation says they would weigh 63,242,274,225,032 TONS! Doubt it? Help yourself to a pencil.

As to how wide a highway flat-laid Operation Penny Revel would form to the sun, or to the moon, or how high a wall would result if the coins were stacked on each other in a single rising tier around and around the equator - or to what depth they would sheath the earth if made to hug it snugly - or how huge a cube they would build into - why not try these problems on your own electronic computer.

As for me, I can't bear the thought of our world trapped in a copper tube 104,000 miles long. I want none of that. I feel an attack of claustrophobia coming on. I just can't tolerate being trapped in closed-in places.

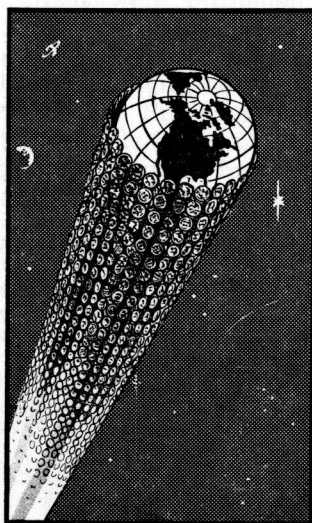


Figure 3

Some Simple Puzzles

Some puzzles and problems can be very simply stated and, after the solution is known, appear to be very elementary. However, arriving at the answer may not always be so simple. We believe the following assortment to lie in the "difficulty simple" category.

1. Dr. H. V. Gosling of Kingston, Ontario asks what is the largest number that can be formed using *only four ones*.

2. Two spheres rest in a cylinder that is 18 inches in diameter. C. E. Branscome of Laccochee, Florida wonders how much water is needed to cover the two spheres which are 8 and 12 inches in diameter.

3. Gerard Mosler of Long Island, who seems to have an unlimited supply of puzzles of all kinds, overheard the following remark by an old man, "Each of my sons has as many sons as I have sons, and the number of my ~~sons~~ and grandsons lies between 50 and 80." Although it appears that the old man gave inadequate information, Mr. Mosler states that all necessary information has been stated so that we can determine how many sons and grandsons the old man has.

4. H. E. Dudeney tells a charming story (which can be found in his *Canterbury Puzzles* published by Dover Publications) of three Japanese sisters who possessed a square ancestral carpet. They had decided to make three square rugs from it so each could have a share in her own house.

Now there are three ways of cutting the carpet to form three square rugs of two different sizes. Supposing the carpet to have been nine square feet then one of the smaller rugs would be one square foot and the other two would be four square feet each. This would require cutting the carpet into only four pieces.

However, the sisters decided on equal sharing and found that by cutting the carpet into no more than six pieces they could sew them together forming three equal sized square rugs.

Unfortunately, Dudeney never heard the rest of the story. It seems the three sisters had three brothers who had inherited another nine square foot rug, but, probably because they were puzzlists, they were able to divide *their* carpet into three equal sized squares by cutting it into only four pieces and having one of their sisters sew two of the pieces together.

The reader now has three different puzzles to wile away his time. How could the carpets be cut up and resewn in the different ways described?

The Commoner's Dilemma

The King did not like the commoner that his daughter loved. But the young man refused to stay away and in order to get rid of him the King preyed on the young man's gambling instinct. One day the King summoned the commoner and told him:

"Scoundrel, you won't stay away from my daughter. Now, I'm a fair man and though I could have you beheaded immediately for disobedience I'm going to give you a chance to marry her and live happily ever after. Are you willing to gamble?" The young man, being very much in love, accepted and the King explained, "Tomorrow, I will call the whole Court and you will draw out of a bag, held by me, one of two peas which I will place therein. One of the peas will be white and if you draw out that one you can marry my daughter. However, the other pea will be black and if you draw THAT one you will be decapitated on the spot."

The next day as the court assembled the Princess saw her father slip two black peas into the bag. She managed to signal her lover of the nefarious plot but, before the young man could register protest, the King spoke:

"Now, young man, there is no turning back. As you can see I have brought the Royal Headsman so the execution, should you have bad luck, can take place instantly. If you decline to draw, now, I'll have you beheaded for insulting my daughter by implying she isn't worth the risk".

The young man was certainly in a quandary. He could not refuse nor could he accuse the King of being a villain. Yet, if he drew, he was bound to draw a deadly black pea.

What did the young man do? Did he save himself?

* * *

A Logic Puzzle

Eustace Stewart has four friends whose occupations are butcher, baker, tailor and carpenter and whose names are Mr. Butcher, Mr. Baker, Mr. Tailor, and Mr. Carpenter. Each man has a son and a daughter but no son practices the same trade as his father and none of the eight males practices a trade corresponding to his name.

Each son marries one of the daughters whose maiden name does not suggest her husband's or his father's trade. Each girl changed her last initial when she married. The baker's son married Miss Butcher.

Mr. Butcher, Sr. is not a baker. The trade of Mr. Carpenter, Sr. is the same as young Mrs. Butcher's maiden name.

Mr. Stewart would like to know the trade of each of the eight males and the maiden names of the sons' wives. Can you help him?

Alphametics

ALPHAMETICS is the name applied to mathematical puzzles in which most, or all, of the numbers are replaced by letters. The name was invented by J. A. H. Hunter and is used by him, and others, to refer specifically to those puzzles in which the combinations of letters used make sense.

Mr. Hunter has devised a couple of alphametics especially for the first issue of RECREATIONAL MATHEMATICS MAGAZINE (RMM).

R M M
O U R
R M M
O U R
R M M
F U N

This is a rather easy alphametic. Each letter stands for a particular, but different figure.

What do you make of our FUN?

T H E) F I R S T (R M M

x x x x
x x x x
T O U R
x x x x
x x x x
x M

This is a really difficult alphametic. Each letter stands for a particular, but different, figure and the little crosses indicate figures about which we are told nothing.

What do you think of this FIRST RMM?

Gerald Mosler of Long Island sends in the following alphametics for your further enjoyment - or frustration. The same rules apply as outlined by Mr. Hunter above.

Here's a breakfast order which, when added up as shown to the right, may not be to your liking.

H A M
E G G S
M A S H

D O G
C A T
A D O

When these two get together there's certain to be much ado about something!

Now, let's see if you can add animals together as shown here. Darwinists won't go along with this, we're sure.

B E A V E R
T I G E R
R A B B I T

Numbers. Numbers. Numbers

"All results of the profoundest mathematical investigations must ultimately be expressible in the simple form of properties of the integers."
— Leopold Kronecker (1823 - 1891)

RECREATIONAL MATHEMATICS MAGAZINE certainly would not live up to its name without a section devoted entirely to numbers and their properties. There is endless fascination in mere numbers that can be appreciated only by those who indulge, even a little, in their study.

* * *

Prime numbers have intrigued man for a long, long time. Euclid proved that there are an infinite number of prime numbers and, ever since, mathematicians seem bent on finding *all of them*. A prime number, p , is an integer which has no integral divisors except p and 1 (or $-p$ and -1). The primes less than 25 are 2, 3, 5, 7, 11, 13, 17, 19, and 23. The other numbers, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, and 24, are all divisible by some smaller integer without leaving a remainder.

Now here are some prime number phenomena and oddities:

1. Maxey Brooke sends in the following identity containing the first ten primes:

$$(2)(3)(5)(7)(11) = 11^2 + 13^2 + 17^2 + 19^2 + 23^2 + 29^2$$

2. Below 100 there are five pairs of reversible primes:

11, 11; 13, 31; 17, 71; 37, 73; 79, 97

If we go above 100 there are not only many more reversible primes, but also find some interesting sets of permutable primes:

113, 131, 311; 199, 919, 991; 137, 173, 317; 157, 571, 751

1021, 1201, 2011; 1249, 1429, 4129; 1559, 5519, 5951

1013, 1031, 1301, 3011; 3779, 7793, 7937, 9377

And, amazingly, in the twelve possible permutations of the digits in 1123 and in 1139 there are eight primes:

1123, 1213, 1231, 1321, 2113, 2131, 2311, 3121

and 1193, 1319, 1913, 1931, 3119, 3191, 3911, 9311

7	61	43
73	37	1
31	13	67

111

83	29	101
89	71	53
41	113	59

213

103	79	37
7	73	139
109	67	43

219

1669	199	1249
619	1039	1459
829	1879	409

3117

3. Here are four magic squares made up of primes only composed by Henry E. Dudeney (1847 - 1930) who was one of the great puzzlists of all time. The number below each of the squares is the constant for that square - the sum of the rows, the columns and the two main diagonals.

The fourth magic square is especially interesting in that the primes used, 199, 409, 619, 829, 1039, 1249, 1459, 1669 and 1879, have a common difference of 210.

4. To close this little excursion in the realm of prime numbers we'd like to list a few unusual primes and a list of the latest LARGE primes.

Here are two that are *not* divisible by 11 - try it!

$$\frac{10^{19} - 1}{9} = 1,111,111,111,111,111,111$$

$$\frac{10^{23} - 1}{9} = 11,111,111,111,111,111,111,111$$

Some other unusual primes:

$$(2)(10^5) - 1 = 199,999$$

$$10^{12} - 909 = 999,999,999,091$$

$$10^{12} - 11 = 999,999,999,989$$

The formula $2^p - 1$ gives prime numbers for certain values of p . $2^p - 1$ is prime for p equal to

2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 521, 607, 1279, 2203, 2281, 3217

The magnitude of these last few numbers might be appreciated if you consider:

$$2^{2281} - 1 \text{ contains 685 digits and}$$

$$2^{3217} - 1 \text{ contains 969 digits!!}$$

5. Maxey Brooke submits the following consecutive numbers whose squares contain the same digits:

$$\begin{array}{lll} 13^2=169 & 157^2=24649 & 913^2=833,569 \\ 14^2=196 & 158^2=24964 & 914^2=835,396 \end{array}$$

6. Dr. H. V. Gosling, of Kingston, Ontario, notes these curious equivalents:

$$\begin{array}{l} (10989)(9)=(98901)(1) \\ (21978)(8)=(87912)(2) \\ (32967)(7)=(76923)(3) \\ (43956)(6)=(65934)(4) \\ (54945)(5)=(54945)(5) \end{array}$$

and

$$\begin{array}{ll} 1^2+0^2=1 & 1^2=1^3 \neq 0^3 \quad (-) \\ 2^2+3^2=13 & 13^2=8^3 \neq 7^3 \quad (-) \\ 9^2+10^2=181 & 181^2=105^3 \neq 104^3 \quad (-) \\ 35^2+36^2=2521 & 2521^2=1456^3 \neq 1455^3 \quad (-) \end{array}$$

* * *

7. Back in August 1935 the magazine *Sphinx*, devoted to recreational mathematics, published a table by G. Fistic of more than 100 ways of writing 100 by using the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and mathematical symbols and operations. Here are a few of those equations:

$$\begin{array}{l} (7-5)^2+96+8-4-3-1=100 \\ 3^2+91+7+8-6-5-4=100 \\ 67^2-4385-1-\sqrt{9}=100 \end{array}$$

The editor of RMM now adds a list of his own using these same digits *in order*. All of the following expressions equal 100. Some of the symbols are defined:

$$n! = n \text{ factorial} = (1)(2)(3)(4) \dots (n)$$

$$\Sigma n = \text{The sum of the integers to } n = 1+2+3+4+\dots+n$$

1. $1+2+3+4+5+6+7+(8)(9)$
2. $1+2+3-4+5+6+78+9$
3. $-1+2+3+(4)(5)(6)-7-8-9$
4. $-1+2+(3)(45-6-7-8+9)$
5. $1+2+(3)(4+5)(6)-(7)(8)-9$
6. $1+2+34-5+67-8+9$
7. $1+2+[(3)(4)(5) \div 6]+78+9$
8. $-1+2+34+[(56) \div (-7+8)]+9$
9. $[(-1-2-3-4) \div 5]+6+7+89$
10. $[(1+2) \div 3]+4+5-6+7+89$
11. $[(1+2) \div (-3)]+4+[(56) \div 7]+89$
12. $1+23-4+56+7+8+9$
13. $1+23-4+5+6+78-9$
14. $1+(2)(3)+4+5+67+8+9$
15. $1+(2)(3)+(4)(5)-6+7+(8)(9)$
16. $1-(2)(3)+(4)(5)+6+7+(8)(9)$
17. $1-(2)(3)-4+(5)(6)+7+(8)(9)$
18. $1+(23)(4)-5+6+7+8-9$
19. $1+(23)(4)+5-6+7-8+9$
20. $1+(23)(45)-(6+7)(8)(9)$
21. $(1+23+4+5+67) \div (-8+9)$
22. $-1+2+3+4[(5)(6)-7-8+9]$
23. $-1+2+3-4[(-5)(6)+7+8-9]$
24. $1+[(2)(3)(4)(5) \div 6]+7+(8)(9)$
25. $(1+2+3+4)(5)+67-8-9$
26. $-1+[(2)(3)+4] \div 5+6(7+8)+9$
27. $(1+2-3-4)(5-6-7-8-9)$
28. $[1-2+(3)(4)+5-6][-7+8+9]$
29. $[(1)(2)][(3)(4)+(5)(6)+7-8+9]$
30. $[(1)(2)+3][45-(6)(7)+8+9]$
31. $[12 \div 3][4+5+6-7+8+9]$
32. $[(-1)(2)+(3)(4)] \div [-5]+6+7+89$
33. $[(1)(2)-(3)(4)] \div [5]+6+7+89$
34. $(-1)(2)+3+4+(5)(6)-7+(8)(9)$
35. $(-1)(2)-3-4+(5)(6)+7+(8)(9)$

36. $(-1)(2) + 3 + 4 + (5)(6) + (7)(8) + 9$
37. $(1)(2) + 34 + 56 + 7 - 8 + 9$
38. $[(1)(2)(3) + 4] \div [-5] + 6 + 7 + 89$
39. $12 + 3 + 4 + 5 - 6 - 7 + 89$
40. $12 - 3 - 4 + 5 - 6 + 7 + 89$
41. $12 + 3 - 4 + 5 + 67 + 8 + 9$
42. $12 + 34 + (5)(6) + 7 + 8 + 9$
43. $12 + 3(45) + (6)(7) - 89$
44. $[(12 - 3 - 4) \div 5] + 6(7 + 8) + 9$
45. $123 - 4 - 5 - 6 - 7 + 8 - 9$
46. $123 + 4 - 5 + 67 - 89$
47. $123 + 45 - 67 + 8 - 9$
48. $123 - 45 - 67 + 89$
49. $1 - 2.3 + 45 - 6 + 7(8.9)$
50. $1 + (2)(3 + 4.5) + 67 + 8 + 9$
51. $1 + [(2)(3)(4.5)(-6 + 7)] + (8)(9)$
52. $[(1 + 2) \div 3] + 4.5(-67 + 89)$
53. $-1.2 - 3 + 4 + (5)(6) + (7.8)(9)$
54. $(1.2 - 3 + 4)(5) + (-6 + 7)(89)$
55. $12 + 3.4 + 5.6 + 7 + (8)(9)$
56. $12 + (3.4)(5) + 6 + (7)(8) + 9$
57. $-12 + (3.4)(5)(6) - 7 + 8 + 9$
58. $12(3.4 + 5.6) - 7 + 8 - 9$
59. $-123 + (4)(5)(6.7) + 89$
60. $(1)(2)(3 + 4.5) + 6 + 7 + (8)(9)$
61. $(1)(2)(3 + 4.5)(6) - 7 + 8 + 9$
62. $-(1)(2) + 3 + 4.5(-67 + 89)$
63. $-1 + 2^3 + 4 + 5 + 67 + 8 + 9$
64. $1 - 2 + 3 + 4 + 5^{(-6+7)} + 89$
65. $12 + 3^4 + 5 - 6 + 7 - 8 + 9$
66. $12 + 3^4 - 5 + 6 + 7 + 8 - 9$
67. $-12 - 3 - 4^5 + 67(8 + 9)$
68. $-1^{2^3} + 4 - 5 + 6 + 7 + 89$
69. $1^{2^3} + 4 + 5 - 6 + 7 + 89$
70. $1^{2^3 4^5} + 6(7 + 8) + 9$
71. $-1 + 2 + 3 - \sqrt{4} + 5 + 6 + 78 + 9$
72. $1 + \sqrt{2 + 34} + 5 + 6 - 7 + 89$
73. $-1 - \sqrt{2 + 34} + 5 + 6 + 7 + 89$

74. $-1 + \sqrt{2 + 34} + 5 - 6 + 7 + 89$
75. $1 + 2 + \sqrt{-3 + 4 + 56 + 7} + 89$
76. $1 + 2 - 3 + 4\sqrt{56 - 7} + (8)(9)$
77. $1 + 2 - 3 + 4 + \sqrt{56 - 7} + 89$
78. $-1 + 2 + 34 - \sqrt{56 - 7} + (8)(9)$
79. $1 - 2 + (3)(4)(\sqrt{56 - 7}) + 8 + 9$
80. $-1 + 23 \div 4 - \sqrt{56 - 7} + 89$
81. $-(1)(23) + 4 + (\sqrt{56 - 7})(8 + 9)$
82. $-1 - 2 + \sqrt{-3 - 4 + 56} + 7 + 89$
83. $-1 + (2)(\sqrt{-3 - 4 + 56}) + 78 + 9$
84. $(1 + 2)(\sqrt{-3 - 4 + 56}) + 7 + (8)(9)$
85. $(\sqrt{(12)(3)} + 4)(5 + 6) + 7 - 8 - 9$
86. $\sqrt{(12)(3)(4)} - 5 + 6 + 78 + 9$
87. $\sqrt{12 - 3 + 45 + 67} + 89$
88. $\sqrt{-(1)(23) + 45} \div 6 + 7 + 89$
89. $\sqrt{1 + (2)(3)(4)} + 5 - 6 + 7 + 89$
90. $1 + 23 + \sqrt{4} + 56 + 7 + 8 + \sqrt{9}$
91. $[(12.3)(4)(5)] \div [6] + (7)(8) + \sqrt{9}$
92. $12 \div 3 + \sqrt{4^5} - 6 - 7 + (8)(9)$
93. $(\sqrt{(1+2)(3)})^4 + 5 + 6 + 7 - 8 + 9$
94. $-1 - 2 - (3)(4) + \sqrt{5^6} + 7 - 8 - 9$
95. $-(1)(2) - 3 + 4 + \sqrt{5^6} - 7 - 8 - 9$
96. $-123 + 4^5 - 6! - 78 - \sqrt{9}$
97. $-1 - 2 - 3 + 4 + 5! - 6! + (78)(9)$
98. $12 + 3 + [(4!)(5!) \div (6!)] + 78 + \sqrt{9}$
99. $12^{-3+4} - 5 + 6 + 78 + 9$
100. $(1.2 + \sum 3 + 4!)(5) - 67 + 8 + \sqrt{9}$

Of course, an even one hundred expressions does not exhaust the possibilities using even the simplest mathematical operations. Here are an additional thirteen, some of which are interesting in that they utilize a similar operation throughout, viz., the tenth one utilizes factorials, the eleventh summations, and the twelfth and thirteenth are both entirely under the radical. A repeating decimal, e.g. $0.444\ldots$, is represented by a dot over the digit. $0.111\ldots = 1/9$, $0.777\ldots = 7/9$, etc.

$$\begin{aligned}
 & -1 - 2^{-3} - \sqrt{4} + 5 + \left[(6)(7) - 8 \right] / \sqrt{9} \\
 & (1)(\sum 2) + 3! - \sum 4 + \sum 5 + \sum 6 + \sqrt{7! - 8 + 9} \\
 & -1 + 2 + \sqrt[3]{(4.5)(6)} + 7 + 89 \\
 & 1 + \sqrt{2^{(3)(4)}} + 5 + 6 + 7 + 8 + 9 \\
 & (1)(\sqrt{2^{3+4+5}}) + 6(7+8-9) \\
 & (.1)(234) + 56 + 7 + 8 + \sqrt{9} \\
 & (1)(.2 + .3)(45) + 67 + 8(.9) \\
 & -1 - 2 + (.3)(45) + 6 - 7 + 89 \\
 & (.1)(2.34)(-5! + 6!) - (7)(8)(.9) \\
 & (1!)(2!)(3! + 4!) - 5! + 6! - \left[(7!)(8!) \right] / \left[9! \right] \\
 & -\sum 1 + \left[\sum 2 \right] \left[\sum 3 \right] + \sum 4 + \sum 5 + \sum 6 + \sum 7 - \sum 8 + \sum 9 \\
 & \sqrt{\left[(1)(2) \right] \left[-3 + 4! + 5 + 6 + 7! - (8)(9) \right]} \\
 & \sqrt{\left[(12 \div 3)(\sum 4)(5!) \right] + 6 + \left[(7!)(8) \right] / \left[9 \right]}
 \end{aligned}$$

Manual of the Dozen System

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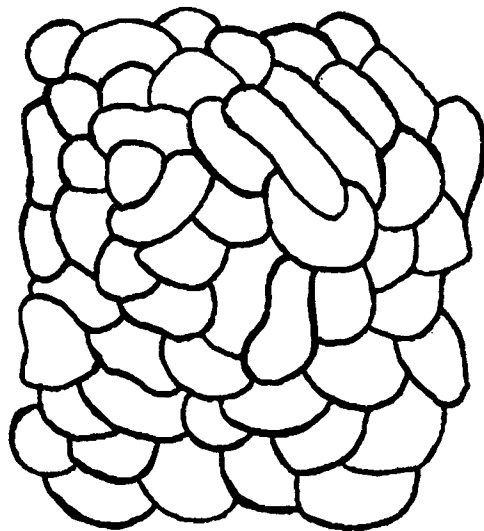
The First 750 Prime Numbers - 2 to 5693

2	233	547	877	1229	1597	1993	2371	2749	3187	3581	4001	4421	4861	5281
3	239	557	881	1231	1601	1997	2377	2753	3191	3583	4003	4423	4871	5297
5	241	563	883	1237	1607	1999	2381	2767	3203	3593	4007	4441	4877	5303
7	251	569	887	1249	1609	2003	2383	2777	3209	3607	4013	4447	4889	5309
11	257	571	907	1259	1613	2011	2389	2789	3217	3613	4019	4451	4903	5323
13	263	577	911	1277	1619	2013	2393	2791	3221	3617	4021	4457	4909	5333
17	269	587	919	1279	1621	2027	2399	2797	3229	3623	4027	4463	4919	5347
19	271	593	929	1283	1627	2029	2411	2801	3251	3631	4049	4481	4931	5351
23	277	599	937	1289	1637	2039	2417	2803	3253	3637	4051	4483	4933	5381
29	281	601	941	1291	1657	2053	2423	2819	3257	3643	4057	4493	4937	5387
31	283	607	947	1297	1663	2063	2437	2833	3259	3659	4073	4507	4943	5393
37	293	613	953	1301	1667	2069	2441	2837	3271	3671	4079	4513	4951	5399
41	307	617	967	1303	1669	2081	2447	2843	3299	3673	4091	4517	4957	5407
43	311	619	971	1307	1693	2083	2459	2851	3309	3677	4093	4519	4967	5413
47	313	631	977	1319	1697	2087	2467	2857	3307	3691	4099	4523	4969	5417
53	317	641	983	1321	1699	2089	2473	2861	3313	3697	4111	4547	4973	5419
59	331	643	991	1327	1709	2099	2477	2879	3319	3701	4127	4549	4987	5431
61	337	647	997	1361	1721	2111	2503	2887	3323	3709	4129	4561	4993	5437
67	347	653	1009	1367	1723	2113	2521	2897	3329	3719	4133	4567	4999	5441
71	349	659	1013	1373	1733	2129	2531	2903	3331	3727	4139	4583	5003	5443
73	353	661	1019	1381	1741	2131	2539	2909	3343	3733	4153	4591	5009	5449
79	359	673	1021	1399	1747	2137	2543	2917	3347	3739	4157	4597	5011	5471
83	367	677	1031	1409	1753	2141	2549	2927	3359	3761	4159	4603	5021	5477
89	373	683	1033	1423	1759	2143	2551	2939	3361	3767	4177	4621	5023	5479
97	379	691	1039	1427	1777	2153	2557	2953	3371	3769	4201	4637	5039	5483
101	383	701	1049	1429	1783	2161	2579	2957	3373	3779	4211	4639	5051	5501
103	389	709	1051	1433	1787	2179	2591	2963	3389	3793	4217	4643	5059	5503
107	397	719	1061	1439	1789	2203	2593	2969	3391	3797	4219	4649	5077	5507
109	401	727	1063	1447	1801	2207	2609	2971	3407	3803	4229	4651	5081	5519
113	409	733	1069	1451	1811	2213	2617	2999	3413	3821	4231	4657	5087	5521
127	419	739	1087	1453	1823	2221	2621	3001	3433	3823	4241	4663	5099	5527
131	421	743	1091	1459	1831	2237	2633	3011	3449	3833	4243	4673	5101	5531
137	431	751	1093	1471	1847	2239	2647	3019	3457	3847	4253	4679	5107	5557
139	433	757	1097	1481	1861	2243	2657	3023	3461	3851	4259	4691	5113	5563
149	439	761	1103	1483	1867	2251	2659	3037	3463	3853	4261	4703	5119	5569
151	443	769	1109	1487	1871	2267	2663	3041	3467	3863	4271	4721	5147	5573
157	449	773	1117	1489	1873	2269	2671	3049	3469	3877	4273	4723	5153	5581
163	457	787	1123	1493	1877	2273	2677	3061	3491	3881	4283	4729	5167	5591
167	461	797	1129	1499	1879	2281	2683	3067	3499	3889	4289	4733	5171	5623
173	463	809	1151	1511	1889	2287	2687	3079	3511	3907	4297	4751	5179	5639
179	467	811	1153	1523	1901	2293	2689	3083	3517	3911	4327	4759	5189	5541
181	479	821	1163	1531	1907	2297	2693	3089	3527	3917	4337	4783	5197	5547
191	487	823	1171	1543	1913	2309	2699	3109	3529	3919	4339	4787	5209	5651
193	491	827	1181	1549	1931	2311	2707	3119	3533	3923	4349	4789	5227	5653
197	499	829	1187	1553	1933	2333	2711	3121	3539	3929	4357	4793	5231	5657
199	503	839	1193	1559	1949	2339	2713	3137	3541	3931	4363	4799	5233	5659
211	509	853	1201	1567	1951	2341	2719	3163	3543	3943	4373	4801	5237	5669
223	521	857	1213	1571	1973	2347	2729	3167	3557	3947	4391	4813	5261	5683
227	523	859	1217	1579	1979	2351	2731	3169	3559	3967	4397	4817	5273	5689
229	541	863	1223	1583	1987	2357	2741	3181	3571	3989	4409	4831	5279	5693

The difference between the minor puzzles on pages 7 and 8 and these appearing here is not a matter of training or education but one of insight, patience and, maybe, just plain luck. Before tackling these, try warming up on the other puzzles appearing elsewhere in RMM.

1. AN UNLUCKY MAP-COLORING PROBLEM.

Somewhere in the vast Pacific is an island of rather clannish folk. So clannish are they that they have even split-up amongst themselves and have now established a total of 52 countries on this one island! Of course, they've drawn up a map and you can see it to your right. Having heard that no more than four colors are necessary to distinguish areas on a map so that no two areas have a common border with a common color these people decided that four would be all they would use to color their map. But they didn't want any color to appear on more countries than any other and stipulated to the map-maker that each of the four colors must cover only 13 countries. The poor map-maker has been having a hard time - can you help him out? If you find it easy at the start, don't give up! It gets harder as you go on! (G.M.)



2. SOME NUMBER TOUGHIES.

What two integers, neither containing any zeros, when multiplied together equal *exactly* 1,000,000,000? If that was too easy (or too hard) what two integers, also containing no zeros, multiply to give *exactly* one quintillion? (i.e. 1,000,000,000,000,000,000) (H. V. Gosling).

3. SHADOWS.

How far, on a bright sunny day, does the perfect shadow of a horizontal bar four inches in diameter extend? H. V. Gosling, who gave us the Number Toughies above, assures us that this can readily be solved but that the answer may be a bit of a surprise.

4. THE FOULED BASEBALL TEAMS — OR THE PROFESSOR DIDN'T HELP MUCH.

Not too far from where he lives Francis L. Miksa, of Aurora, Illinois, tells us there is a group of seven small towns whose names, oddly enough, are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday. Each of the towns has a baseball team having the same name as their home town.

It had been decided that these teams play each other but since they had but a week to play a committee was formed to arrange a schedule so that all the towns and teams could participate equally in the games.

After much wrangling in the committee the rules of the games were drawn by Professor Gee Whiz, an expert in combinatorial arrangements, and all the teams pledged to abide by the rules. It was left to the Professor to arrange a schedule according to the following rules:

1. Each team is to play only once with each of the other teams.
2. No team ever plays in its home town, nor on a day that bears its name. Further, no team can play more than once in a town during the week.
3. Three games are played in each town during the week and three games are played each day of the week in some three towns.
4. No games are allowed on consecutive days in any town.
5. The schedule starts on Sunday and ends on Saturday.

Unfortunately, the good Professor disappeared a few days later and the teams could find no more amongst his papers than a series of patterns which had been labeled as schedules for the teams. The patterns gave only the towns in which the games were to be held, but gave no indication which teams were to play in those towns. The committee has gone wild trying to fit the teams to at least one of the patterns in conformity with the rules. With only a few weeks left before the season is to start the committee still has not been able to set up the schedule. One of the patterns is on the next page. The names across the top are the names of the towns and the names down the side are the days of the week. The little x's are Professor Gee Whiz' indication that two teams are to play in a given town on a given day - but the Professor did not indicate *which* two teams. Can you help the committee?

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Sun	x		x				x
Mon		x		x		x	
Tue	x		x		x		
Wed		x		x			x
Thu			x		x	x	
Fri	x	x		x			
Sat					x	x	x

Some Absolutely Amazing Afghan Bands

or What Hath Moebius Wrought?

by Maxey Brooke and Joseph S. Madachy

When old Augustus Moebius invented the Moebius band in 1863, he knew what he was doing. It took the mathematical world nearly fifty years to recognize that he had invented *Analysis Situs* or *Topology*. For once, the non-mathematical world was faster on the uptake. Magicians seized upon the idea. For years, they have entertained audiences by forming incredible loops and links with cloth and paper. Somewhere along the line they changed the name to *Afghan Bands*.

Few abstract mathematical ideas have had practical uses. But Moebius' one-sided surfaces are exceptions. For example, tape recorders can get twice the mileage out of a given strip of tape by forming a Moebius strip. Both 'sides' of the tape can be used without flipping the reel. Another 'practical' use is the fun and education you can get out of strips of paper, scotch tape, and a pair of scissors.

Take a strip of paper, say one inch wide and twelve inches long. Glue the ends together to form a simple band (see figure 1). If you

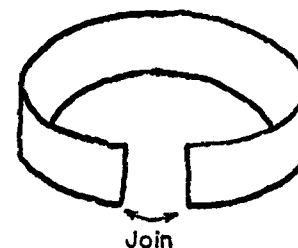


Figure 1

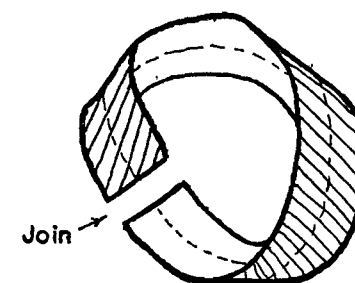


Figure 2

cut along the middle you get two rings. Hardly amazing. But, take another strip of paper and, before glueing the ends together, give one of the ends a half twist (see figure 2). You now have a Moebius band - a band that has the remarkable property of having only *one side*! If you don't believe this, trace with a pencil along the middle of the band until you come back to the starting point. You will find that both 'sides' of the band have a pencil line. And if that isn't enough, the Moebius band has only *one edge*!

Now take your band and cut along the center. Instead of two bands you will have only one. Now take another Moebius band and start cutting one-third the distance from an edge and cut all the way around (you'll do it twice) until you come back to your starting point, and end up with . . . We won't give away any secrets here. Try it yourself and see. You can get a similar effect by giving the band a full

twist before glueing the ends and cutting it down the middle.

Variations on these simple tricks can be devised by twisting the paper strip in different ways and cutting the secondary rings formed after the first cutting.

If we slit the ends of a strip of paper and then join the loose ends in certain ways we can produce a variety of seemingly incredible effects. For example, slit the ends as shown in figure 3 and then join ends A and D directly. Pass B under A and join to E. Now pass C over B and under A; pass F over D and under E. Join ends C and F. If you complete both cuts around the band you'll have an interesting arrangement of three rings.

Or try an effect developed by Ellis Stanyon in 1930. Start with the strip in figure 3 again. Turn over E to the right and join to C; turn over F to the right and join to B; finally pass A under B and join to D without turning over. Cut along the two slits and see what you get.

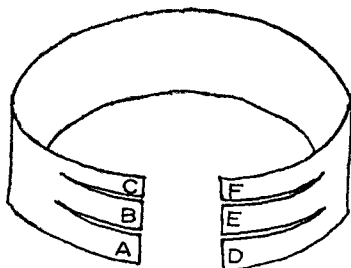


Figure 3

By now, you should have a table top full of rings and bands. And by now, you should have noticed that it takes one cut to make two rings and two cuts to make three rings. If you want four rings you need three cuts, and so on.

Now to the real purpose of this article. Your authors have developed a technique whereby you can produce any number of rings in a chain with *only one cut*. The secret lies in first folding the paper strip lengthwise before slitting and joining the loose ends. Begin by folding a strip of paper once, twisting the folded strip through 360° and pasting the ends together (see figure 4). Three interlocking rings

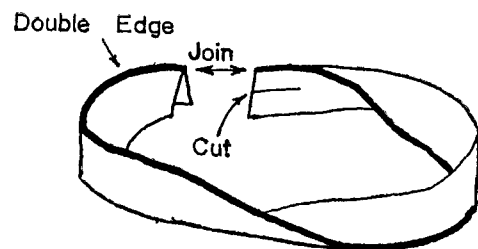


Figure 4



Figure 5

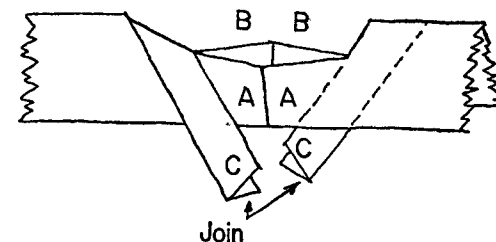


Figure 6

are produced when the band is cut all the way around through the double thickness.

Or, try *not* twisting the folded strip, but slit the ends as shown in figure 5. By joining the ends as in figure 6, a chain of three rings is produced by cutting the slit around the band, again through the double thickness.

For the rest of the work you'll need wider strips of paper so that the multiple folding will still leave you some working space. For the 5-ring chain, to be described next, a paper strip at least four inches wide will be needed.

A 5-ring chain can be formed by accordion-folding a paper strip as shown in figure 7, slitting the ends, forming a circle and joining the ends as follows: Join ends C to C, D to D and E to E - all directly, without passing over or under any rings or ends. Now take one end of B and pass it under both of the rings C and D and join to the other end of B. Take one end of A, pass it under rings C and E and join to the other end of A. By finishing the cut (through all the folds) along the middle of the band you will have a 5-ring chain.

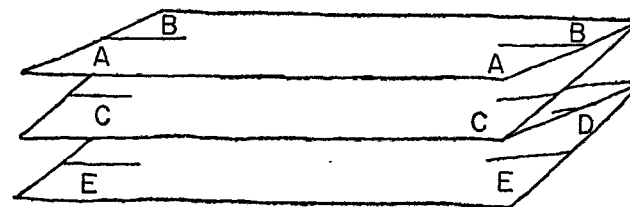


Figure 7

If you study the general technique indicated above you have the fundamental processes involved in the formation of paper chains of any number of rings - all formed with a single cut after proper preparation.

Before we finish this little excursion into Ph.D.-level paper-doll

cutting, let's make a 9-ring chain. We'll need a paper strip about eight to ten inches wide and about two feet long. A piece of newspaper or, for strength, the cover from a large magazine will do excellently.

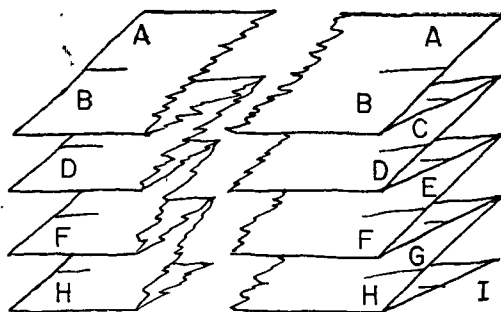


Figure 8

Prepare an accordion-fold of seven folds as shown above in figure 8, slitting the ends and marking as indicated. Join ends F to F, G to G, H to H and I to I directly, without passing over or under any rings or ends. Pass one end of E under rings G and I and join to the other end of E. Pass one end of D under the rings F and H and join to the other end of D. Now pass one end of C under ring G and join to the other end of C. Pass one end of A over ring G and under ring C and then join to the other end of A.

Now at this point of the process you'll see that the ends of B are still left over. One side of the band, if cut along the middle, would fall out separately to form a 5-ring chain and the other side would fall out to form a 3-ring chain. Strip B would fall out loosely. However, we will use strip B to join the 5-ring and 3-ring chains together. Ring A is an end-ring in the 5-ring chain and ring F is an end-ring in the 3-ring chain. Pass one end of B under ring A, over ring D, under ring F and then join to the other end of B. The two end-rings, A and F, have now been joined by ring B and the result, though looking like a real mess, will form a neat 9-ring chain after the cut is completed around the band and through the seven folds. The rings should be in the order H, D, F, B, A, C, G, E, I.

In case you've gotten confused and failed to follow directions exactly, you'll end up with something entirely different. In fact, the fun is in the surprises you get when you do it wrong deliberately.

That's all the further we're going to lead you. You are now on your own. We will suggest some problems you might try. They can all be done with one cut on a properly folded, slit, and joined band.

1. A 4-ring chain.
2. Two separate chains of 2 and 3 rings each.
3. Three rings in a chain with a fourth ring looped through the middle ring.

4. Three separate chains of 3 rings each.

5. Two separate chains of 4 and 5 rings each.

May fun, and frustration, be yours!!

Readers' Research Department

The concept of a Readers' Research Department, where various unsolved problems or the analyses of certain problems could be posed for study by the readers of RMM, was suggested by Bob Underwood of the University of North Carolina.

We are not going to suggest work on a formula to generate primes, a proof or disproof of the map-coloring theorem, proof of Fermat's Last Theorem, or any such problems that have kept mathematicians busy for many, many years. Rather, we will present problems of a nature open to study by almost anyone with an analytical bent of mind.

Readers who know of unsolved puzzles or problems - or those having no analyses of known answers - may submit such work. Perhaps, among the readers of RMM is the one person who can see through the maze of information and data pertaining to a given problem and supply the solution or the analysis.

For this first issue of RMM and its first Readers' Research problem we would like to have an analysis of the solution to a problem. For the benefit of those who like challenges we will not give the formula requested in the problem - the April issue of RMM will have the correct formula and, we hope, the analysis giving the derivation of the formula. Now, here is the problem:

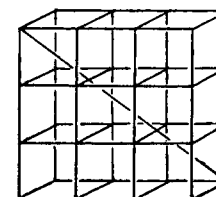
A parallelepiped is built up from regular cubes. If an imaginary straight line, the main diagonal, is constructed from one corner to the opposite corner through how many of the component cubes will this line pass? The answer should be expressible in terms of the three dimensions of the parallelepiped.

This problem came to the editor from J. A. H. Hunter who received it from the Head of a famous Scottish school.

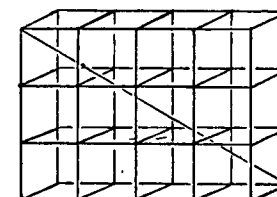
To give you an idea of the complexity of the problem let's start first with a solid in which one of the dimensions is 1. Several figures are shown below and the numbers under each figure give the dimensions of the solid and the number of cubes cut by the main diagonal.



2 x 1 x 1 2

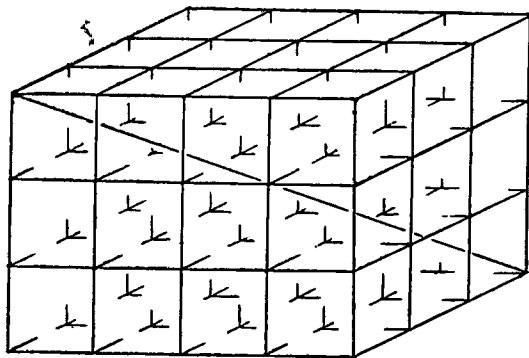


3 x 3 x 1 3



4 x 3 x 1 6

Here is another figure which, though containing three times as many cubes as the last of the previous figures, still shows that only 6 cubes are cut by the main diagonal.



4 x 3 x 3 6

All the numbers given for the number of penetrated cubes can be worked out by means of accurate drawings and extraordinary spacial visualization.

Well, there's the problem. The answer is known but the editor does not know of a quick neat derivation yielding the formula for any dimensions. Who will come up with the formula and proof?

For the benefit of those with poor drawing pens or fatigued imaginations, here is a table of figure dimensions and the number of cubes penetrated by the main diagonal.

1x1x1 1	2x2x2 2	3x3x3 3
2x1x1 2	3x2x2 4	4x3x3 6
2x2x1 2	3x3x2 4	4x4x3 6
3x2x1 4	4x3x2 6	5x4x3 10
5x4x1 8	4x4x2 4	5x5x3 5
9x5x1 13	5x4x2 8	6x5x3 10
4x4x4 4	5x5x5 5	
6x5x4 12	7x6x5 16	
8x7x5 18	13x11x7 29	

Book Review

by Joseph S. Madachy

FIGURES ARE FUN - 5 booklets and a Teacher's Manual by J. A. H. Hunter; published by the Copp Clark Publishing Co., Ltd. Toronto; \$1.68 for all 6 booklets.

Here is a delightful set of booklets for teachers of arithmetic in grades 4 through 8. Mr. Hunter has taken his theme from his own recreational mathematics books, *Fun with Figures* and *Figurets*, and written an excellent supplementary text to the arithmetic books used in the classroom.

No misunderstanding, please. These are little booklets meant to be used once a week as enrichment material for the younger set. My only regret is that I was not exposed to such treatment in my own grade-school days. These little problems and teasers are bound to catch on in the classroom and I doubt very much that one problem a week, the pace suggested in the Teacher's Manual, will suffice for some classes. Every teacher who decides to use these manuals should have a ready supply of other puzzles to take care of those days when the problem in one of Mr. Hunter's books is dispatched with ease.

The level of difficulty gradually increases through the series starting with items of simple addition and subtraction. Problem 3 in Book 1 is certainly a delightful way of teaching addition and subtraction. Probably most classes will have more fun reading the puzzle than in working it:

Fanny Flinders found forty-five flamingo feathers, but Fannie's friend Freddy filched fourteen flamingo feathers from Fanny.

Fortunately, Fanny Flinders' father found fifty-four flamingo feathers.

So how many flamingo feathers did Fanny Flinders and Fanny Flinders' father have then?

Not all of the material is as whimsical as this, most of it is in plausible story form. Simple little poems, too, add considerably to the telling of problems, as do the generous sprinkling of sketches. Since Mr. Hunter is a specialist in Alphametics it is expected that he would have considerable space devoted to such puzzles. But the reviewer personally believes Alphametics an extremely valuable form of mathematical exercise. Here is one that appears in Book 5:

W E) S E W (I T
S O
— E W
W E
— S

Each of the Books has from 36 to 45 problems and should last the school year at the rate of one problem a week. The Teacher's Manual gives suggestions on the use of the booklets and works out, in detail some of the problems that may present some difficulty to the students.

It is hard to find fault with such a pleasant way of practicing the basic arithmetic operations with integers and fractions. Warmly recommended.

Bibliography

Some readers may wish to delve further into some of the ideas presented in some of the articles and departments in this issue of RMM. We give a brief list of suggested reference below.

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